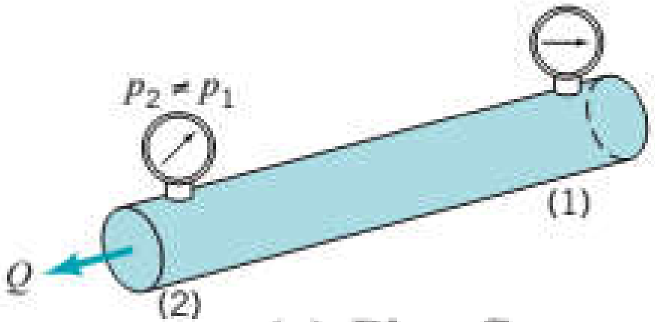
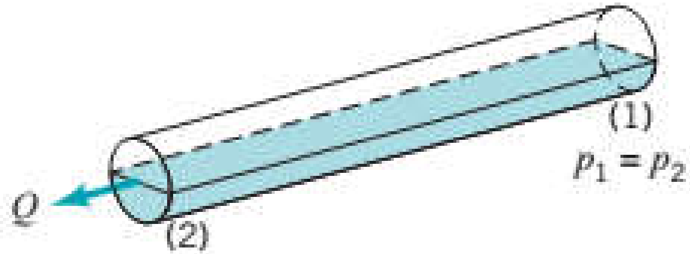
جریان داخل لوله

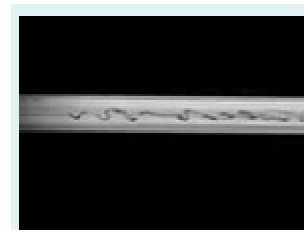
General Characteristics of Pipe Flow



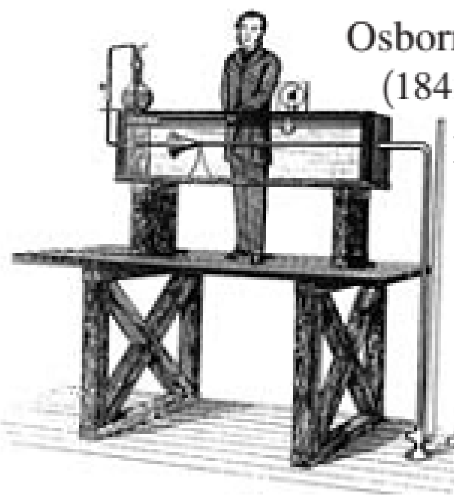
(a) Pipe flow.



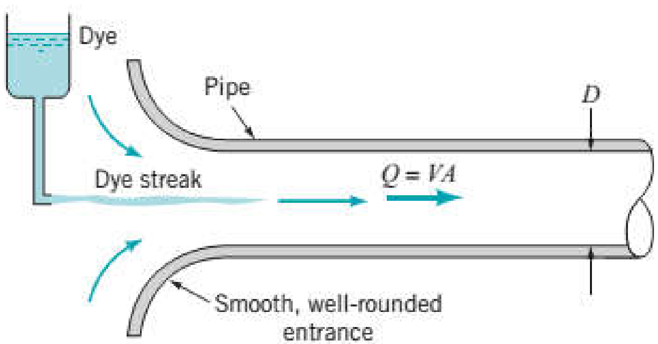
(b) Open-channel flow.

