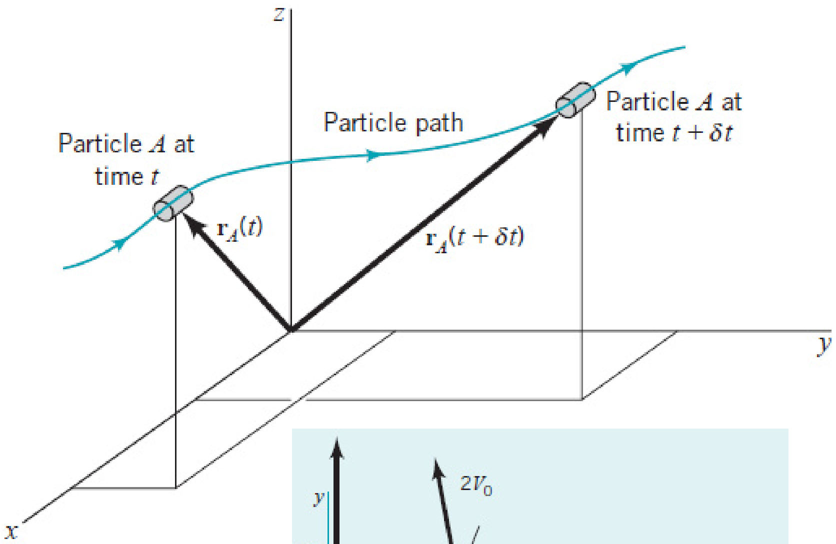
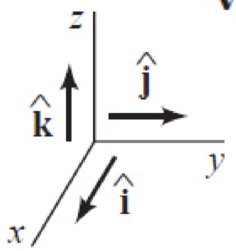
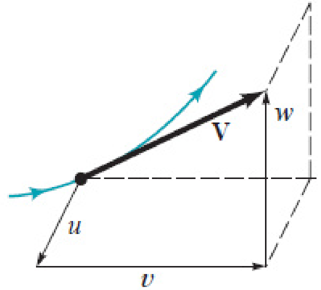
حرکت سیال



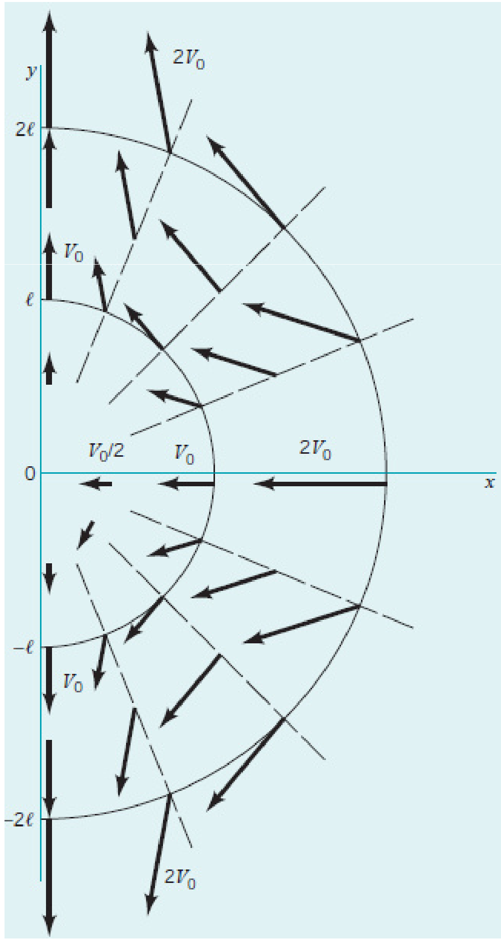
the ***velocity field,***

$$\mathbf{V} = u(x, y, z, t)\hat{\mathbf{i}} + v(x, y, z, t)\hat{\mathbf{j}} + w(x, y, z, t)\hat{\mathbf{k}}$$





مثال: برای سرعت سیال با رابطه $\mathbf{V} = (V_0/\ell)\,(-x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$ در کدام نقطه سرعت برابر V_0 است. بردارهای سرعت را نمایش دهید.

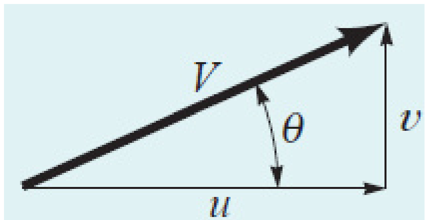


$$u = -V_0 x/\ell,\ v = V_0 y/\ell, \text{ and } w = 0$$

$$V = (u^2 + v^2 + w^2)^{1/2} = \frac{V_0}{\ell}(x^2 + y^2)^{1/2}$$

$V = V_0$ دایره ای است به شعاع ℓ

$$\left[(x^2 + y^2)^{1/2} = \ell\right]$$



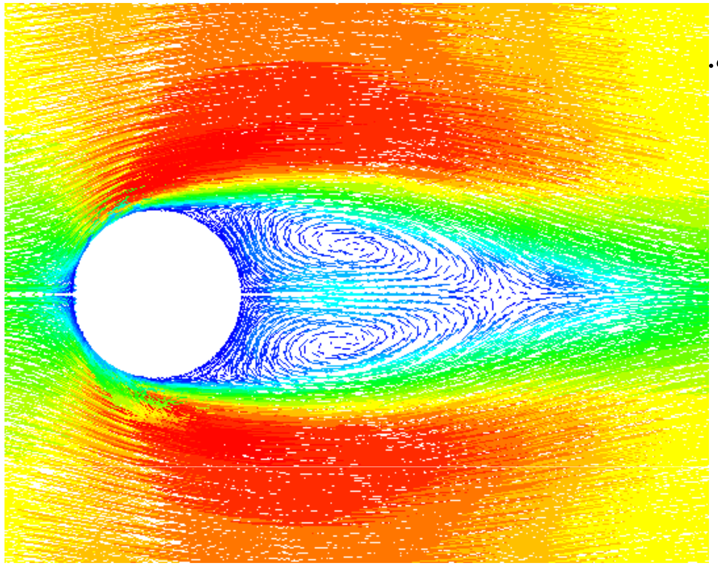
راستای بردار سرعت با محور ی x با استفاده از رابطه زیر قابل محاسبه است. $\theta = \arctan(v/u)$

$$\tan\theta = \frac{v}{u} = \frac{V_0 y/\ell}{-V_0 x/\ell} = \frac{y}{-x}$$

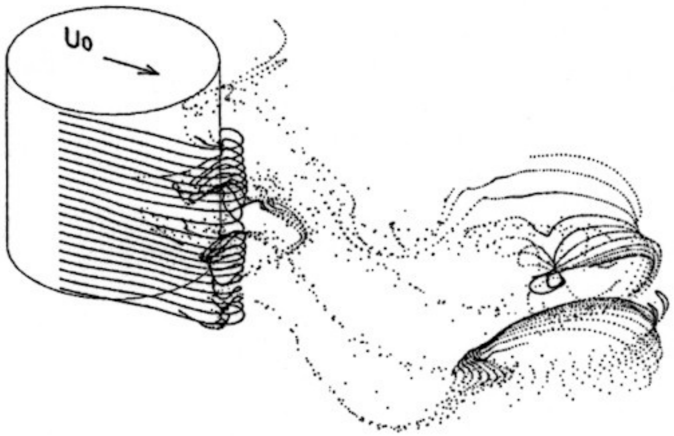
Eulerian and Lagrangian Flow Descriptions

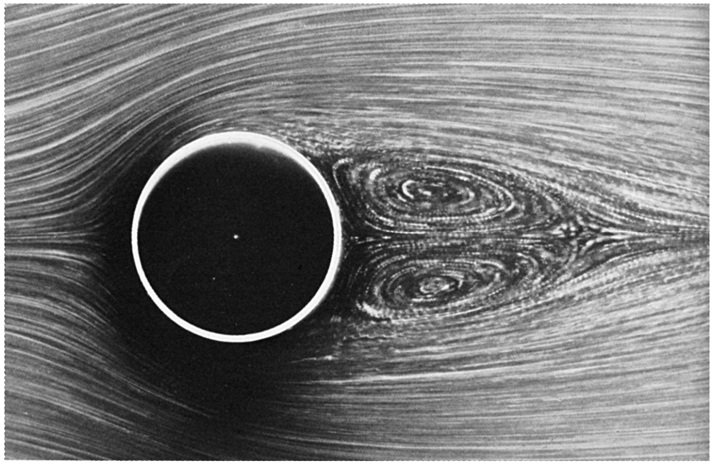
Eulerian method,

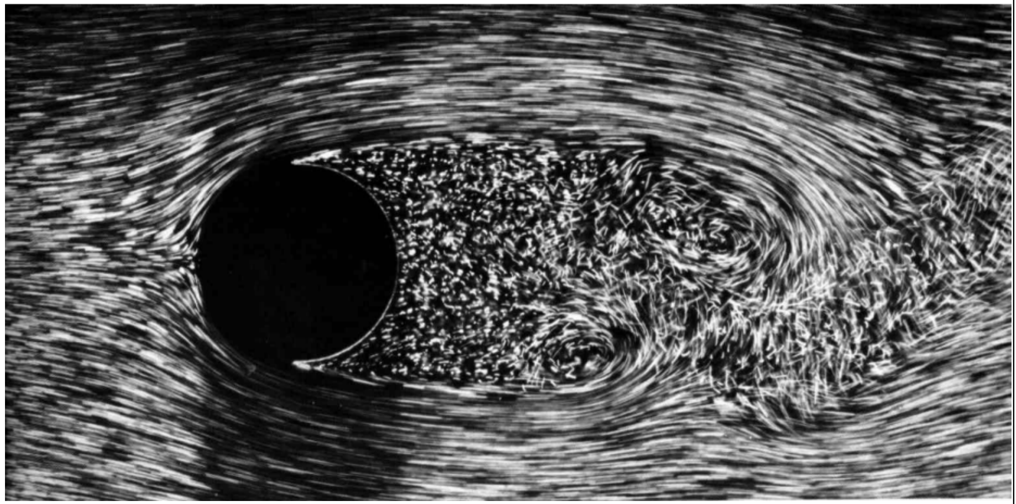
در نگاه اولری (اویلری) ، حرکت سیال بر اساس خواص ضروری مثل فشار، سرعت، دانسیته و غیره بصورت تابعی از مکان و زمان بیان می شود. در این نگاه اتفاقات رخ داده شده برای سیال در یک نقطه ثابت از فضا که سیال از آن می گذرد، مورد بررسی قرار می گیرد.



نگاه لاگرانژی بر اساس حرکت ذرات قرار گرفته و رفتار ذرات بر اساس زمان تغییر می کند.







