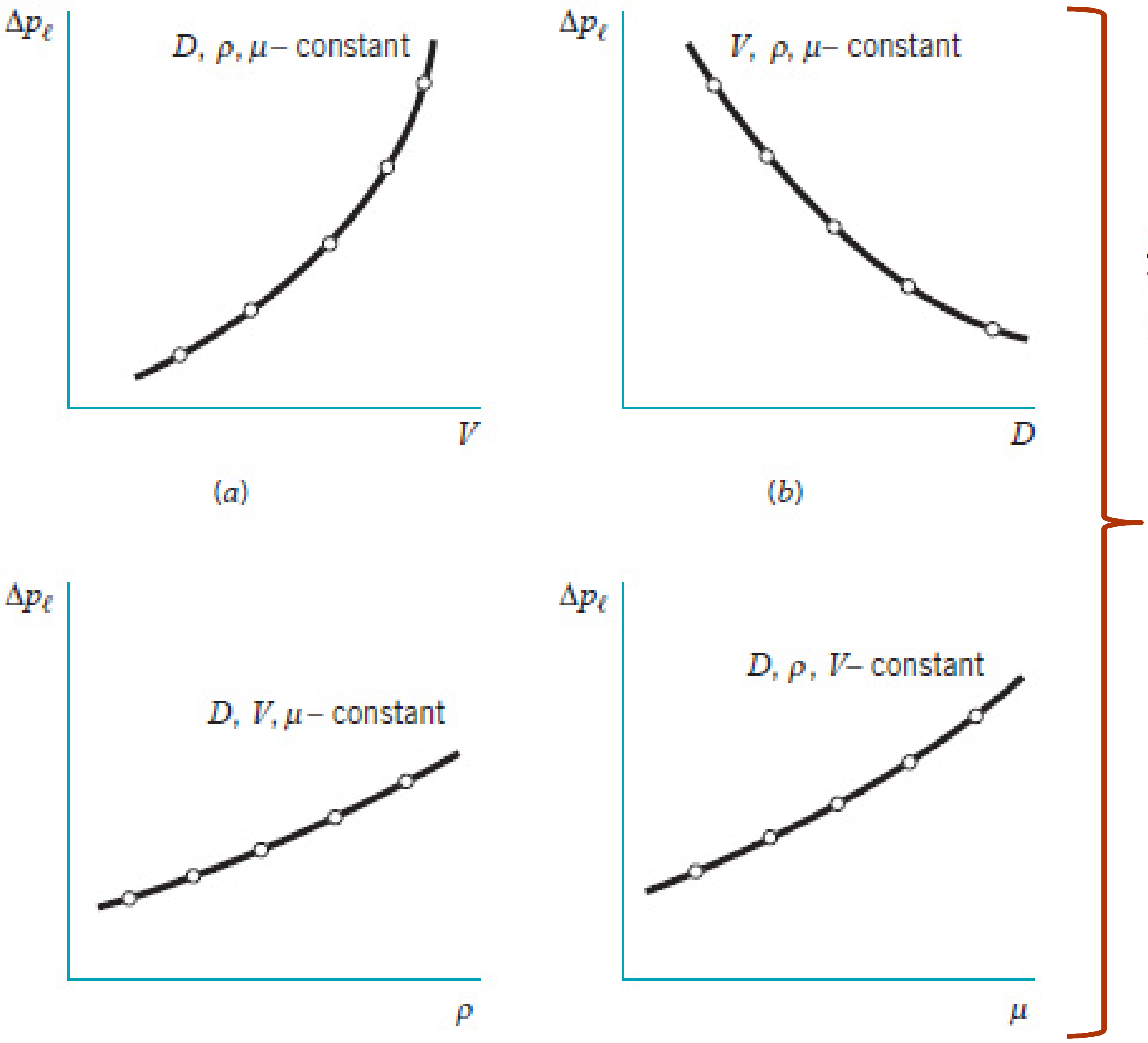
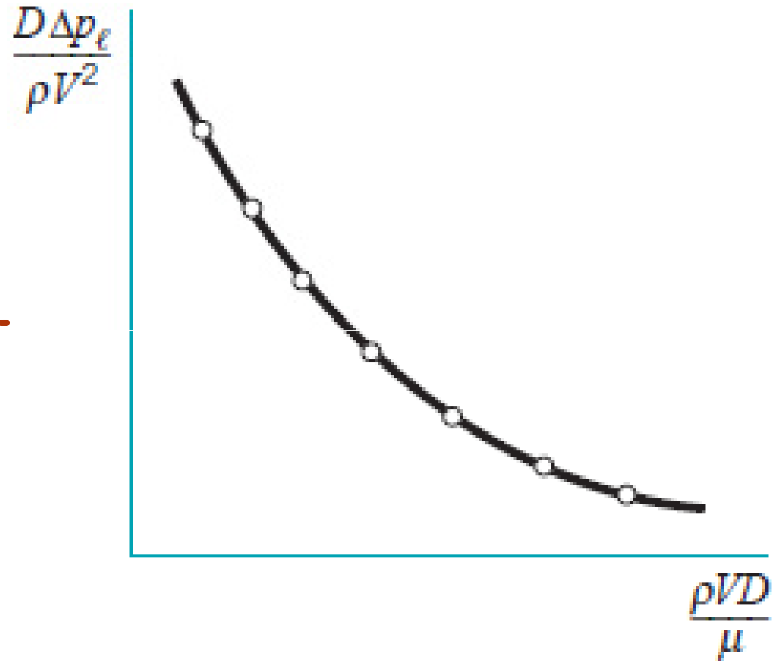
$$\Delta p_\ell = f(D, \rho, \mu, V)$$