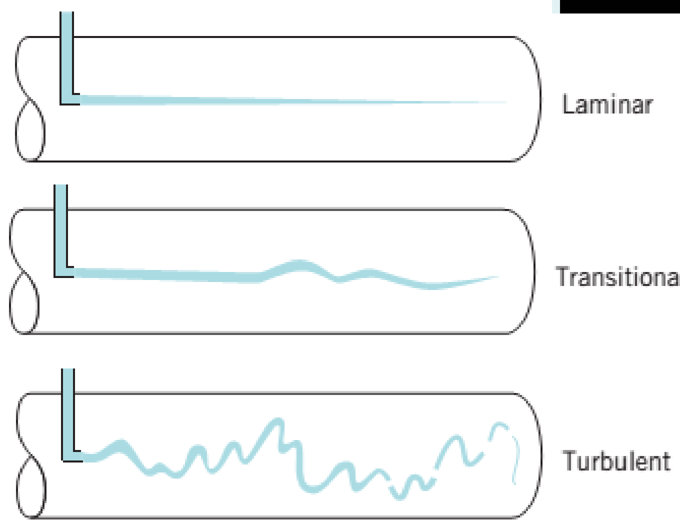


Laminar or Turbulent Flow



Osborne Reynolds
(1842–1912)
British scientist and mathematician





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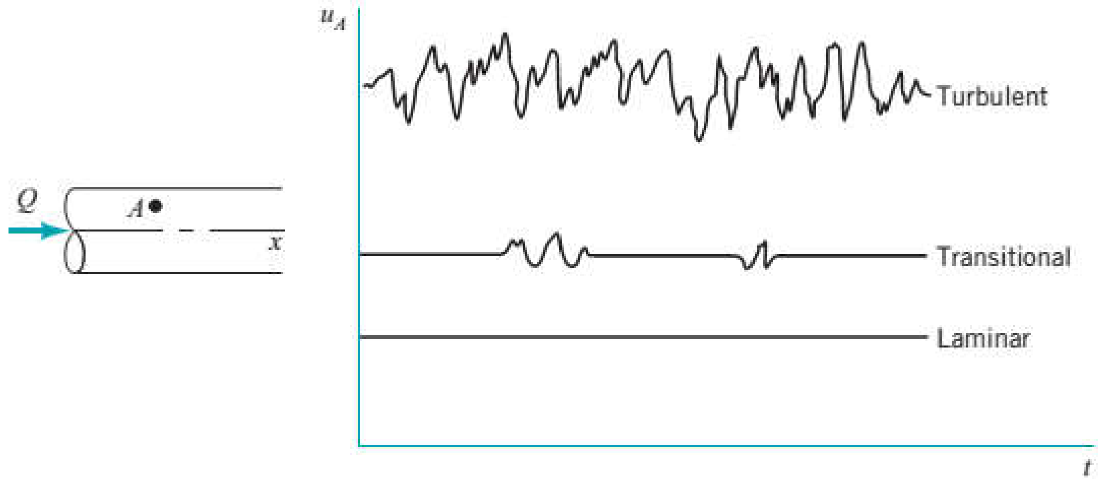
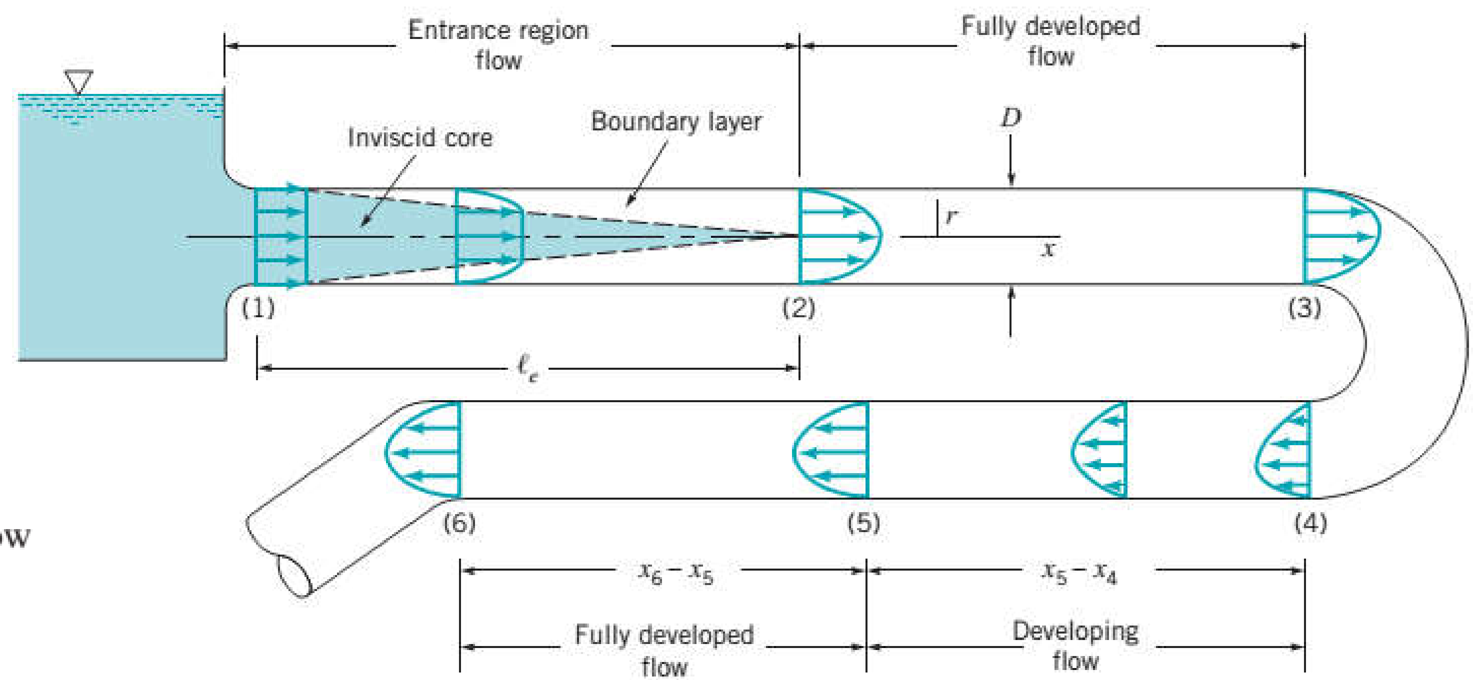