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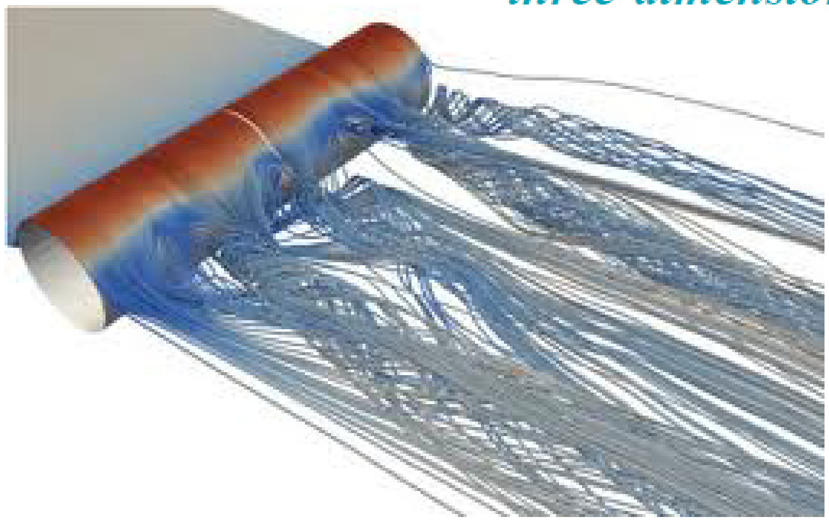
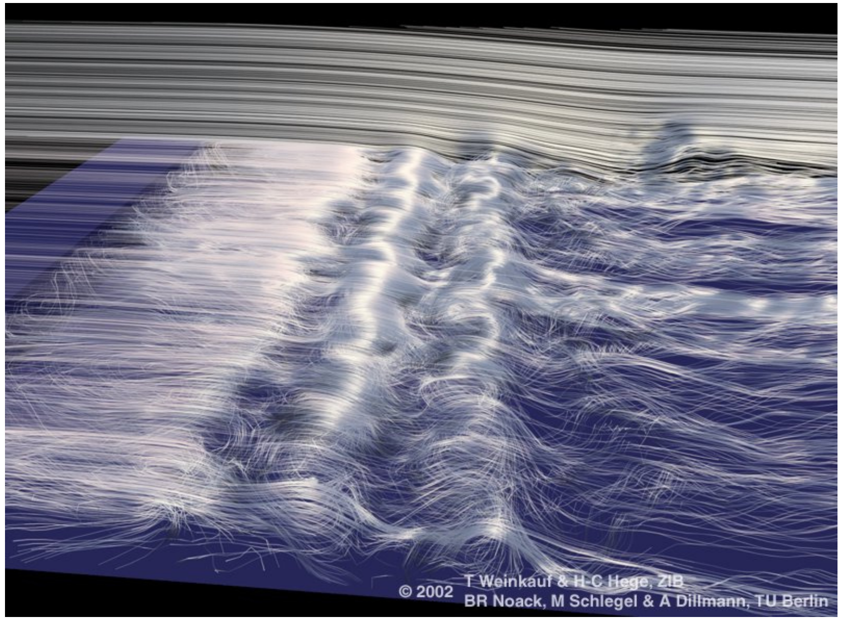
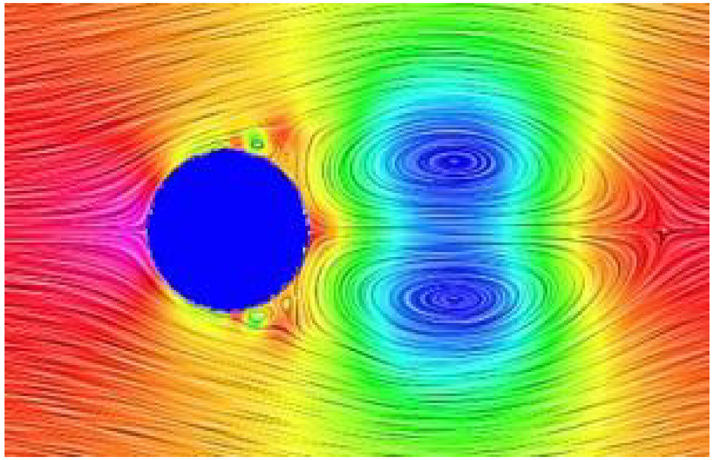
جریان می تواند بصورت چند بعدی بررسی گردد.

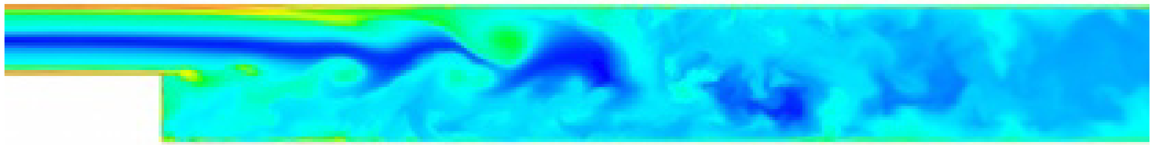
three-dimensional flow



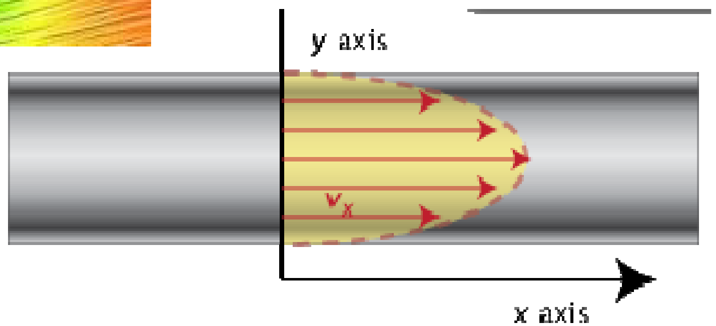


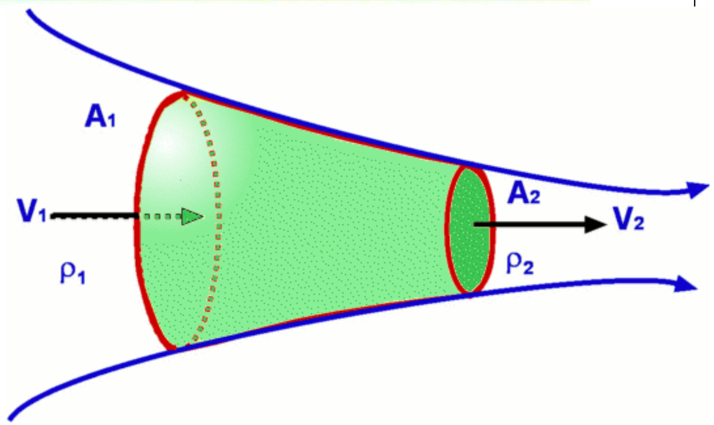
two-dimensional flow.