dimensionless products

$$\frac{D\,\Delta p_\ell}{\rho V^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$



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$$\Delta p_\ell \doteq FL^{-3}, D \doteq L, \rho \doteq FL^{-4}T^2, \mu \doteq FL^{-2}T, \; V \doteq LT^{-1}$$

$$\Delta p_\ell = f(D, \rho, \mu, V) \Rightarrow \frac{D\,\Delta p_\ell}{\rho V^2} \doteq \frac{L(F/L^3)}{(FL^{-4}T^2)(LT^{-1})^2} \doteq F^0L^0T^0$$

$$\frac{\rho VD}{\mu} \doteq \frac{(FL^{-4}T^2)(LT^{-1})(L)}{(FL^{-2}T)} \doteq F^0L^0T^0$$

$$\frac{D\,\Delta p_\ell}{\rho V^2} = \phi\left(\frac{\rho VD}{\mu}\right)$$

با نگاه جدید تعداد متغیرها از ۵ به ۲ تقلیل یافته (یعنی فقط دو دو متغیر فوق محاسبه و بررسی می گردد) و نکته مهمتر این متغیرها بدون بعد نیز می باشند. این بدان معنی است که از سیستم اندازه گیری مستقل می گردد

This type of analysis is called *dimensional analysis*.

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Buckingham Pi Theorem

تئوری پی بوکینگهام بیان می دارد که برای یک معادله با k متغیر که از نظر ابعادی مشابه هستند، می توان به یک معادله با تعداد k-r متغیر بدون بعد کاهش یابد. r مینیمم تعداد متغیر با بعد است که می تواند برای بیان ابعادی سایر متغیرها استفاده نمود.

$$u_1 = f(u_2, u_3, \ldots, u_k) \longrightarrow$$

dimensionless products (pi terms)

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots, \Pi_{k-r})$$

$\phi(\Pi_2, \Pi_3, \ldots, \Pi_{k-r})$ is a function of Π_2 through Π_{k-r}

غالبا متغیرهای r بر اساس ابعاد اصلی انتخاب می شوند مانند M, L, T یا F و از این قبیل که می توانند سایر متغیرها را با استفاده از این ابعاد اصلی بصورت ابعادی بیان نمود

چگونگی دست یابی به متغیرهای بدون بعد و همچنین انتخاب متغیرهای پایه و اصلی، در ۸ مرحله تئوری پی بوکینگهام تشریح شده است

Step 1 List all the variables that are involved in the problem.

Typically the variables will include

are necessary to describe the *geometry* of the system (such as a pipe diameter)

any *fluid properties* (such as a fluid viscosity)

External effects that influence the system (such as a driving pressure drop per unit length)

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Step 2 Express each of the variables in terms of basic dimensions.