Entrance Region and Fully Developed Flow

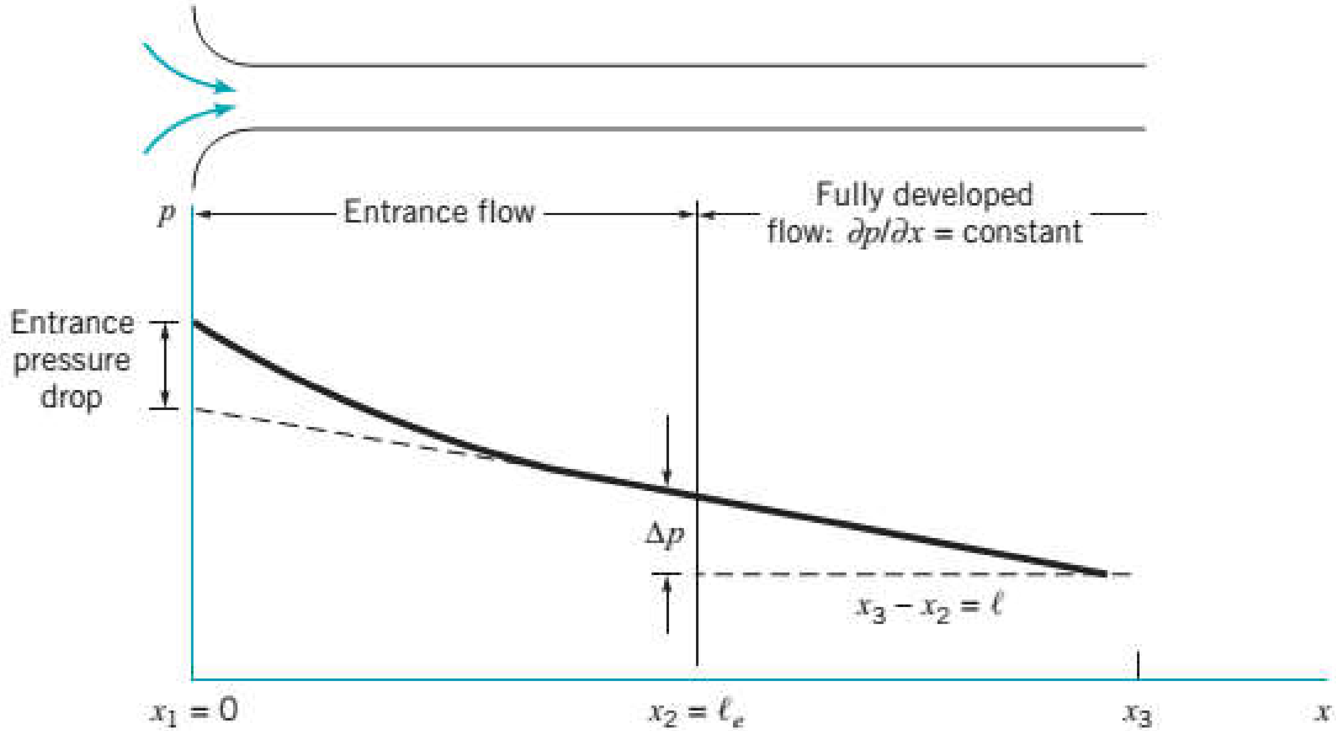
The entrance length is a function of the Reynolds number.

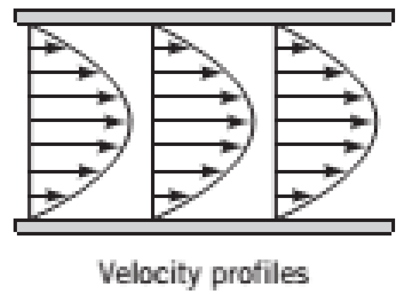


$$\frac{\ell_e}{D} = 0.06 \text{ Re for laminar flow}$$

$$\frac{\ell_e}{D} = 4.4 \,(\text{Re})^{1/6} \text{ for turbulent flow}$$

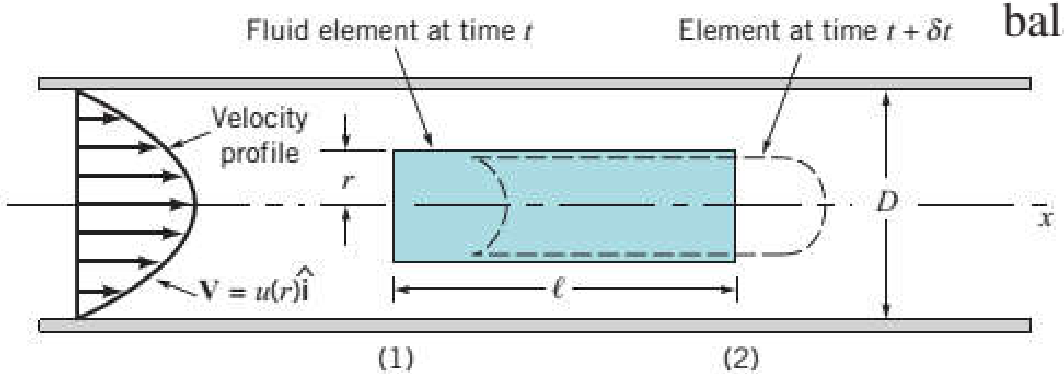
For many practical engineering problems, $10^4 < \text{Re} < 10^5$ → $20D < \ell_e < 30D$.





$a_x = 0$

$$F_x = ma_x$$



balance between pressure and viscous forces

$$(p_1)\pi r^2 - (p_1 - \Delta p)\pi r^2 - (\tau)2\pi r\ell = 0$$

$$\frac{\Delta p}{\ell} = \frac{2\tau}{r}$$

افت فشار و طول تابعی از شعاع نبوده لذا سمت راست رابطه نیز تابعی از شعاع نیست در نتیجه:

$\tau = Cr$, where C is a constant

$p = p_1$ at section (1), it is $p_2 = p_1 - \Delta p$ at section (2)