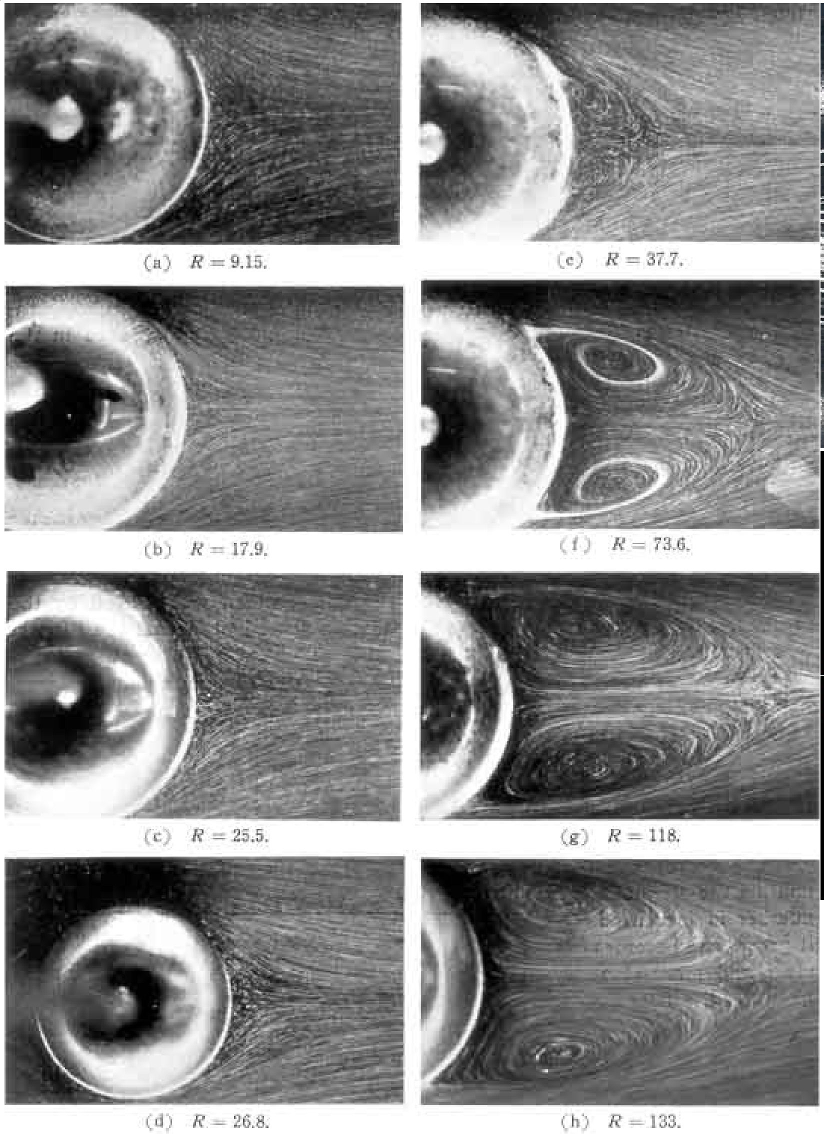


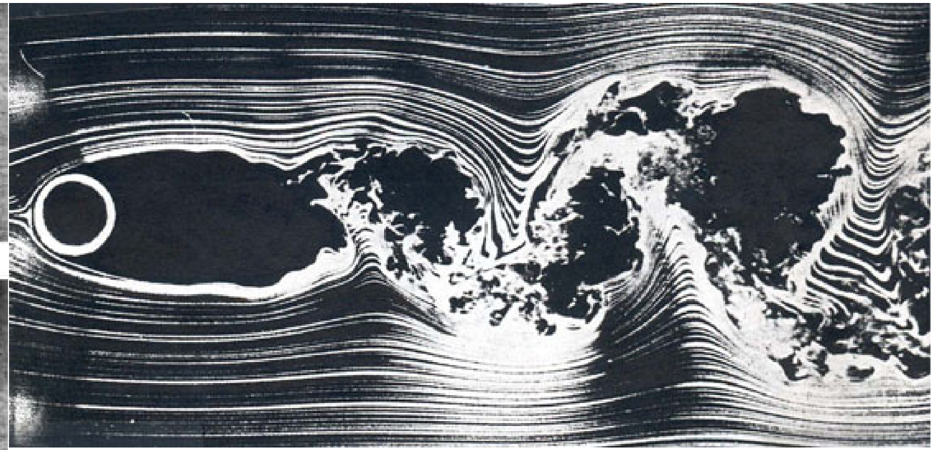
One dimensional flow

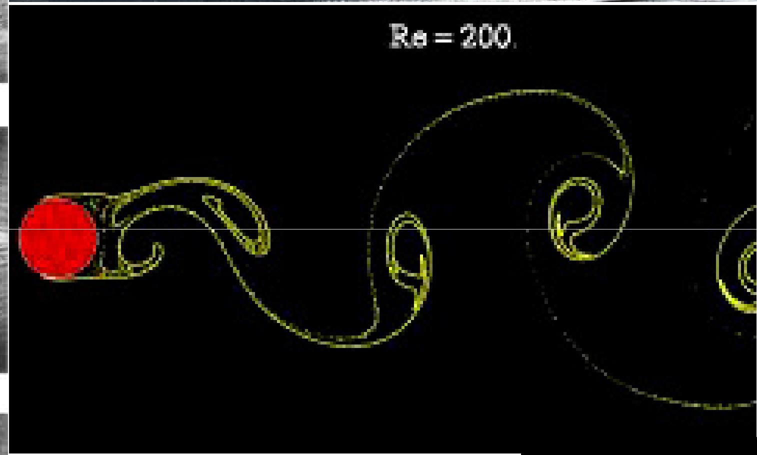


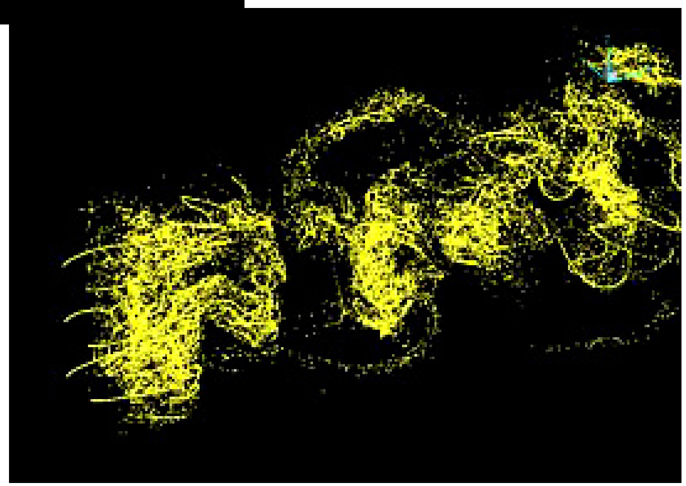


دائم و غیر دائم









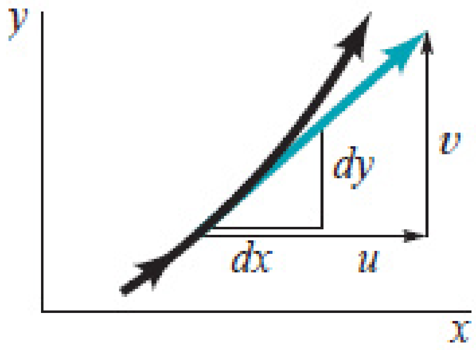
Streamlines, Streaklines, and Pathlines

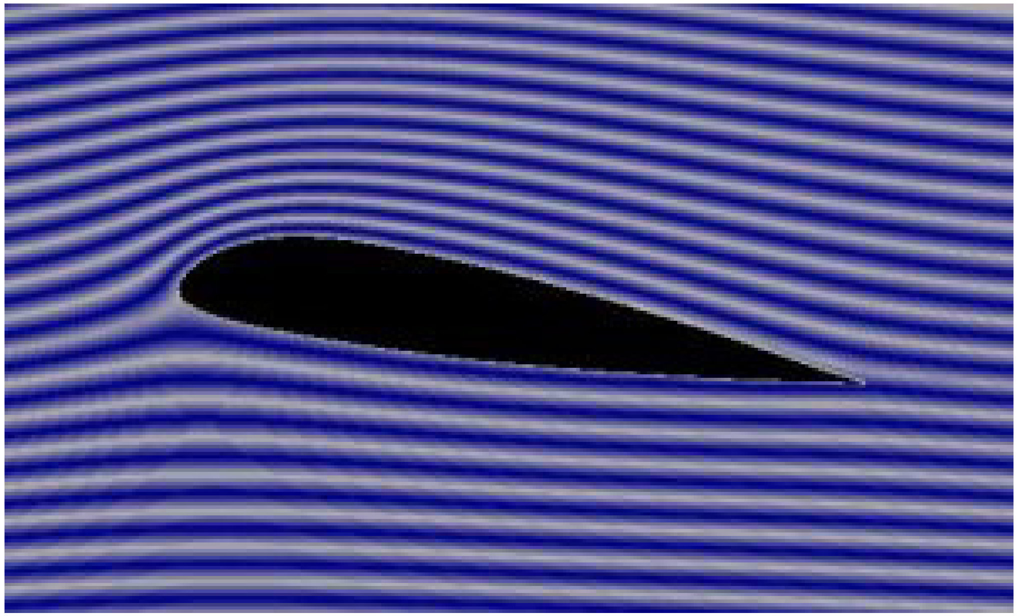
خط جریان، خط اثر و خط مسیر

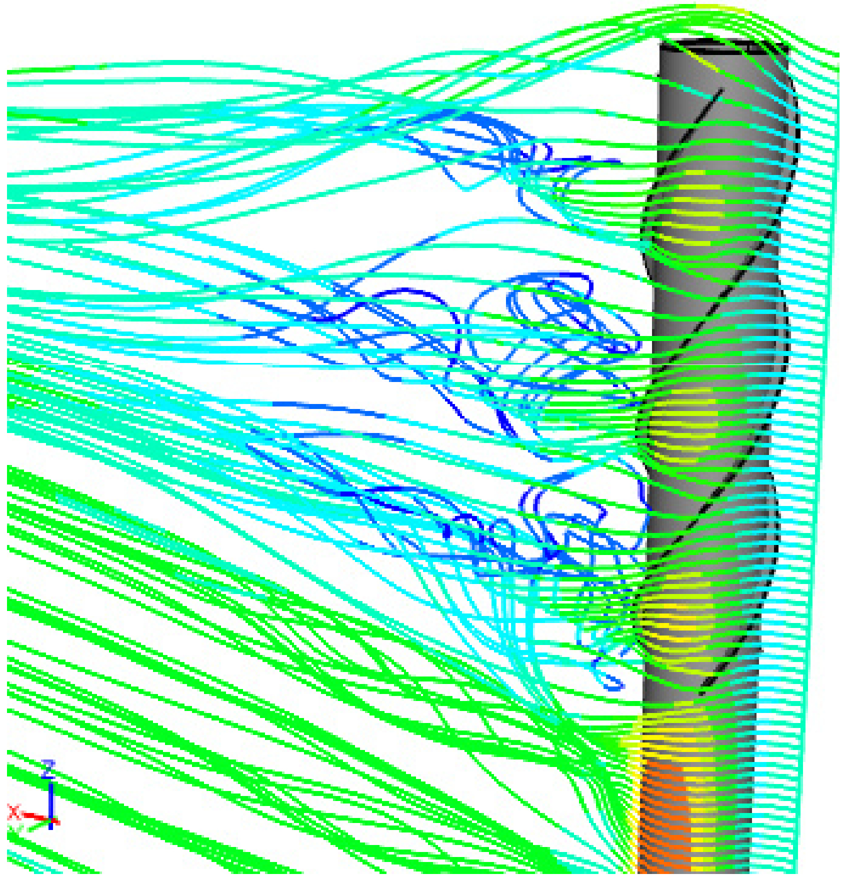
خط جریان خطی است که در هر نقطه بردار سرعت بر آن مماس باشد

$$\frac{dy}{dx} = \frac{v}{u}$$

$$\left.\frac{dy}{dx}\right)_{\text{streamline}} = \frac{v(x, y)}{u(x, y)}$$







مثال: برای سرعت سیال با رابطه $\mathbf{V} = (V_0/\ell)(-x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$ ، رابطه خط جریان را بدست آورید

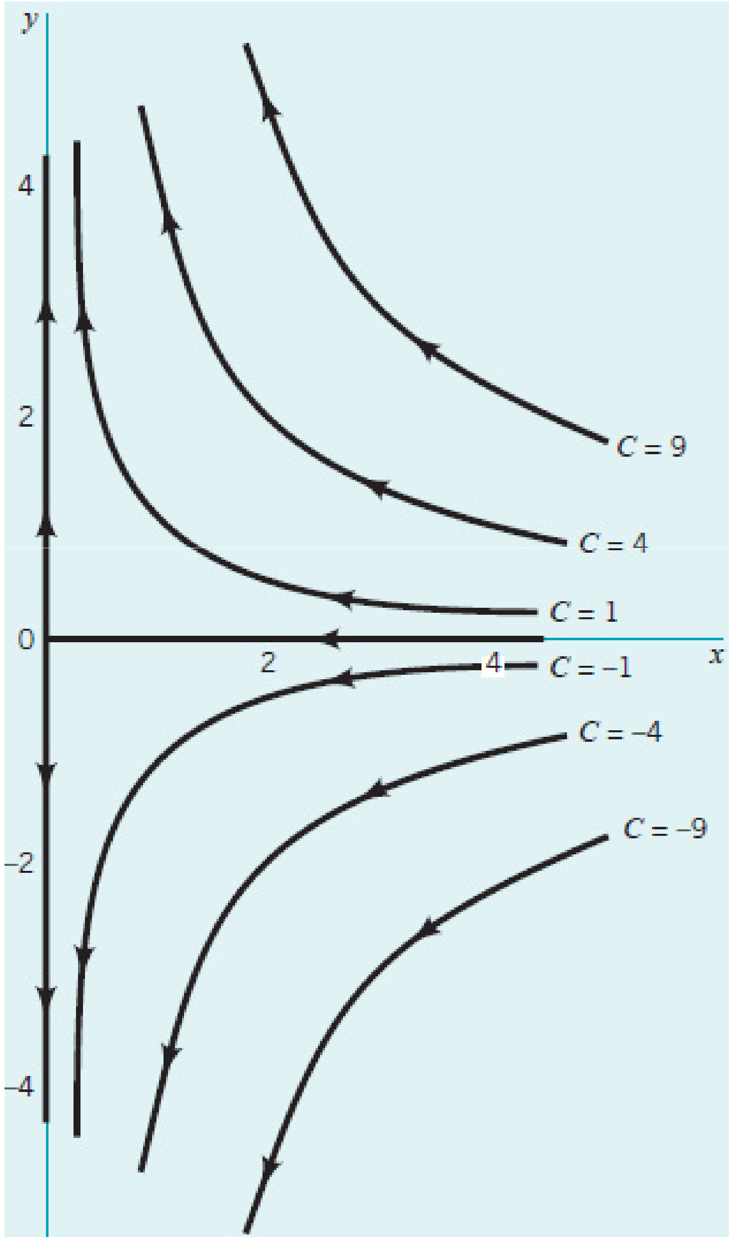
$$u = (-V_0/\ell)x \text{ and } v = (V_0/\ell)y$$

$$\frac{dy}{dx} = \frac{v}{u} = \frac{(V_0/\ell)y}{-(V_0/\ell)x} = -\frac{y}{x}$$

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\ln y = -\ln x + \text{constant}$$

$$xy = C, \qquad \text{where } C \text{ is a constant}$$



با در نظر گرفتن مقادیر مختلف برای C می توان خطوط جریان را رسم نمود

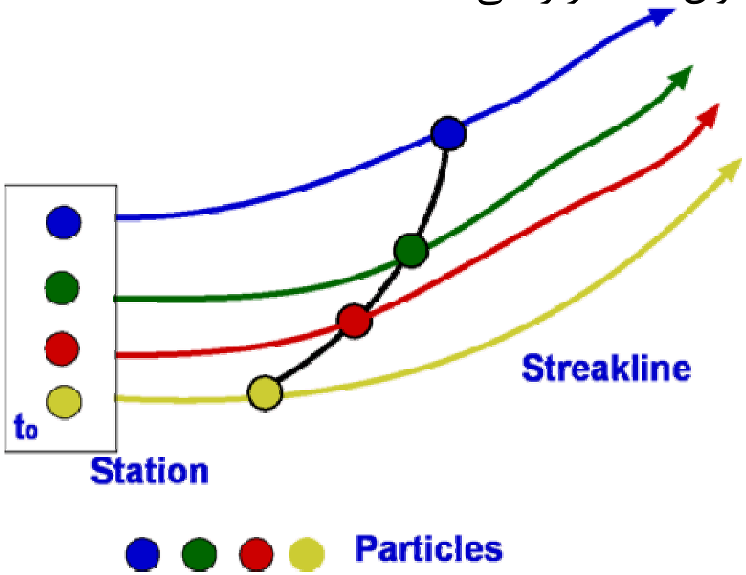
خط اثر (Streakline)