M, L, and T or F, L, and T ➡ $(\mathbf{F} = m\mathbf{a})$ $F \doteq MLT^{-2}$ ➡ $\rho \doteq ML^{-3}$ or $\rho \doteq FL^{-4}T^{2}$

TABLE 1.1

Dimensions Associated with Common Physical Quantities

	FLT System	*MLT* System
Acceleration	LT^{-2}	LT^{-2}
Angle	$F^0L^0T^0$	$M^0L^0T^0$
Angular acceleration	T^{-2}	T^{-2}
Angular velocity	T^{-1}	T^{-1}
Area	L^2	L^2
Density	$FL^{-4}T^2$	ML^{-3}
Energy	FL	ML^2T^{-2}
Force	F	MLT^{-2}
Frequency	T^{-1}	T^{-1}
Heat	FL	ML^2T^{-2}
Length	L	L
Mass	$FL^{-1}T^2$	M
Modulus of elasticity	FL^{-2}	$ML^{-1}T^{-2}$
Moment of a force	FL	ML^2T^{-2}
Moment of inertia (area)	L^4	L^4
Moment of inertia (mass)	FLT^2	ML^2
Momentum	FT	MLT^{-1}
Power	FLT^{-1}	ML^2T^{-3}
Pressure	FL^{-2}	$ML^{-1}T^{-2}$
Specific heat	$L^2T^{-2}\Theta^{-1}$	$L^2T^{-2}\Theta^{-1}$
Specific weight	FL^{-3}	$ML^{-2}T^{-2}$
Strain	$F^0L^0T^0$	$M^0L^0T^0$
Stress	FL^{-2}	$ML^{-1}T^{-2}$
Surface tension	FL^{-1}	MT^{-2}
Temperature	Θ	Θ
Time	T	T
Torque	FL	ML^2T^{-2}
Velocity	LT^{-1}	LT^{-1}
Viscosity (dynamic)	$FL^{-2}T$	$ML^{-1}T^{-1}$
Viscosity (kinematic)	L^2T^{-1}	L^2T^{-1}
Volume	L^3	L^3
Work	FL	ML^2T^{-2}

Step 3 Determine the required number of pi terms

$k - r$
- k is the number of variables in the problem (which is determined from Step 1)
- r is the number of reference dimensions required to describe these variables → from Step 2

Step 4 Select a number of repeating variables, where the number required is equal to the number of reference dimensions.

در این قسمت متغیرهایی انتخاب می شوند که سایر متغیرها با تکرار آنها بدست آیند.
نکته: متغیر های انتخاب شده باید از نظر ابعادی مستقل از یکدیگر باشند. یعنی با ترکیب خودشان نتوانند ابعاد یکدیگر را بدست آورند

Step 5 Form a pi term by multiplying one of the nonrepeating variables by the product of the repeating variables, each raised to an exponent that will make the combination dimensionless.

$$u_i u_1^{a_i} u_2^{b_i} u_3^{c_i}$$

u_i is one of the nonrepeating variables; u_1, u_2, and u_3 are the repeating variables; a_i, b_i, and c_i are determined so that the combination is dimensionless

Step 6 Repeat Step 5 for each of the remaining nonrepeating variables.

Step 7 Check all the resulting pi terms to make sure they are dimensionless

Step 8 Express the final form as a relationship among the pi terms, and think about what it means. Typically the final form can be written as

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots, \Pi_{k-r})$$

(Step 1)

$\Delta p_\ell = f(D, \rho, \mu, V)$

D is the pipe diameter,
ρ and μ are the fluid density and viscosity,
V is the mean velocity

(Step 2)

$\Delta p_\ell \doteq FL^{-3}$
$D \doteq L$
$\rho \doteq FL^{-4}T^2$
$\mu \doteq FL^{-2}T$
$V \doteq LT^{-1}$

$\rho \doteq ML^{-3}$
$\rho \doteq FL^{-4}T^2$
} $F \doteq MLT^{-2}$

F, L, and T or M, L, and T.

ابعاد اصلی برای این مسئله

(Step 3)

$(k = 5)$
$(r = 3)$
} $(5 - 3)$, or two pi terms required

(Step 4) repeating variables

from the D, ρ, μ, and V.

we will use D, V, and ρ as repeating variables

(Step 5)

$\Pi_1 = \Delta p_\ell D^a V^b \rho^c \Rightarrow (FL^{-3})(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$

a, b, and c must be determined

$1 + c = 0$ (for F)
$-3 + a + b - 4c = 0$ (for L)
$-b + 2c = 0$ (for T)

$a = 1, b = -2, c = -1$

$$\Pi_1 = \frac{\Delta p_\ell D}{\rho V^2}$$

(Step 6)

repeated for the remaining nonrepeating variables

$\Pi_2 = \mu D^a V^b \rho^c$

$(FL^{-2}T)(L)^a(LT^{-1})^b(FL^{-4}T^2)^c \doteq F^0L^0T^0$

$1 + c = 0$ (for F)
$-2 + a + b - 4c = 0$ (for L)
$1 - b + 2c = 0$ (for T)