At $r = 0$ → $(\tau = 0)$ At $r = D/2$ (the pipe wall) → τ = maximum,

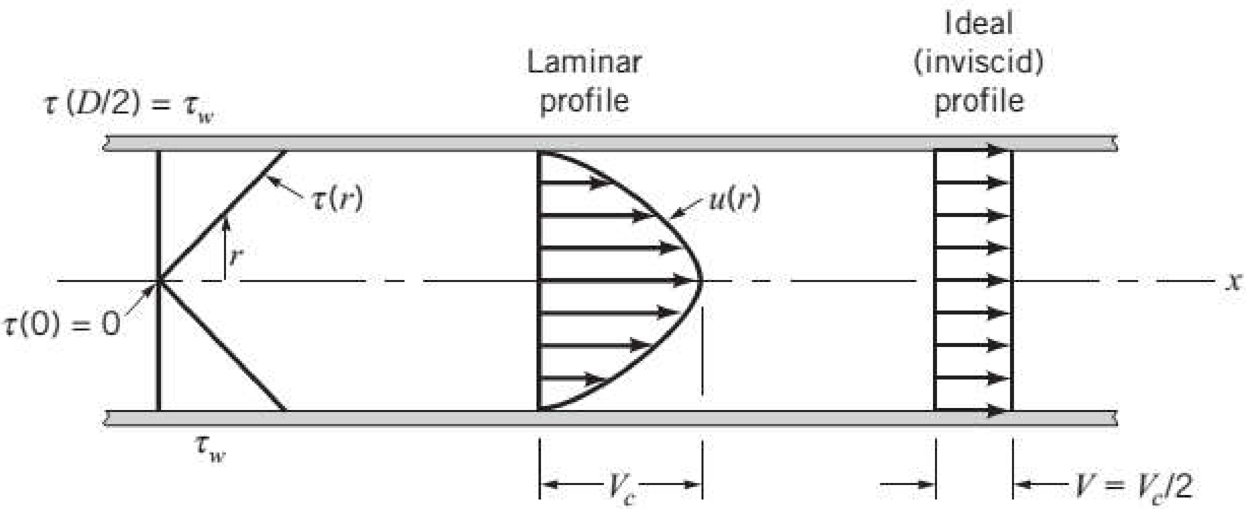
τ_w, the ***wall shear stress.***

$\tau = Cr$, At $r = D/2$ } $C = 2\tau_w/D$ → $\tau = \dfrac{2\tau_w r}{D}$

$$\frac{\Delta p}{\ell} = \frac{2\tau}{r}$$

} $\Delta p = \dfrac{4\ell\tau_w}{D}$

این رابطه بیان می دارد که تنش برشی کوچک می تواند در لوله های طویل، افت فشار بزرگی را نتیجه دهد



$\tau = \mu\, du/dy$

pipe flow $\tau = -\mu \dfrac{du}{dr}$, $du/dr < 0$ } to give $\tau > 0$

$$\Delta p = \frac{4\ell\tau_w}{D}$$

$$\frac{du}{dr} = -\left(\frac{\Delta p}{2\mu\ell}\right) r$$

$$\int du = -\frac{\Delta p}{2\mu\ell}\int r\,dr \rightarrow u = -\left(\frac{\Delta p}{4\mu\ell}\right) r^2 + C_1$$

$u = 0$ $r = D/2$ → $C_1 = (\Delta p/16\mu\ell)D^2$.

$$u(r) = \left(\frac{\Delta p D^2}{16\mu\ell}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right] = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$$

$V_c = \Delta p D^2/(16\mu\ell)$ is the centerline velocity

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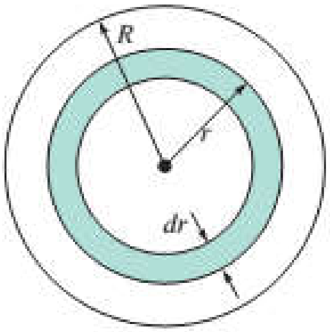
$$u(r) = \frac{\tau_w D}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

This flow is termed ***Hagen–Poiseuille flow.***



$dA = 2\pi r\, dr$

$$Q = \int u\, dA = \int_{r=0}^{r=R} u(r) 2\pi r\, dr = 2\pi V_c \int_0^R \left[1 - \left(\frac{r}{R}\right)^2\right] r\, dr \rightarrow Q = \frac{\pi R^2 V_c}{2}$$

$V = Q/A = Q/\pi R^2$,

the average velocity

$$V = \frac{\pi R^2 V_c}{2\pi R^2} = \frac{V_c}{2} = \frac{\Delta p D^2}{32\mu\ell} \rightarrow \boxed{Q = \frac{\pi D^4 \Delta p}{128\mu\ell}}$$

Poiseuille's law.

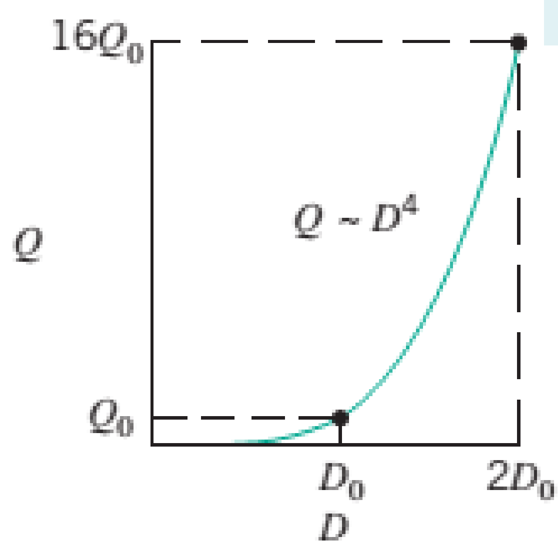
Poiseuille's law is valid for laminar flow only.