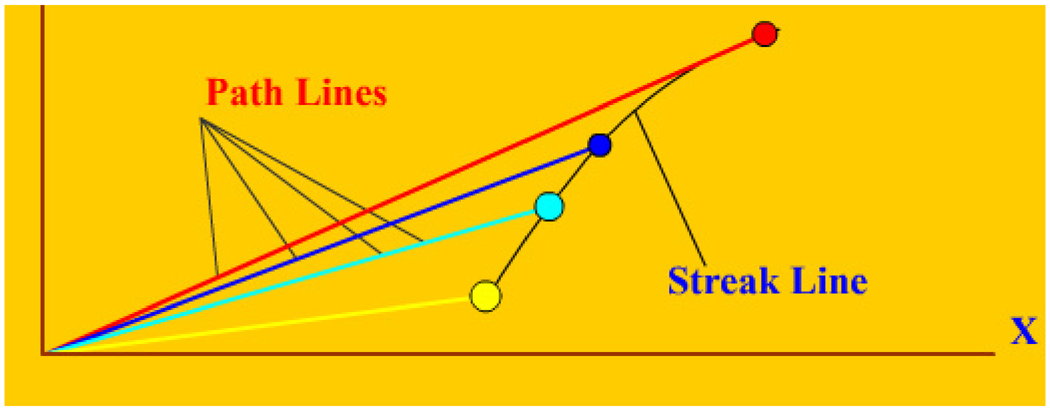
اتصال ذرات عبوری از یک محل مشخص در زمانهای مختلف، تشکیل خطی تحت عنوان خط اثر را می دهند

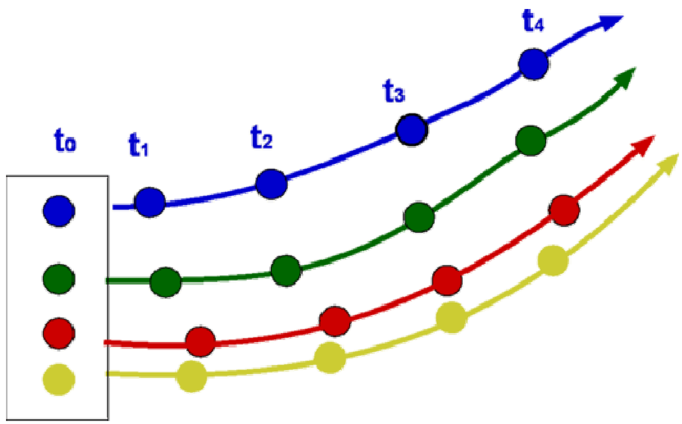
خط مسیر (Pathlines)

مسیر حرکت ذره در زمانهای مختلف را خط مسیر می نامند.

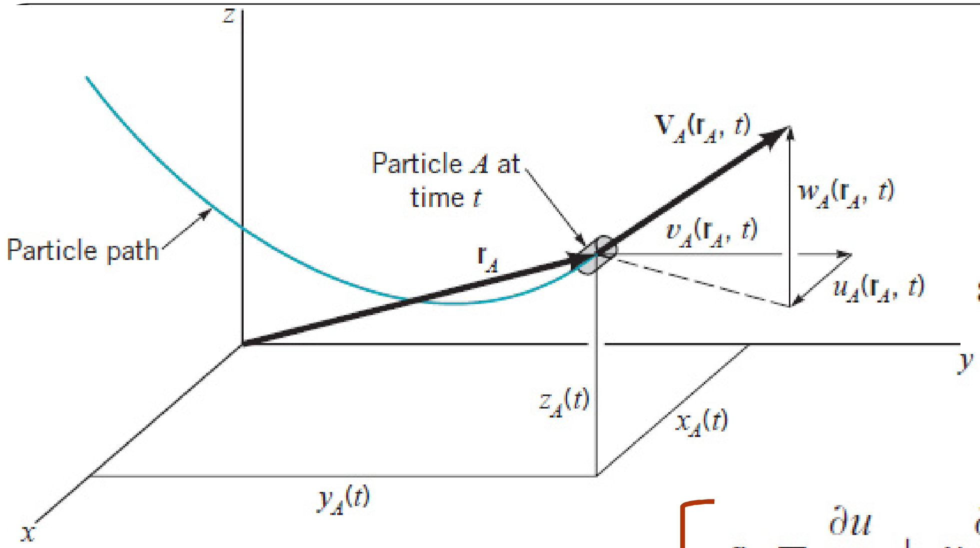
نکته: در شرایط دائم، خط مسیر، خط جریان و خط اثر بر هم منطبق است







سرعت و شتاب



$$\mathbf{V}_A = \mathbf{V}_A(\mathbf{r}_A, t) = \mathbf{V}_A[x_A(t), y_A(t), z_A(t), t]$$

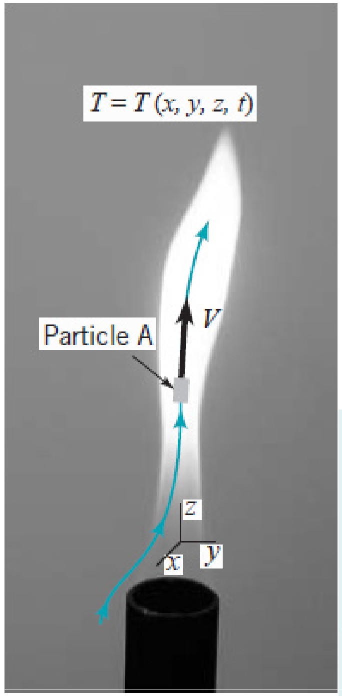
$$\mathbf{a}_A(t) = \frac{d\mathbf{V}_A}{dt} = \frac{\partial \mathbf{V}_A}{\partial t} + \frac{\partial \mathbf{V}_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial \mathbf{V}_A}{\partial z}\frac{dz_A}{dt}$$

$$\mathbf{a}_A = \frac{\partial \mathbf{V}_A}{\partial t} + u_A\frac{\partial \mathbf{V}_A}{\partial x} + v_A\frac{\partial \mathbf{V}_A}{\partial y} + w_A\frac{\partial \mathbf{V}_A}{\partial z}$$

$$\mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z} \left\{ \begin{aligned} a_x &= \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \\ a_y &= \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \\ a_z &= \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \end{aligned} \right\} \mathbf{a} = \frac{D\mathbf{V}}{Dt}$$

$$\frac{D(\ )}{Dt} \equiv \frac{\partial(\ )}{\partial t} + u\frac{\partial(\ )}{\partial x} + v\frac{\partial(\ )}{\partial y} + w\frac{\partial(\ )}{\partial z} \Rightarrow \frac{D(\ )}{Dt} = \frac{\partial(\ )}{\partial t} + (\mathbf{V}\cdot\nabla)(\ )$$

material derivative or *substantial derivative*

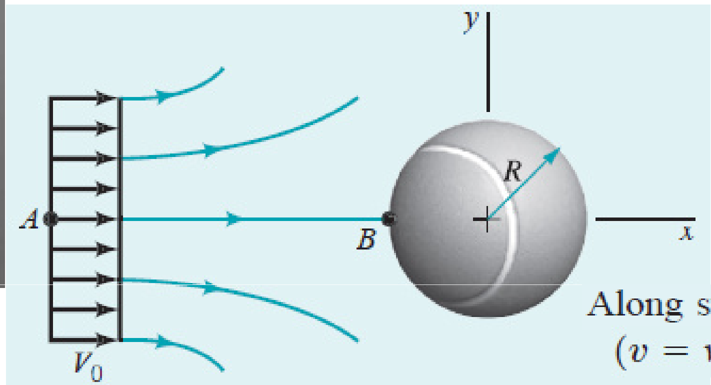


$$T = T(x, y, z, t) \qquad \mathbf{V} = \mathbf{V}(x, y, z, t)$$

(particle A) rate of change of temperature

$$T = T(x, y, z, t) \rightarrow \frac{dT_A}{dt} = \frac{\partial T_A}{\partial t} + \frac{\partial T_A}{\partial x}\frac{dx_A}{dt} + \frac{\partial T_A}{\partial y}\frac{dy_A}{dt} + \frac{\partial T_A}{\partial z}\frac{dz_A}{dt}$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{\partial T}{\partial t} + \mathbf{V}\cdot\nabla T$$



مثال: برای جریان تراکم ناپذیر، غیر لزج و دائم عبوری از روی یک توپ با توزیع سرعت داده شده، تغییرات شتاب در راستای خط جریان A-B را بدست آورید

$$\mathbf{V} = u(x)\hat{\mathbf{i}} = V_0\left(1 + \frac{R^3}{x^3}\right)\hat{\mathbf{i}}$$

Along streamline A–B $(v = w = 0)$

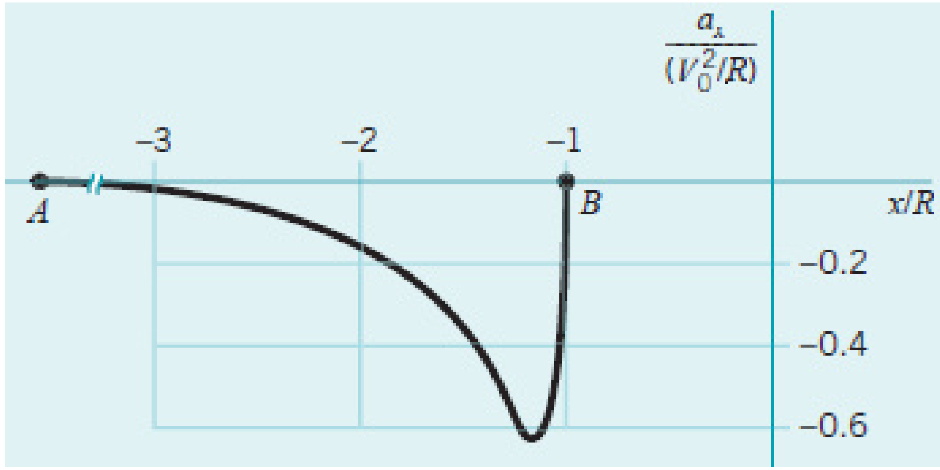
$$\rightarrow \mathbf{a} = \frac{\partial \mathbf{V}}{\partial t} + u\frac{\partial \mathbf{V}}{\partial x} = \left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}\right)\hat{\mathbf{i}}$$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x}, \qquad a_y = 0, \qquad a_z = 0$$

flow is steady $\rightarrow$ $\partial u/\partial t = 0$

$$a_x = u\frac{\partial u}{\partial x} = V_0\left(1 + \frac{R^3}{x^3}\right)V_0[R^3(-3x^{-4})]$$

$$a_x = -3(V_0^2/R)\frac{1 + (R/x)^3}{(x/R)^4}$$



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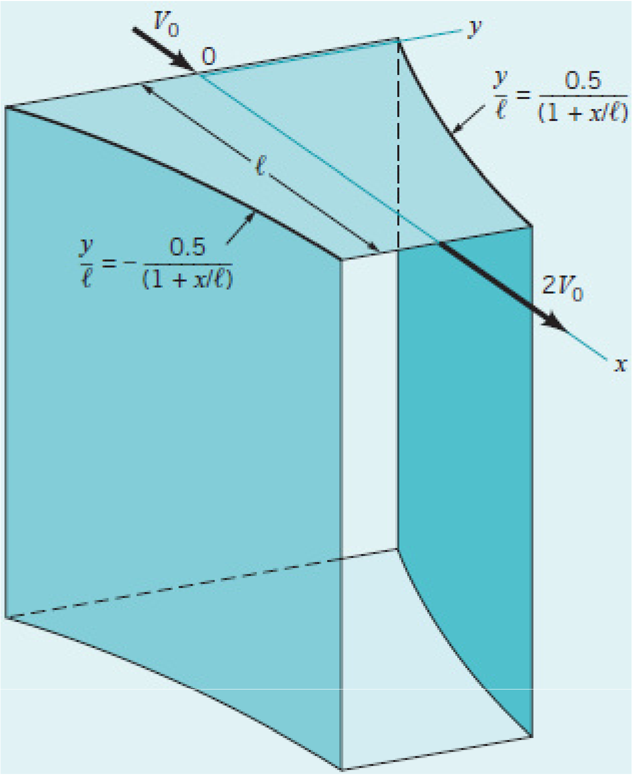
مثال: برای یک نازل در شرایط دائم به طول l (دو بعدی) با صرفنظر از اثر لزجت،
برای توزیع فشار و سرعت داده شده، تغییرات فشار بر اساس زمان (نرخ تغییر فشار)
برای هر نقطه را بدست آورید