$a = -1, b = -1, c = -1$

$$\Pi_2 = \frac{\mu}{DV\rho}$$

(Step 7) check to make sure the pi terms are actually dimensionless

We will check using both *FLT* and *MLT* dimensions.

$$\Pi_1 = \frac{\Delta p_\ell D}{\rho V^2} \doteq \frac{(FL^{-3})(L)}{(FL^{-4}T^2)(LT^{-1})^2} \doteq F^0L^0T^0 \qquad \Pi_2 = \frac{\mu}{DV\rho} \doteq \frac{(FL^{-2}T)}{(L)(LT^{-1})(FL^{-4}T^2)} \doteq F^0L^0T^0$$

$$\Pi_1 = \frac{\Delta p_\ell D}{\rho V^2} \doteq \frac{(ML^{-2}T^{-2})(L)}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0 \qquad \Pi_2 = \frac{\mu}{DV\rho} \doteq \frac{(ML^{-1}T^{-1})}{(L)(LT^{-1})(ML^{-3})} \doteq M^0L^0T^0$$

(Step 8)

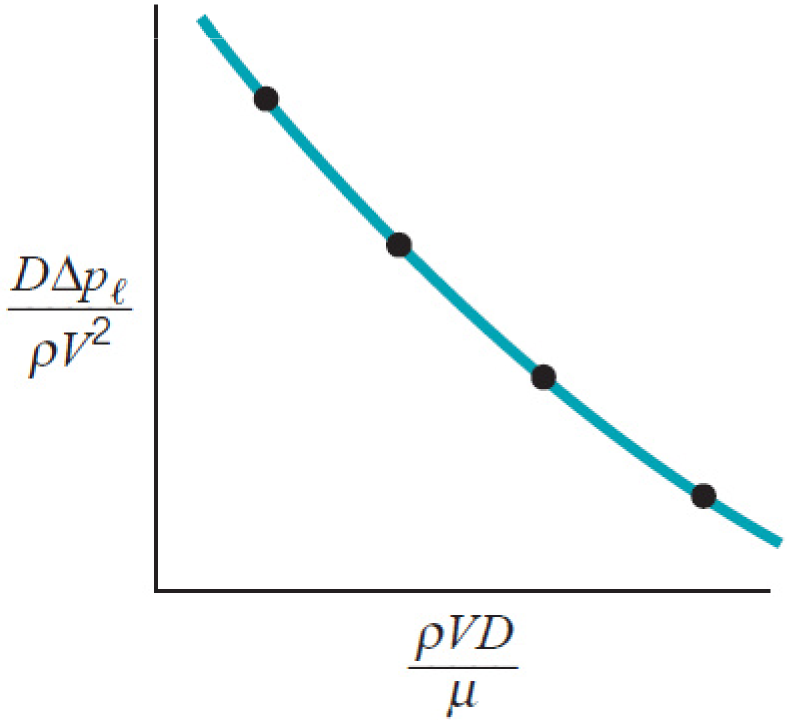
$$\frac{\Delta p_\ell D}{\rho V^2} = \tilde{\phi}\left(\frac{\mu}{DV\rho}\right)$$

در صورت نیاز می توان تغییرات جزیی در ترتیب نوشتن ایجاد نمود. توجه شود که تغییرات نباید اصل بی بعد بودن را برهم زند