$(Q \sim D^4$ or $\delta Q \sim 4D^3\,\delta D$, so that $\delta Q/Q = 4\,\delta D/D)$

2% error in diameter gives an 8% error in flowrate

G. Hagen (1797–1884) in 1839

J. Poiseuille (1799–1869) in 1840



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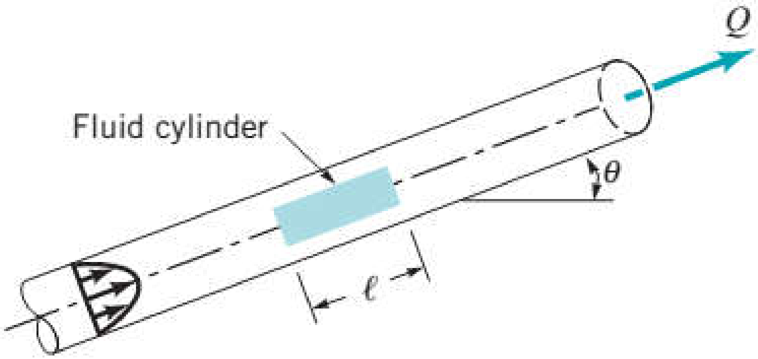

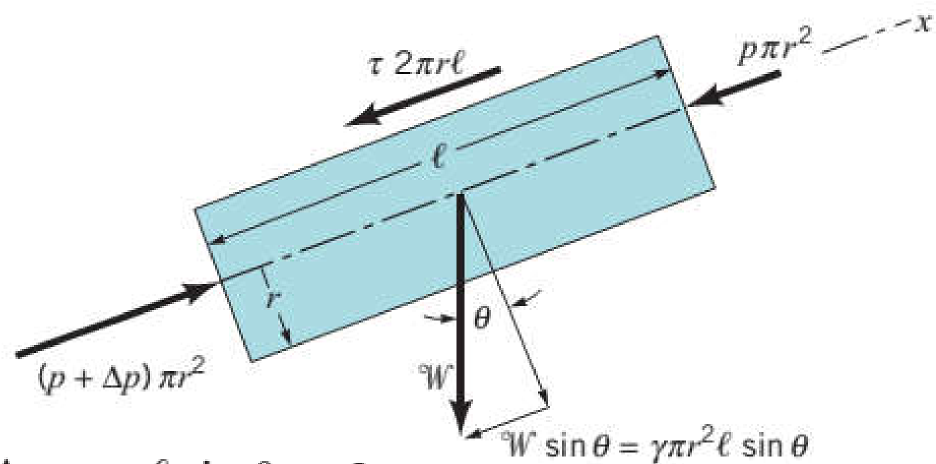

$$\frac{\Delta p - \gamma \ell \sin\theta}{\ell} = \frac{2\tau}{r}$$

$$V = \frac{\Delta p D^2}{32\mu\ell}$$

$$\Delta p - \gamma \ell \sin\theta$$

$$V = \frac{(\Delta p - \gamma \ell \sin\theta) D^2}{32\mu\ell}$$

(a smaller pressure drop is required; $\sin\theta < 0$)

(a larger pressure drop is required; $\sin\theta > 0$).

$$Q = \frac{\pi(\Delta p - \gamma \ell \sin\theta) D^4}{128\mu\ell}$$

GIVEN An oil with a viscosity of $\mu = 0.40\ \text{N} \cdot \text{s/m}^2$ and density $\rho = 900\ \text{kg/m}^3$ flows in a pipe of diameter $D = 0.020$ m.

FIND **(a)** What pressure drop, $p_1 - p_2$, is needed to produce a flowrate of $Q = 2.0 \times 10^{-5}\ \text{m}^3\text{/s}$ if the pipe is horizontal with $x_1 = 0$ and $x_2 = 10$ m?

the flow is laminar

$V = Q/A = (2.0 \times 10^{-5}\ \text{m}^3\text{/s})/[\pi(0.020)^2\text{m}^2/4] = 0.0637\ \text{m/s}$ Reynolds number is $\text{Re} = \rho V D/\mu = 2.87 < 2100$

$\ell = x_2 - x_1 = 10$ m

$$\Delta p = p_1 - p_2 = \frac{128\mu\ell Q}{\pi D^4} = \frac{128(0.40\ \text{N} \cdot \text{s/m}^2)(10.0\ \text{m})(2.0 \times 10^{-5}\ \text{m}^3\text{/s})}{\pi(0.020\ \text{m})^4}$$

$$\Delta p = 20{,}400\ \text{N/m}^2 = 20.4\ \text{kPa}$$

(b) If the pipe is on a hill of angle θ such that $\Delta p = p_1 - p_2 = 0$

$$Q = \frac{\pi(\Delta p - \gamma\ell \sin\theta)D^4}{128\mu\ell}$$

$\Delta p = 0$

$$\rightarrow \sin\theta = -\frac{128\mu Q}{\pi\rho g D^4} \rightarrow \sin\theta = \frac{-128(0.40\ \text{N} \cdot \text{s/m}^2)(2.0 \times 10^{-5}\ \text{m}^3\text{/s})}{\pi(900\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.020\ \text{m})^4}$$

$$\rightarrow \theta = -13.34°$$

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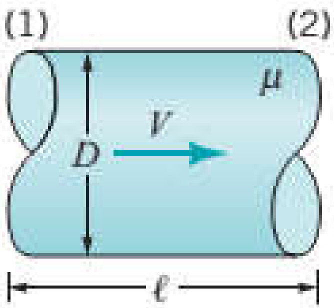
From Dimensional Analysis



$\Delta p = p_1 - p_2 = F(V, \ell, D, \mu)$

$\Delta p = F(V, \ell, D, \mu)$ $\quad k - r = 5 - 3 = 2$ dimensionless groups.