$$u = V_0[1 + x/\ell], \; v = -V_0 y/\ell$$

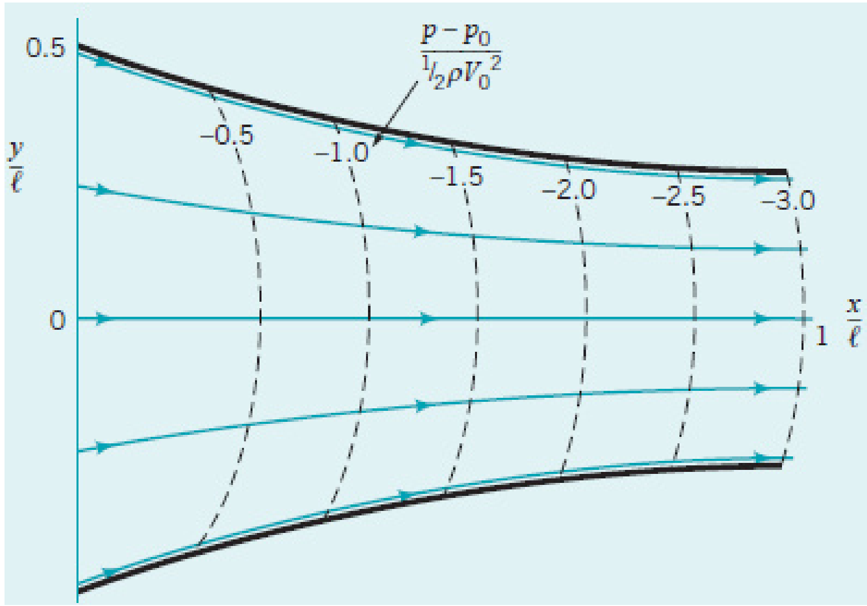
$$p - p_0 = -(\rho V_0^2/2)[(x^2 + y^2)/\ell^2 + 2x/\ell]$$

$$\frac{Dp}{Dt} = \overbrace{\frac{\partial p}{\partial t}}^{\text{Steady}=0} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} = u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}$$

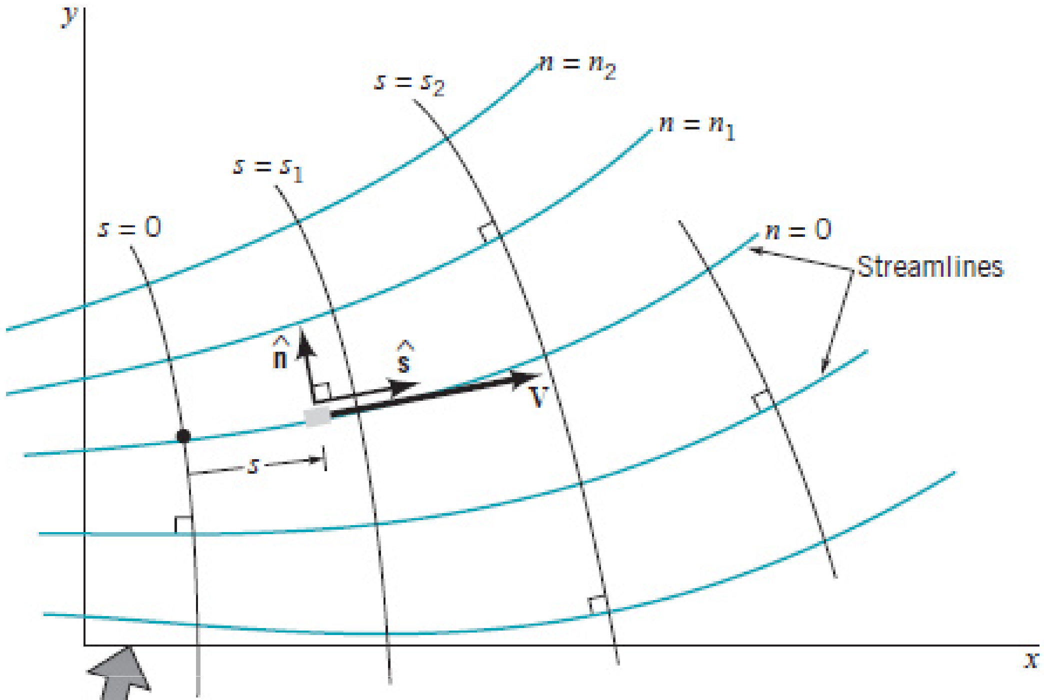
$$\frac{\partial p}{\partial x} = -\frac{\rho V_0^2}{\ell}\left(\frac{x}{\ell} + 1\right) \qquad \frac{\partial p}{\partial y} = -\frac{\rho V_0^2}{\ell}\left(\frac{y}{\ell}\right)$$

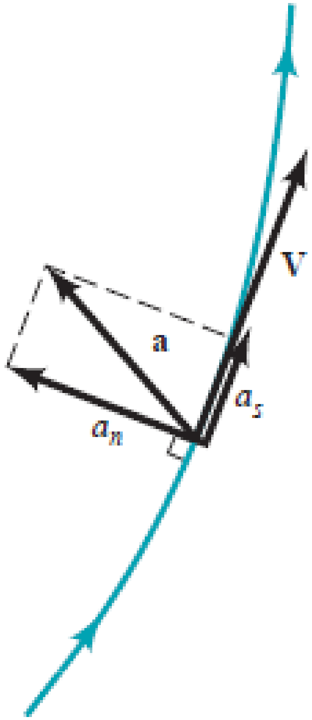
$$\frac{Dp}{Dt} = V_0\left(1 + \frac{x}{\ell}\right)\left(-\frac{\rho V_0^2}{\ell}\right)\left(\frac{x}{\ell} + 1\right) + \left(-V_0\frac{y}{\ell}\right)\left(-\frac{\rho V_0^2}{\ell}\right)\left(\frac{y}{\ell}\right)$$

$$\frac{Dp}{Dt} = -\frac{\rho V_0^3}{\ell}\left[\left(\frac{x}{\ell} + 1\right)^2 - \left(\frac{y}{\ell}\right)^2\right]$$



مختصات خط جریان

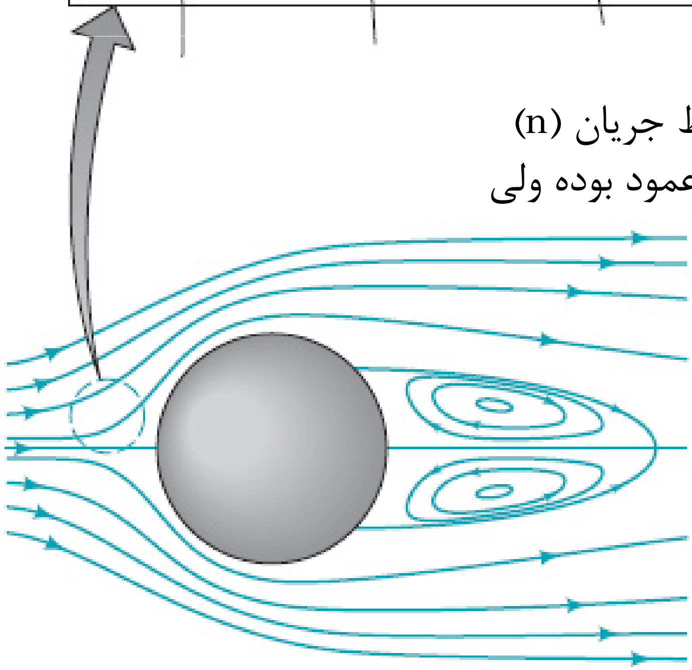




$$\mathbf{V} = V\hat{\mathbf{s}}$$

$$\mathbf{a} = \frac{D\mathbf{V}}{Dt} = a_s\hat{\mathbf{s}} + a_n\hat{\mathbf{n}}$$

با استفاده از این مختصات راستای s (راستای خط جریان) و را ستای عمود بر خط جریان (n) مبنای بررسی بوده و تطبیق بر راستای جریان دارد. این خطوط (s, n) بر یکدیگر عمود بوده ولی این خطوط می توانند مستقیم نباشند



In general, for steady flow

$$V = V(s, n) \text{ and } \hat{\mathbf{s}} = \hat{\mathbf{s}}(s, n)$$

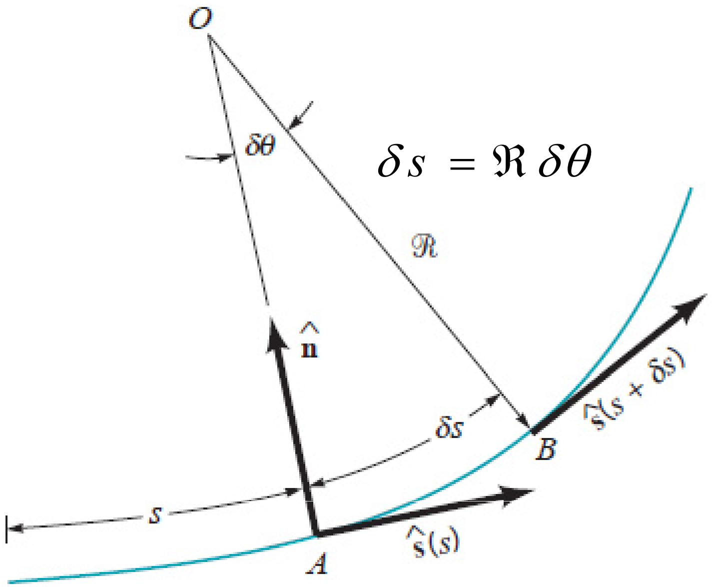
$$\mathbf{a} = \frac{D(V\hat{\mathbf{s}})}{Dt} = \frac{DV}{Dt}\hat{\mathbf{s}} + V\frac{D\hat{\mathbf{s}}}{Dt}$$

$$\mathbf{a} = \left(\frac{\partial V}{\partial t} + \frac{\partial V}{\partial s}\frac{ds}{dt} + \frac{\partial V}{\partial n}\frac{dn}{dt}\right)\hat{\mathbf{s}} + V\left(\frac{\partial \hat{\mathbf{s}}}{\partial t} + \frac{\partial \hat{\mathbf{s}}}{\partial s}\frac{ds}{dt} + \frac{\partial \hat{\mathbf{s}}}{\partial n}\frac{dn}{dt}\right)$$

for steady flow nothing changes with time $\Rightarrow$ $\partial V/\partial t$ and $\partial \hat{\mathbf{s}}/\partial t$ are zero

the particle remains on its streamline (n = constant) so that $dn/dt = 0$.

$$\mathbf{a} = \left(V\frac{\partial V}{\partial s}\right)\hat{\mathbf{s}} + V\left(V\frac{\partial \hat{\mathbf{s}}}{\partial s}\right)$$