$$\Pi_2 = \frac{\rho VD}{\mu}$$

the relationship between Π_1 and Π_2 as

$$\frac{D\,\Delta p_\ell}{\rho V^2} = \phi\left(\frac{\rho VD}{\mu}\right)$$



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Step 1

$$\Delta p_\ell = f(D, \rho, \mu, V)$$

Step 2

$$\Delta p_\ell \doteq FL^{-3}, \ldots$$

Step 3

$$k - r = 3$$

Step 4

$$D, V, \rho$$

Step 5

$$\Pi_1 = \Delta p_\ell D^a V^b \rho^c$$

Step 6

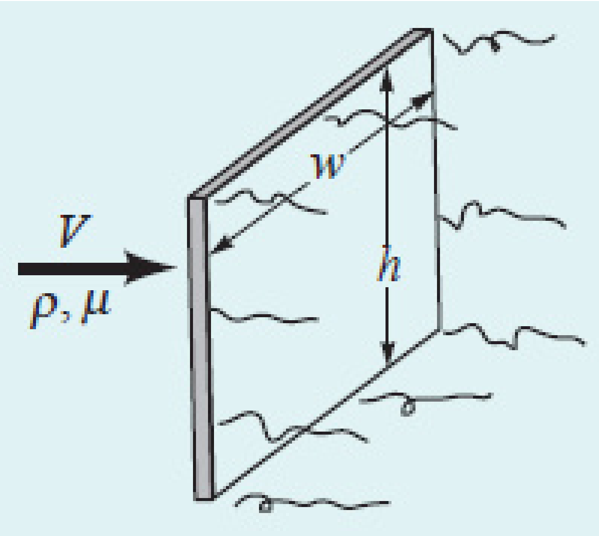
$$\Pi_2 = \mu D^a V^b \rho^c$$

Step 7

$$\frac{\Delta p_\ell D}{\rho V^2} \doteq F^0 L^0 T^0$$

Step 8

$$\frac{\Delta p_\ell D}{\rho V^2} = \tilde{\phi}\left(\frac{\mu}{DV\rho}\right)$$



مثال: نیروی وارده از طرف سیال در برخورد با یک صفحه عمودی (نیروی درگ) تابعی از سرعت سیال، لزجت و دانسیته سیال و ابعاد هندسی پنجره (عرض و ارتفاع) می باشد.
با استفاده از تئوری پی، اعداد بی بعد مورد نیاز برای آزمایش را بدست آورید.

$\mathfrak{D} = f(w, h, \mu, \rho, V)$ (using the MLT system) $V \doteq LT^{-1}$

$\mathfrak{D} \doteq MLT^{-2}$ $w \doteq L$ $h \doteq L$ $\mu \doteq ML^{-1}T^{-1}$ $\rho \doteq ML^{-3}$

$k - r = 6 - 3$ three repeating variables such as w, V, and ρ

$\Pi_1 = \mathfrak{D} w^a V^b \rho^c$ → $(MLT^{-2})(L)^a(LT^{-1})^b(ML^{-3})^c \doteq M^0L^0T^0$

$1 + c = 0$ (for M)
$1 + a + b - 3c = 0$ (for L)
$-2 - b = 0$ (for T)

$a = -2$, $b = -2$, and $c = -1$ → $\Pi_1 = \dfrac{\mathfrak{D}}{w^2V^2\rho}$

$\Pi_2 = h w^a V^b \rho^c$ $(L)(L)^a(LT^{-1})^b(ML^{-3})^c \doteq M^0L^0T^0$ $a = -1, b = 0, c = 0$ → $\Pi_2 = \dfrac{h}{w}$

$\Pi_3 = \mu w^a V^b \rho^c$ $(ML^{-1}T^{-1})(L)^a(LT^{-1})^b(ML^{-3})^c \doteq M^0L^0T^0$ $a = -1, b = -1, c = -1$ → $\Pi_3 = \dfrac{\mu}{wV\rho}$

$$\Pi_1 = \frac{\mathfrak{D}}{w^2V^2\rho} \doteq \frac{(F)}{(L)^2(LT^{-1})^2(FL^{-4}T^2)} \doteq F^0L^0T^0$$

$$\Pi_3 = \frac{\mu}{wV\rho} \doteq \frac{(FL^{-2}T)}{(L)(LT^{-1})(FL^{-4}T^2)} \doteq F^0L^0T^0$$

$$\Pi_2 = \frac{h}{w} \doteq \frac{(L)}{(L)} \doteq F^0L^0T^0$$

$$\frac{\mathfrak{D}}{w^2V^2\rho} = \tilde{\phi}\left(\frac{h}{w}, \frac{\mu}{wV\rho}\right) \rightarrow \frac{\mathfrak{D}}{w^2\rho V^2} = \phi\left(\frac{w}{h}, \frac{\rho V w}{\mu}\right)$$

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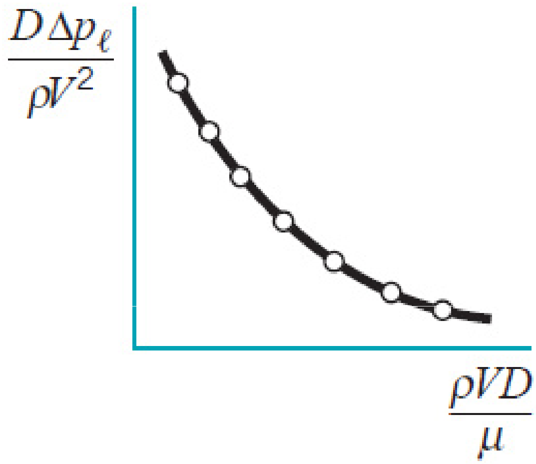
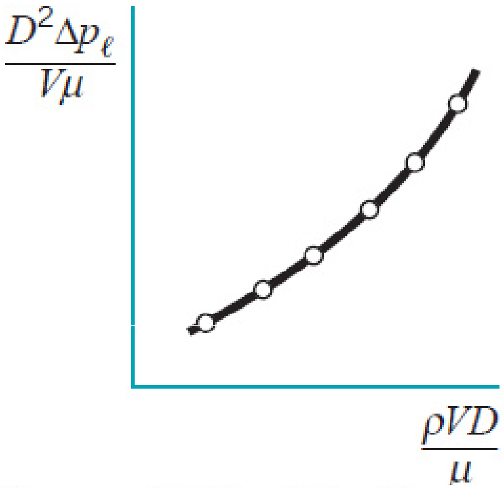
$$\Delta p_\ell = f(D, \rho, \mu, V)$$