$$\frac{D\,\Delta p}{\mu V} = \phi\left(\frac{\ell}{D}\right)$$

if $\phi(\ell/D) = C\ell/D$ $\rightarrow$ $\frac{D\,\Delta p}{\mu V} = \frac{C\ell}{D}$ $\rightarrow$ $\frac{\Delta p}{\ell} = \frac{C\mu V}{D^2}$

$$Q = AV = \frac{(\pi/4C)\,\Delta p D^4}{\mu\ell}$$

The value of C must be determined

For a round pipe, $C = 32$.

$$\Delta p = 32\mu\ell V/D^2$$

divide both sides by the dynamic pressure, $\rho V^2/2$

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{(32\mu\ell V/D^2)}{\frac{1}{2}\rho V^2} = 64\left(\frac{\mu}{\rho V D}\right)\left(\frac{\ell}{D}\right) = \frac{64}{\text{Re}}\left(\frac{\ell}{D}\right)$$

$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$$

$$f = \Delta p(D/\ell)/(\rho V^2/2)$$

friction factor,

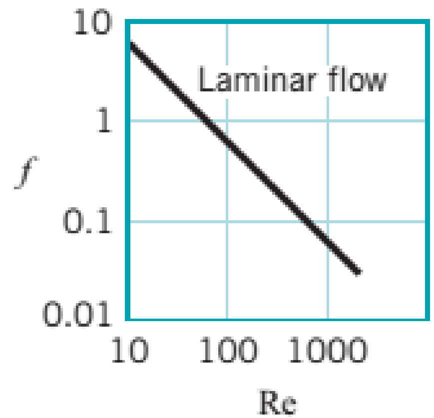
sometimes the *Darcy friction factor* [H. P. G. Darcy (1803–1858)]

$$f = \Delta p(D/\ell)/(\rho V^2/2)$$

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{64}{\text{Re}}\left(\frac{\ell}{D}\right)$$

the friction factor for laminar fully developed pipe flow

$$f = \frac{64}{\text{Re}}$$



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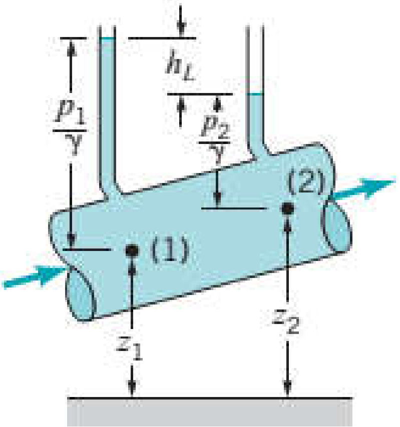
$$\Delta p = \frac{4\ell\tau_w}{D}$$

$$f = \Delta p(D/\ell)/(\rho V^2/2)$$

$$f = \frac{8\tau_w}{\rho V^2}$$

Energy Considerations



$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

for fully developed flow $(\alpha_1 V_1^2/2 = \alpha_2 V_2^2/2)$

$$\left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right) = h_L$$

(recall $p_1 = p_2 + \Delta p$ and $z_2 - z_1 = \ell \sin\theta$).

$$\frac{\Delta p - \gamma\ell \sin\theta}{\ell} = \frac{2\tau}{r}$$

$$h_L = \frac{2\tau\ell}{\gamma r}$$

$$h_L = \frac{2\tau\ell}{\gamma r}$$

$$\tau = \frac{2\tau_w r}{D}$$

$$h_L = \frac{4\ell\tau_w}{\gamma D}$$

it is valid for both laminar and turbu lent flow.