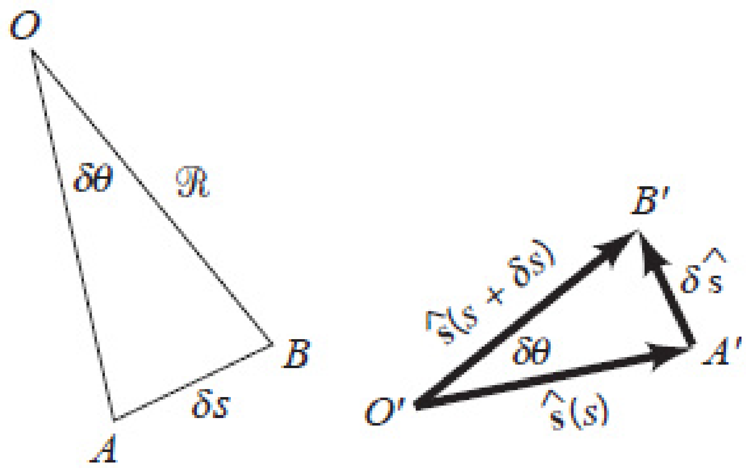


$$\left.\begin{aligned} \hat{\mathbf{s}}(s+\delta s) &= \hat{\mathbf{s}}(s) + \delta\hat{\mathbf{s}} \\ \delta s/\mathcal{R} &= |\delta\hat{\mathbf{s}}|/|\hat{\mathbf{s}}| = |\delta\hat{\mathbf{s}}| \end{aligned}\right\} \quad |\delta\hat{\mathbf{s}}/\delta s| = 1/\mathcal{R}$$

$|\hat{\mathbf{s}}| = 1$; it is a unit vector

$$\frac{\partial \hat{\mathbf{s}}}{\partial s} = \lim_{\delta s \to 0} \frac{\delta \hat{\mathbf{s}}}{\delta s} = \frac{\hat{\mathbf{n}}}{\mathcal{R}}$$

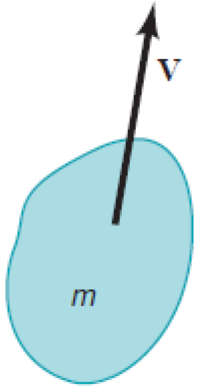
$$\mathbf{a} = V\frac{\partial V}{\partial s}\hat{\mathbf{s}} + \frac{V^2}{\mathcal{R}}\hat{\mathbf{n}} \quad \left\{\begin{aligned} a_s &= V\frac{\partial V}{\partial s}, \\ a_n &= \frac{V^2}{\mathcal{R}} \end{aligned}\right.$$



The Reynolds Transport Theorem

$B = mb$

B	$b = B/m$
m	1
$m\mathbf{V}$	$\mathbf{V}$
$\frac{1}{2}mV^2$	$\frac{1}{2}V^2$



برای مثال   if $B = m$, the mass, it follows that $b = 1$

For infinitesimal fluid particles of size $\delta\forall$

mass $\rho\,\delta\forall$.

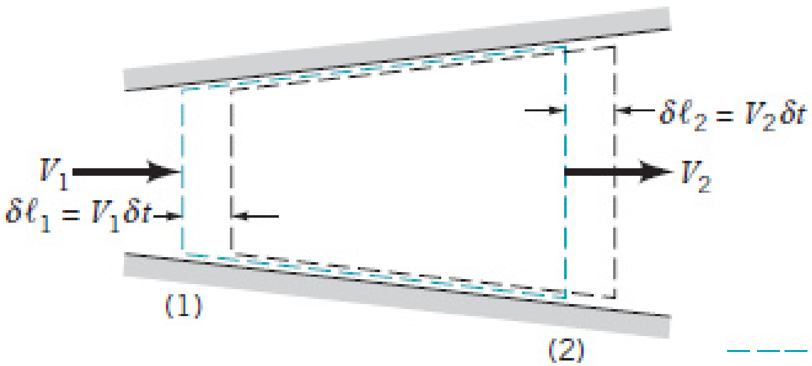
$$\delta B = b\rho\,\delta\forall.$$

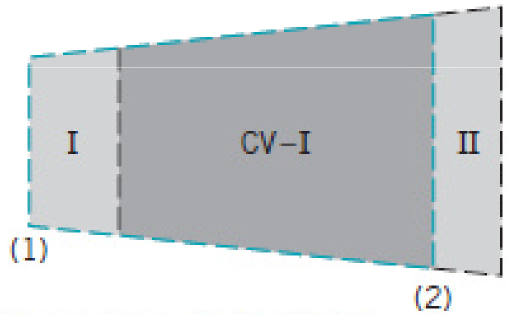
$$B_{sys} = \lim_{\delta\forall\to 0} \sum_i b_i(\rho_i\,\delta\forall_i) = \int_{sys} \rho b\, d\forall$$

$$\frac{dB_{sys}}{dt} = \frac{d\left(\int_{sys} \rho b\, d\forall\right)}{dt}$$

$$\frac{dB_{cv}}{dt} = \frac{d\left(\int_{cv} \rho b\, d\forall\right)}{dt}$$

one-dimensional flow





Fixed control surface and system boundary at time t

System boundary at time $t + \delta t$

the system at time t $B_{sys}(t) = B_{cv}(t)$

at time $t + \delta t$

$$B_{sys}(t + \delta t) = B_{cv}(t + \delta t) - B_{I}(t + \delta t) + B_{II}(t + \delta t)$$

توجه: در یک لحظه از زمان مقداری از سیال را به عنوان سیستم در نظر می گیریم. همان مرز برای حجم کنترل نیز در نظر گرفته می شود. با حرکت سیال، مرزهای سیستم نیز شروع به حرکت می نماید (خط چین مشکی). با جابجا شدن مرز سیستم و مقایسه آن با مرز حجم کنترل، سه ناحیه قابل تشخیص است

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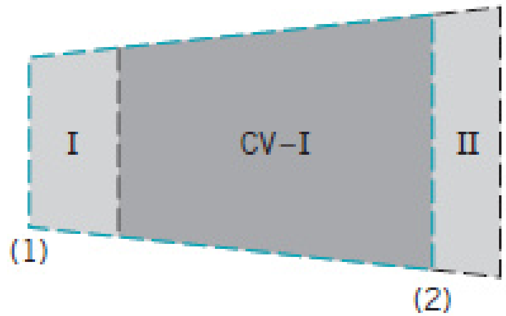
the change in the amount of B in the system



$$\frac{\delta B_{sys}}{\delta t} = \frac{B_{sys}(t+\delta t) - B_{sys}(t)}{\delta t} = \frac{B_{cv}(t+\delta t) - B_{I}(t+\delta t) + B_{II}(t+\delta t) - B_{sys}(t)}{\delta t}$$

at the initial time t we have $B_{sys}(t) = B_{cv}(t)$.

$$\frac{\delta B_{sys}}{\delta t} = \frac{B_{cv}(t+\delta t) - B_{cv}(t)}{\delta t} - \frac{B_{I}(t+\delta t)}{\delta t} + \frac{B_{II}(t+\delta t)}{\delta t}$$

In the limit $\delta t \rightarrow 0$ $\quad \frac{\delta B_{sys}}{\delta t} \Rightarrow DB_{sys}/Dt.$ و $\quad \lim_{\delta t\rightarrow 0} \frac{B_{cv}(t+\delta t) - B_{cv}(t)}{\delta t} = \frac{\partial B_{cv}}{\partial t} = \frac{\partial\left(\int_{cv} \rho b \, d\mathcal{V}\right)}{\partial t}$

$$\delta \mathcal{V}_{II} = A_2 \delta \ell_2 = A_2 (V_2 \delta t) \Rightarrow B_{II}(t+\delta t) = (\rho_2 b_2)(\delta \mathcal{V}_{II}) = \rho_2 b_2 A_2 V_2 \delta t$$

b_2 and ρ_2 are the constant values of b and ρ across section (2)

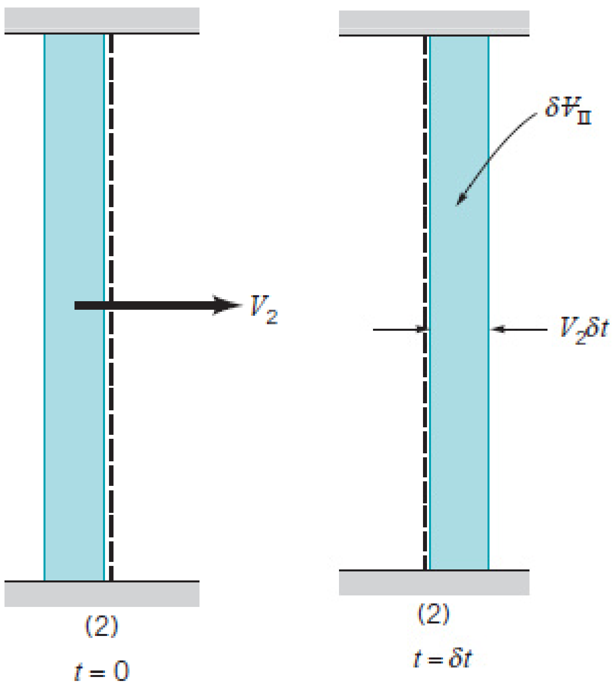
$\dot{B}_{out}$, is given by $\quad \dot{B}_{out} = \lim_{\delta t\rightarrow 0} \frac{B_{II}(t+\delta t)}{\delta t} = \rho_2 A_2 V_2 b_2$

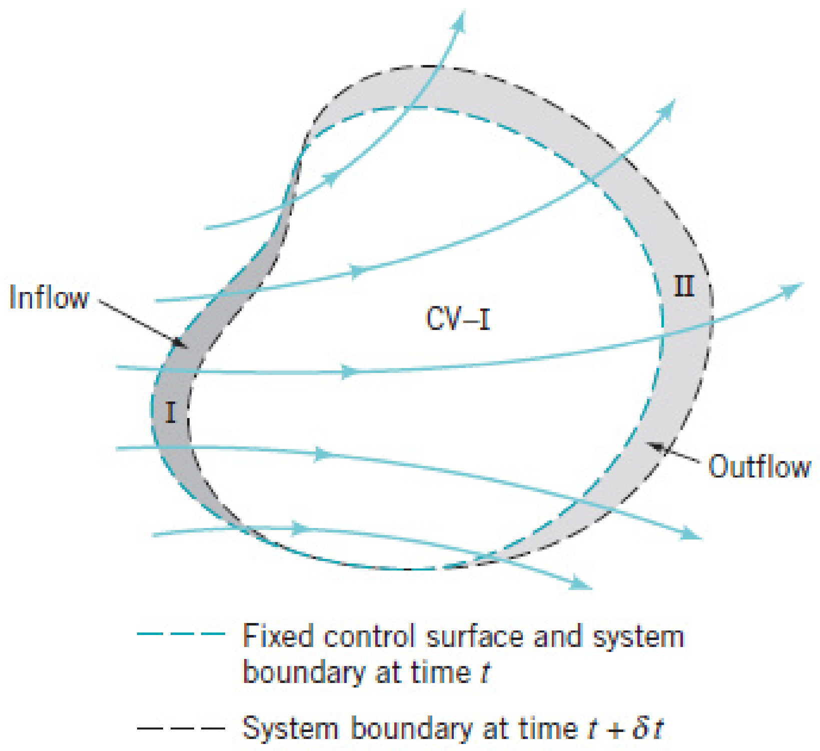
Similarly, the inflow of B into the control volume across section (1)