- D, V, and ρ as repeating variables → $\dfrac{\Delta p_\ell D}{\rho V^2} = \phi\left(\dfrac{\rho V D}{\mu}\right)$
- D, V, and μ as repeating variables → $\dfrac{\Delta p_\ell D^2}{V\mu} = \phi_1\left(\dfrac{\rho V D}{\mu}\right)$





Both results are correct, and both would lead to the same final equation for Δp_ℓ

با ترکیب این اعداد نیز می توان اعداد بدون بعد جدیدی بدست آورد. نتایج حاصل نشان می دهد که اعداد بی بعد منحصر بفرد نمی باشند

$$\Pi_1 = \phi(\Pi_2, \Pi_3)$$

$$\Pi_2' = \Pi_2^a \, \Pi_3^b$$

$$\begin{cases} \Pi_1 = \phi_1(\Pi_2', \Pi_3) \\ \Pi_1 = \phi_2(\Pi_2, \Pi_2') \end{cases}$$

این رابطه نیز یک حالت خاص می باشد

$$\left(\frac{\Delta p_\ell D}{\rho V^2}\right)\left(\frac{\rho V D}{\mu}\right) = \frac{\Delta p_\ell D^2}{V\mu}$$

می توان اعداد بدون بعد را با استفاده از آنالیز ابعادی نیز بدست آورد.

$$\Delta p_\ell = f(D, \rho, \mu, V)$$

$$\Delta p_\ell \doteq FL^{-3}$$
$$D \doteq L$$
$$\rho \doteq FL^{-4}T^2$$
$$\mu \doteq FL^{-2}T$$
$$V \doteq LT^{-1}$$

با تقسیم بر دانسیته، بعد F از رابطه حذف شده است

$$\frac{\Delta p_\ell}{\rho} \doteq \frac{(FL^{-3})}{(FL^{-4}T^2)} \doteq \frac{L}{T^2} \qquad \text{(cancels } F\text{)}$$

به همین ترتیب

$$\left(\frac{\Delta p_\ell}{\rho}\right)\frac{1}{V^2} \doteq \left(\frac{L}{T^2}\right)\frac{1}{(LT^{-1})^2} \doteq \frac{1}{L} \qquad \text{(cancels } T\text{)}$$

$$\left(\frac{\Delta p_\ell}{\rho V^2}\right)D \doteq \left(\frac{1}{L}\right)(L) \doteq L^0 \qquad \text{(cancels } L\text{)} \quad \Rightarrow \quad \Pi_1 = \frac{\Delta p_\ell D}{\rho V^2}$$

$$\mu \;\Rightarrow\; \mu \doteq FL^{-2}T \;\Rightarrow\; \Pi_2 = \frac{\mu}{\rho V D} \doteq \frac{(FL^{-2}T)}{(FL^{-4}T^2)(LT^{-1})(L)} \doteq F^0L^0T^0$$

$$\frac{\Delta p_\ell D}{\rho V^2} = \phi\left(\frac{\mu}{\rho V D}\right)$$

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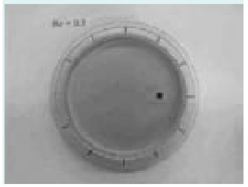
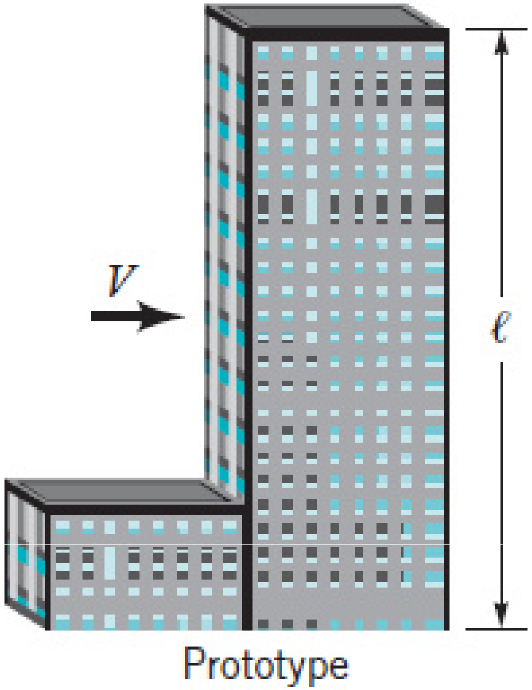
Some Common Variables and Dimensionless Groups in Fluid Mechanics