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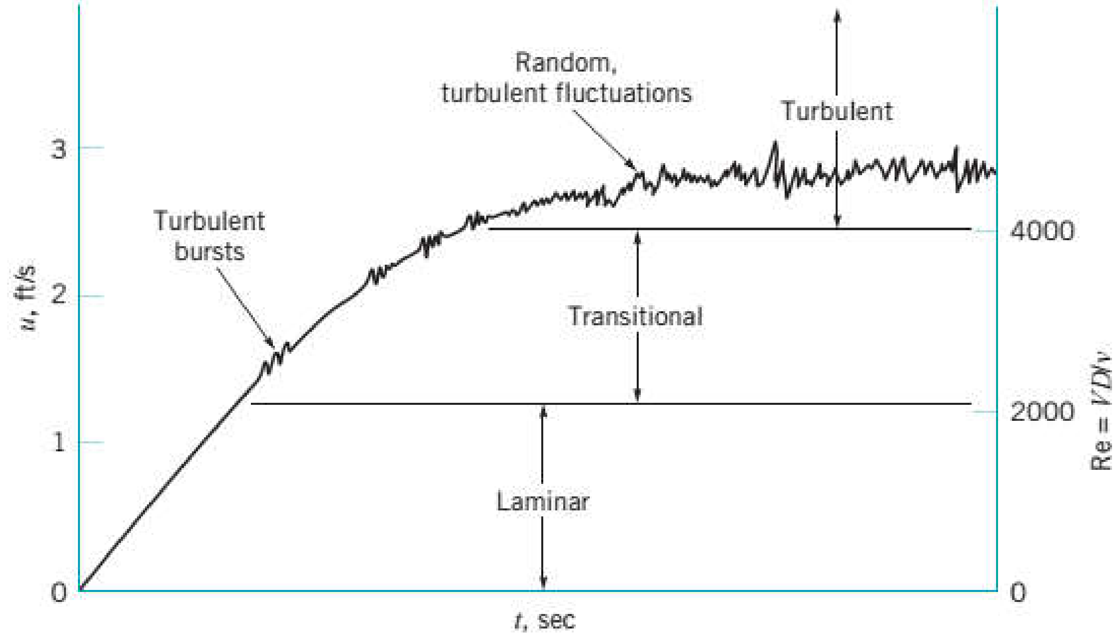
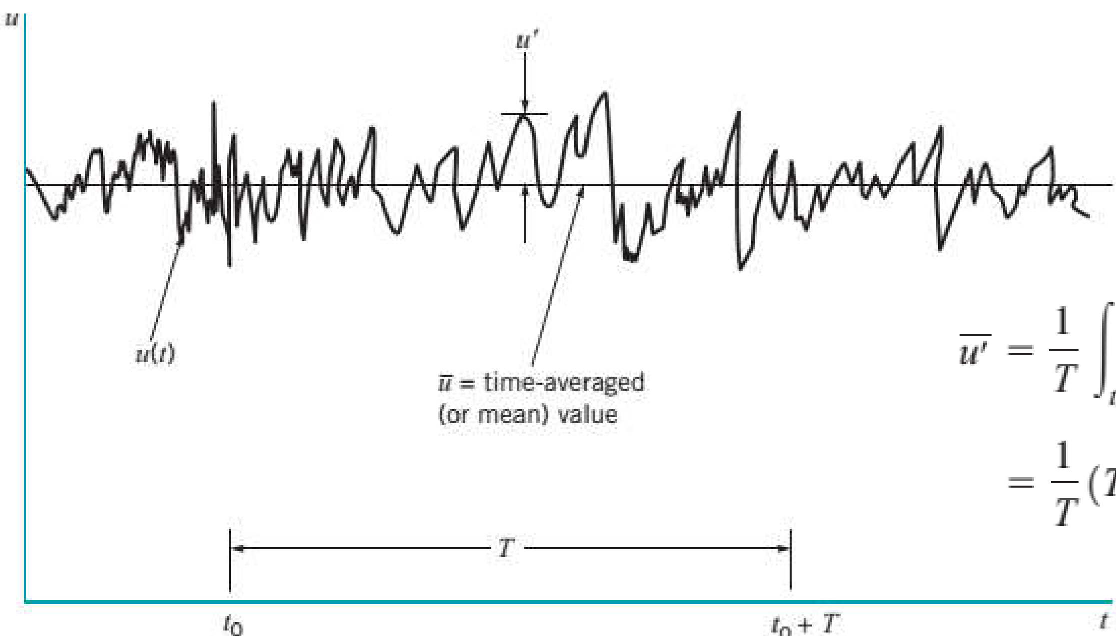
Fully Developed Turbulent Flow

$$\overline{u} = \frac{1}{T}\int_{t_0}^{t_0+T} u(x, y, z, t)\, dt$$

$$u = \overline{u} + u' \quad \text{or} \quad u' = u - \overline{u}$$





$$\overline{u'} = \frac{1}{T}\int_{t_0}^{t_0+T} (u - \overline{u})\, dt = \frac{1}{T}\left(\int_{t_0}^{t_0+T} u\, dt - \overline{u}\int_{t_0}^{t_0+T} dt\right)$$