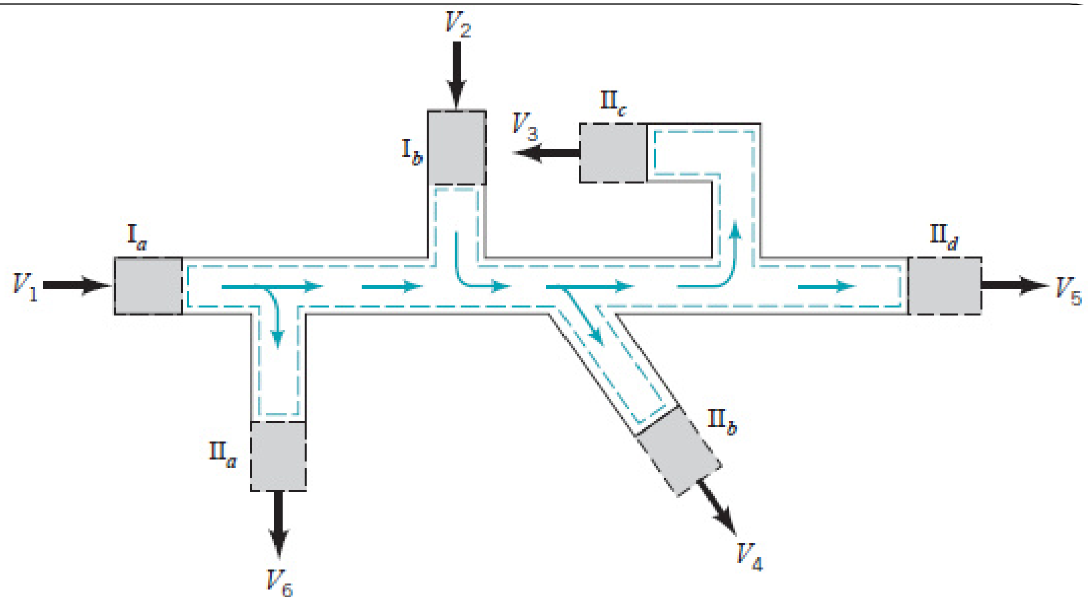
$$\delta \mathcal{V}_{I} = A_1 \delta \ell_1 = A_1 (V_1 \delta t) \Rightarrow B_{I}(t+\delta t) = (\rho_1 b_1)(\delta \mathcal{V}_1) = \rho_1 b_1 A_1 V_1 \delta t$$

$$\dot{B}_{in} = \lim_{\delta t\rightarrow 0} \frac{B_{I}(t+\delta t)}{\delta t} = \rho_1 A_1 V_1 b_1$$

$$\Rightarrow \boxed{\frac{DB_{sys}}{Dt} = \frac{\partial B_{cv}}{\delta t} + \dot{B}_{out} - \dot{B}_{in}}$$







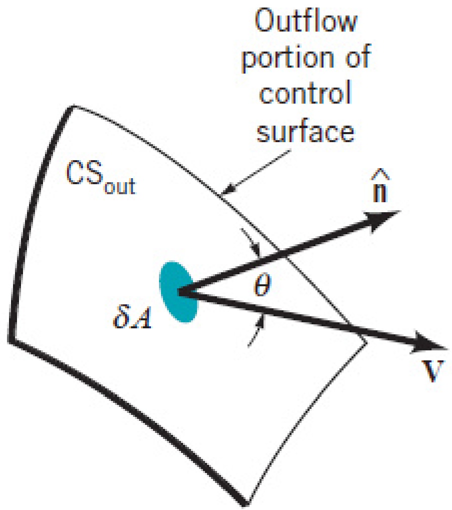
$$\dot{B}_{in} \longrightarrow \quad (I = I_a + I_b + I_c + \cdots)$$

$$\dot{B}_{out} \longrightarrow \quad (II = II_a + II_b + II_c + \cdots)$$

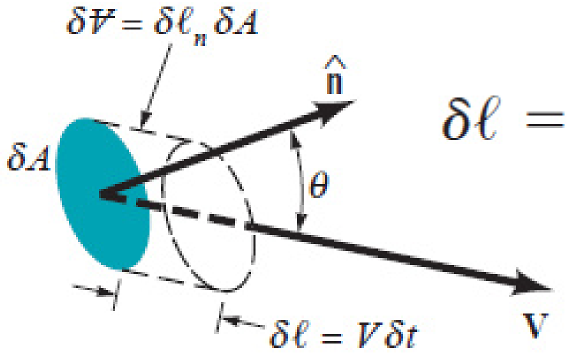
المان حجمی که سیال از آن وارد یا داخل می شود $\delta V\!\!\!\!/ = \delta \ell_n \, \delta A,$

$$\delta \ell_n = \delta \ell \cos \theta$$

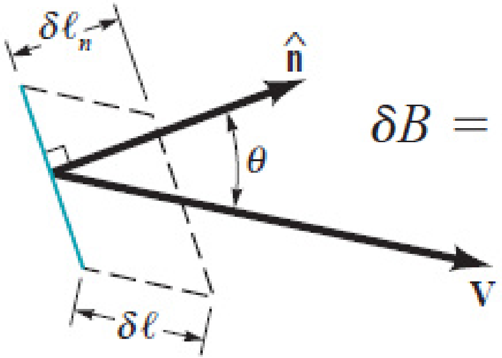
تئوری انتقال رینولدز برای حالت های چند گانه
نیز قابل تعمیم است.



زاویه بین بردار سرعت و بردار
نرمال سطح است



$$\delta\ell = V\,\delta t$$

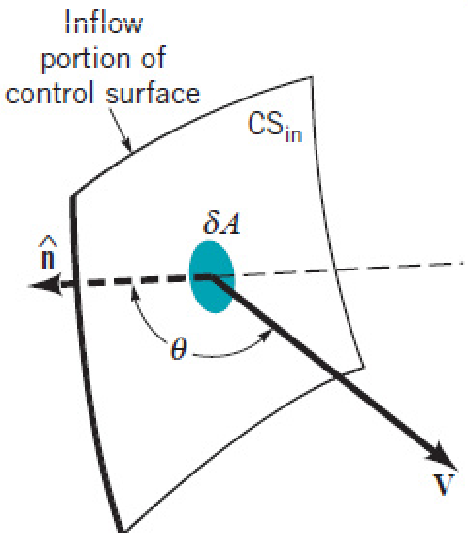


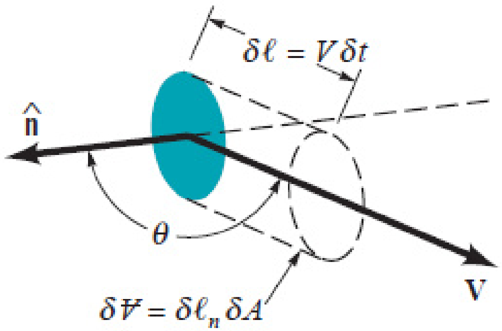
$$\delta B = b\rho\,\delta V\!\!\!\!/ = b\rho(V\cos\theta\,\delta t)\,\delta A$$

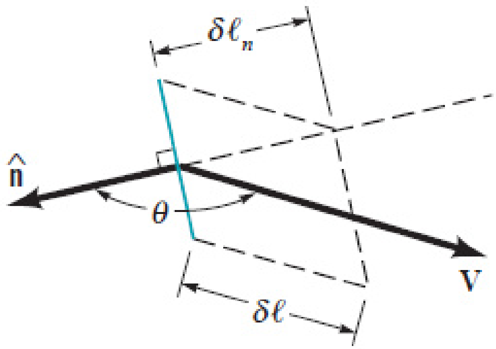
مقدار نرخ عبوری B از سطح المان کوچک

$$\delta\dot{B}_{\text{out}} = \lim_{\delta t\to 0}\frac{\rho b\,\delta V\!\!\!\!/}{\delta t} = \lim_{\delta t\to 0}\frac{(\rho b V\cos\theta\,\delta t)\,\delta A}{\delta t} = \rho b V\cos\theta\,\delta A$$

$$\left.\begin{aligned}&\dot{B}_{\text{out}} = \int_{\text{cs}_{\text{out}}} d\dot{B}_{\text{out}} = \int_{\text{cs}_{\text{out}}} \rho b V\cos\theta\,dA\\ &V\cos\theta = \mathbf{V}\cdot\hat{\mathbf{n}}\end{aligned}\right\}\quad \dot{B}_{\text{out}} = \int_{\text{cs}_{\text{out}}} \rho b\,\mathbf{V}\cdot\hat{\mathbf{n}}\,dA$$



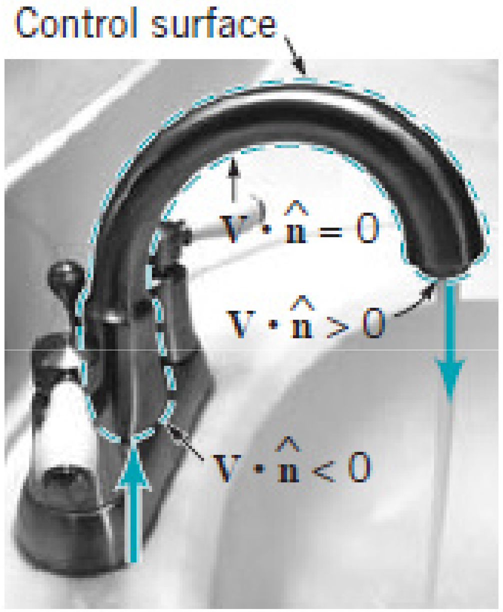


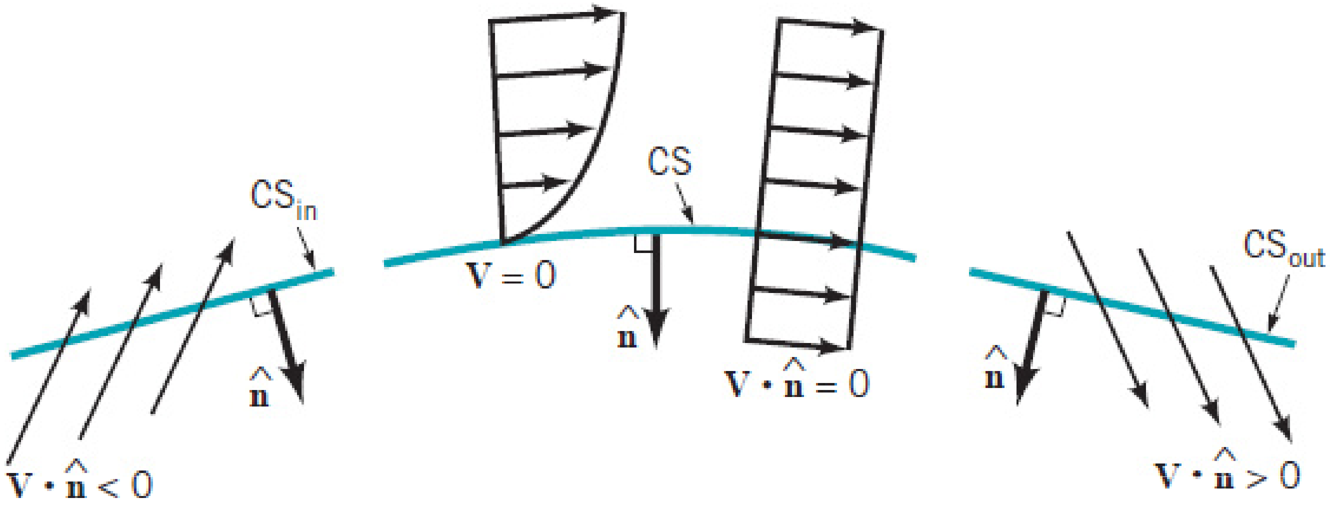


$$\dot{B}_{\text{in}} = -\int_{\text{cs}_{\text{in}}} \rho b V\cos\theta\,dA = -\int_{\text{cs}_{\text{in}}} \rho b\,\mathbf{V}\cdot\hat{\mathbf{n}}\,dA$$

$$\dot{B}_{\text{out}} - \dot{B}_{\text{in}} = \int_{\text{cs}_{\text{out}}} \rho b \mathbf{V}\cdot\hat{\mathbf{n}}\, dA - \left(-\int_{\text{cs}_{\text{in}}} \rho b \mathbf{V}\cdot\hat{\mathbf{n}}\, dA\right)$$

$$= \int_{\text{cs}} \rho b \mathbf{V}\cdot\hat{\mathbf{n}}\, dA$$





$$\left.\begin{aligned}\frac{DB_{\text{sys}}}{Dt} &= \frac{\partial B_{\text{cv}}}{\partial t} + \int_{\text{cs}} \rho b \mathbf{V}\cdot\hat{\mathbf{n}}\, dA \\ B_{\text{cv}} &= \int_{\text{cv}} \rho b\, d\forall\end{aligned}\right\}$$

$$\boxed{\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho b\, d\forall + \int_{\text{cs}} \rho b\, \mathbf{V}\cdot\hat{\mathbf{n}}\, dA}$$