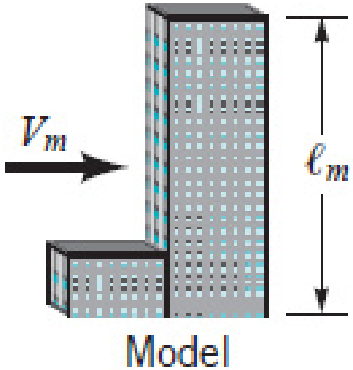
Variables: Acceleration of gravity, g; Bulk modulus, E_v; Characteristic length, ℓ; Density, ρ; Frequency of oscillating flow, ω; Pressure, p (or Δp); Speed of sound, c; Surface tension, σ; Velocity, V; Viscosity, μ

Dimensionless Groups	Name	Interpretation (Index of Force Ratio Indicated)	Types of Applications
$\frac{\rho V \ell}{\mu}$	Reynolds number, Re	$\frac{\text{inertia force}}{\text{viscous force}}$	Generally of importance in all types of fluid dynamics problems
$\frac{V}{\sqrt{g\ell}}$	Froude number, Fr	$\frac{\text{inertia force}}{\text{gravitational force}}$	Flow with a free surface
$\frac{p}{\rho V^2}$	Euler number, Eu	$\frac{\text{pressure force}}{\text{inertia force}}$	Problems in which pressure, or pressure differences, are of interest
$\frac{\rho V^2}{E_v}$	Cauchy number,[a] Ca	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\frac{V}{c}$	Mach number,[a] Ma	$\frac{\text{inertia force}}{\text{compressibility force}}$	Flows in which the compressibility of the fluid is important
$\frac{\omega \ell}{V}$	Strouhal number, St	$\frac{\text{inertia (local) force}}{\text{inertia (convective) force}}$	Unsteady flow with a characteristic frequency of oscillation
$\frac{\rho V^2 \ell}{\sigma}$	Weber number, We	$\frac{\text{inertia force}}{\text{surface tension force}}$	Problems in which surface tension is important



Modeling and Similitude





Theory of Models

$$\Pi_1 = \phi(\Pi_2, \Pi_3, \ldots, \Pi_n) \qquad \text{prototype}$$

$$\Pi_{1m} = \phi(\Pi_{2m}, \Pi_{3m}, \ldots, \Pi_{nm}) \qquad \text{model}$$

تنها در شرایطی نتایج حاصل از مدل با نتایج نمونه واقعی یکسان خواهد بود که

if the model is designed and operated under the following conditions

$$\Pi_{2m} = \Pi_2$$

$$\Pi_{3m} = \Pi_3$$

$$\vdots$$

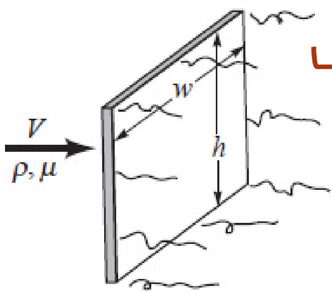
$$\Pi_{nm} = \Pi_n$$

model design conditions,

similarity requirements or ***modeling laws***

$$\mathfrak{D} = f(w, h, \mu, \rho, V)$$

$$\frac{\mathfrak{D}}{w^2\rho V^2} = \phi\left(\frac{w}{h}, \frac{\rho V w}{\mu}\right) \qquad \frac{\mathfrak{D}_m}{w_m^2\rho_m V_m^2} = \phi\left(\frac{w_m}{h_m}, \frac{\rho_m V_m w_m}{\mu_m}\right)$$



$$\frac{w_m}{h_m} = \frac{w}{h} \Rightarrow w_m = \frac{h_m}{h} w$$

$$\frac{\rho_m V_m w_m}{\mu_m} = \frac{\rho V w}{\mu} \Rightarrow V_m = \frac{\mu_m}{\mu}\frac{\rho}{\rho_m}\frac{w}{w_m} V$$