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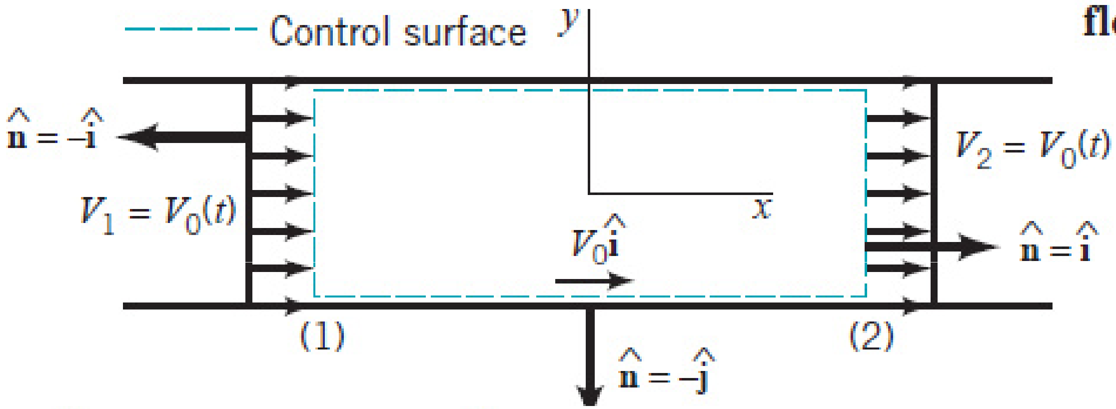
a steady flow $[\partial(\ )/\partial t \equiv 0] \Rightarrow \dfrac{DB_{\text{sys}}}{Dt} = \int_{\text{cs}} \rho b \mathbf{V} \cdot \hat{\mathbf{n}}\, dA$

Unsteady Effects $\boxed{\dfrac{DB_{\text{sys}}}{Dt} = \dfrac{\partial}{\partial t}\int_{\text{cv}} \rho b \, d\forall + \int_{\text{cs}} \rho b\, \mathbf{V} \cdot \hat{\mathbf{n}}\, dA}$



flow through a constant diameter pipe

$$\int_{\text{cs}} \rho b \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = 0$$

$$\frac{DB_{\text{sys}}}{Dt} = \frac{\partial}{\partial t}\int_{\text{cv}} \rho b \, d\forall$$

$$\text{momentum} = m\mathbf{V} = mV_0\hat{\mathbf{i}}$$

$$= \mathbf{B}/m = \mathbf{V} = V_0\hat{\mathbf{i}}$$

$$\int_{\text{cs}} \rho b \mathbf{V} \cdot \hat{\mathbf{n}}\, dA = \int_{\text{cs}} \rho (V_0\hat{\mathbf{i}})(\mathbf{V} \cdot \hat{\mathbf{n}})\, dA$$

$$A_1 = A_2, \qquad = \int_{(1)} \rho (V_0\hat{\mathbf{i}})(-V_0)\, dA + \int_{(2)} \rho (V_0\hat{\mathbf{i}})(V_0)\, dA$$

$$= -\rho V_0^2 A_1 \hat{\mathbf{i}} + \rho V_0^2 A_2 \hat{\mathbf{i}} = 0$$

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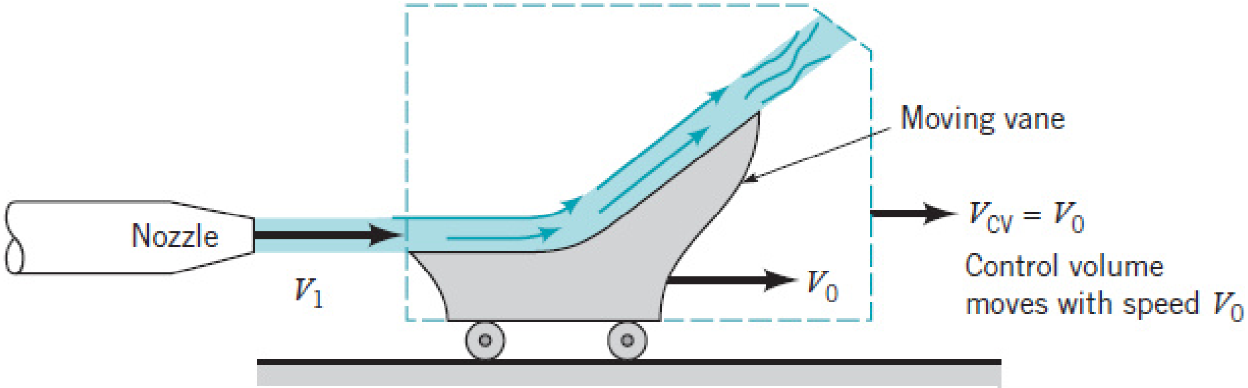
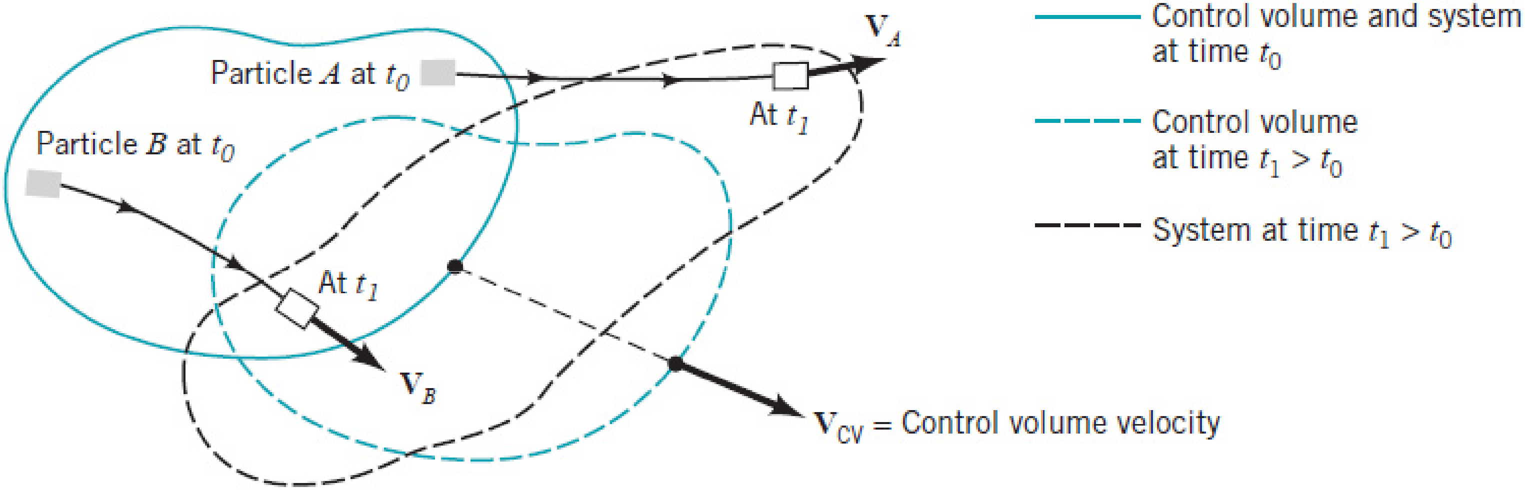
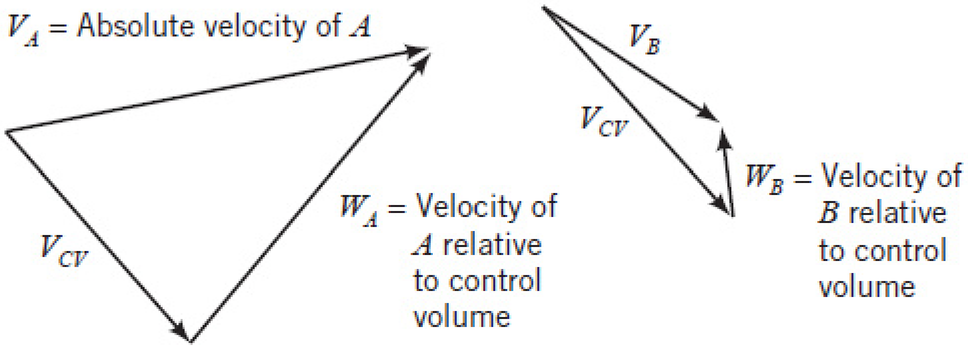
Moving Control Volumes

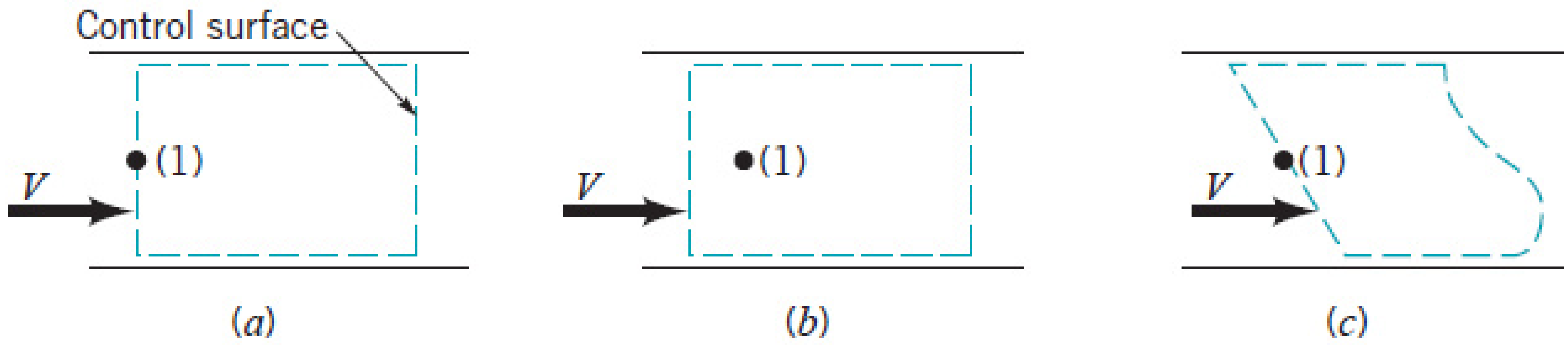






$$\frac{DB_{sys}}{Dt} = \frac{\partial}{\partial t}\int_{cv} \rho b \, dV + \int_{cs} \rho b \, \mathbf{W} \cdot \hat{\mathbf{n}} \, dA$$

Selection of a Control Volume



انتخاب حجم کنترل مناسب می تواند در رسیدن به جواب کمک شایانی نماید. در شکل بالا یافتن پارامتر مورد نظر در نقطه ۱ می توان از سه نوع حجم کنترل استفاده نمود. تنها حجم کنترل مناسب قابل استفاده، حجم کنترل a می باشد.