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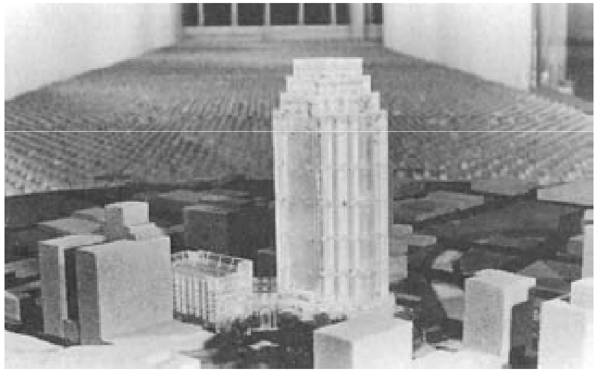
$$\frac{\mathcal{D}}{w^2\rho V^2} = \frac{\mathcal{D}_m}{w_m^2\rho_m V_m^2} \;\Rightarrow\; \mathcal{D} = \left(\frac{w}{w_m}\right)^2\left(\frac{\rho}{\rho_m}\right)\left(\frac{V}{V_m}\right)^2 \mathcal{D}_m$$

Model Scales

$$\frac{\ell_1}{\ell_2} = \frac{\ell_{1m}}{\ell_{2m}} \;\Rightarrow\; \frac{\ell_{1m}}{\ell_1} = \frac{\ell_{2m}}{\ell_2}$$

در بررسی یک مدل، باید ابعاد هندسی مدل دقیقا بر اساس نمونه واقعی باشد.
بعبارت دیگر، مدل یک نمونه کوچک شده از هندسه واقعی است.



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$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial^2 y}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}-\rho g+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

$$u^*=\frac{u}{V}\quad v^*=\frac{v}{V}\quad p^*=\frac{p}{p_0}\quad x^*=\frac{x}{\ell}\quad y^*=\frac{y}{\ell}\quad t^*=\frac{t}{\tau}$$

$$\frac{\partial u}{\partial x}=\frac{\partial Vu^*}{\partial x^*}\frac{\partial x^*}{\partial x}=\frac{V}{\ell}\frac{\partial u^*}{\partial x^*}$$

$$\frac{\partial^2 u}{\partial x^2}=\frac{V}{\ell}\frac{\partial}{\partial x^*}\left(\frac{\partial u^*}{\partial x^*}\right)\frac{\partial x^*}{\partial x}=\frac{V}{\ell^2}\frac{\partial^2 u^*}{\partial x^{*2}}$$

$$\Rightarrow\quad \frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0$$

$$\left[\frac{\rho V}{\tau}\right]\frac{\partial u^*}{\partial t^*}+\left[\frac{\rho V^2}{\ell}\right]\left(u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}\right)=-\left[\frac{p_0}{\ell}\right]\frac{\partial p^*}{\partial x^*}+\left[\frac{\mu V}{\ell^2}\right]\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

$$\underbrace{\left[\frac{\rho V}{\tau}\right]}_{F_{I\ell}}\frac{\partial v^*}{\partial t^*}+\underbrace{\left[\frac{\rho V^2}{\ell}\right]}_{F_{Ic}}\left(u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}\right)=-\underbrace{\left[\frac{p_0}{\ell}\right]}_{F_P}\frac{\partial p^*}{\partial y^*}-\underbrace{[\rho g]}_{F_G}+\underbrace{\left[\frac{\mu V}{\ell^2}\right]}_{F_V}\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

$F_{I\ell}$ = inertia (local) force
F_{Ic} = inertia (convective) force
F_G = gravitational force
F_p = pressure force
F_V = viscous force

$$\left[\frac{\ell}{\tau V}\right]\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\left[\frac{p_0}{\rho V^2}\right]\frac{\partial p^*}{\partial x^*}+\left[\frac{\mu}{\rho V\ell}\right]\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

$$\left[\frac{\ell}{\tau V}\right]\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\left[\frac{p_0}{\rho V^2}\right]\frac{\partial p^*}{\partial y^*}-\left[\frac{g\ell}{V^2}\right]+\left[\frac{\mu}{\rho V\ell}\right]\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

$\left[\frac{\ell}{\tau V}\right]$ → Strouhal number

$\left[\frac{p_0}{\rho V^2}\right]$ → Euler number

$\left[\frac{g\ell}{V^2}\right]$ → مجذور عکس عدد فرود

$\left[\frac{\mu}{\rho V\ell}\right]$ → عکس عدد رینولدز

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