$$= \frac{1}{T}(T\overline{u} - T\overline{u}) = 0$$

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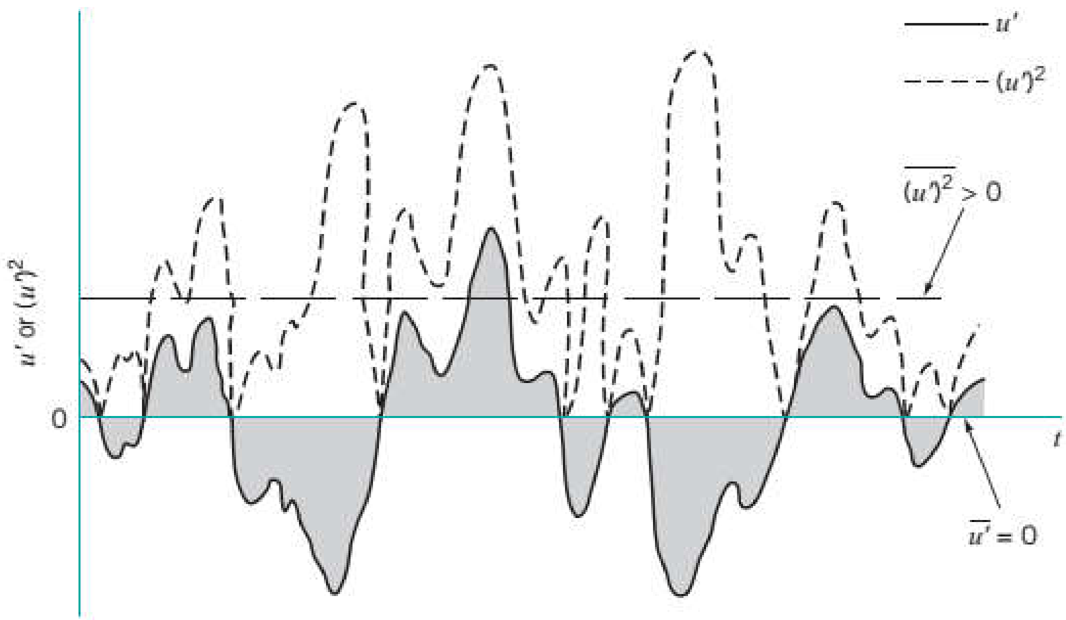
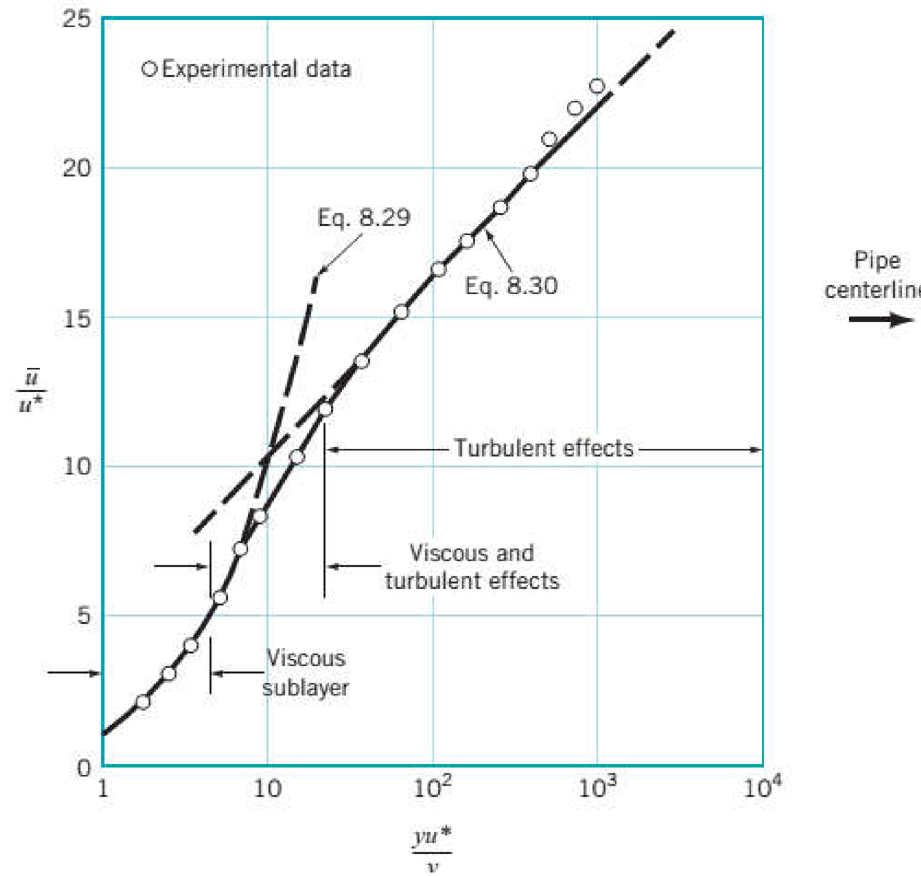


the *turbulence intensity*

$$\mathcal{J} = \frac{\sqrt{\overline{(u')^2}}}{\bar{u}} = \frac{\left[\frac{1}{T}\int_{t_0}^{t_0+T} (u')^2\, dt\right]^{1/2}}{\bar{u}}$$

$$\overline{(u')^2} = \frac{1}{T}\int_{t_0}^{t_0+T} (u')^2\, dt > 0$$



$$\frac{\bar{u}}{u^*} = \frac{yu^*}{\nu}$$

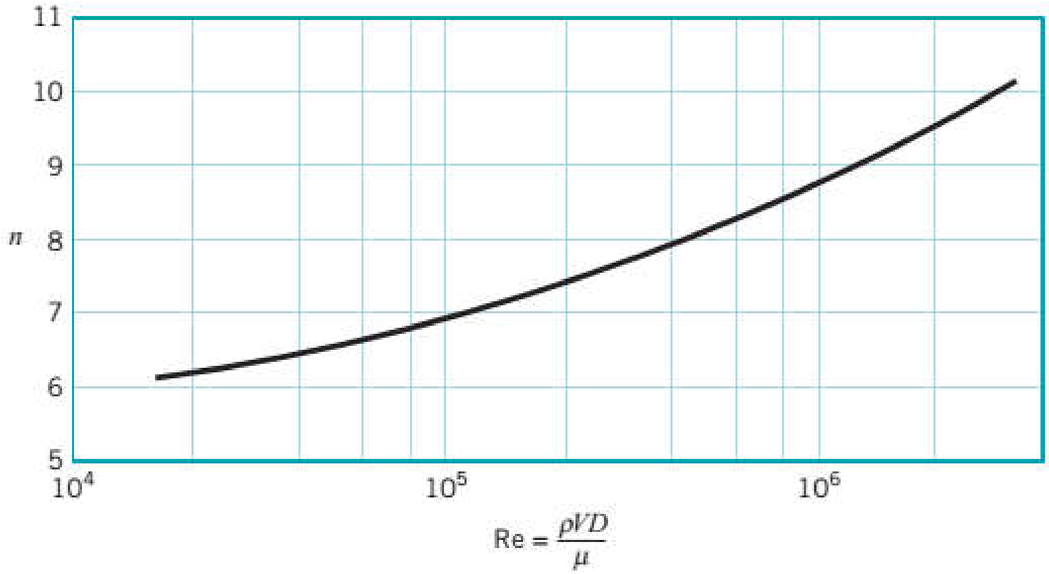
$u^* = (\tau_w/\rho)^{1/2}$ is termed the *friction velocity*.

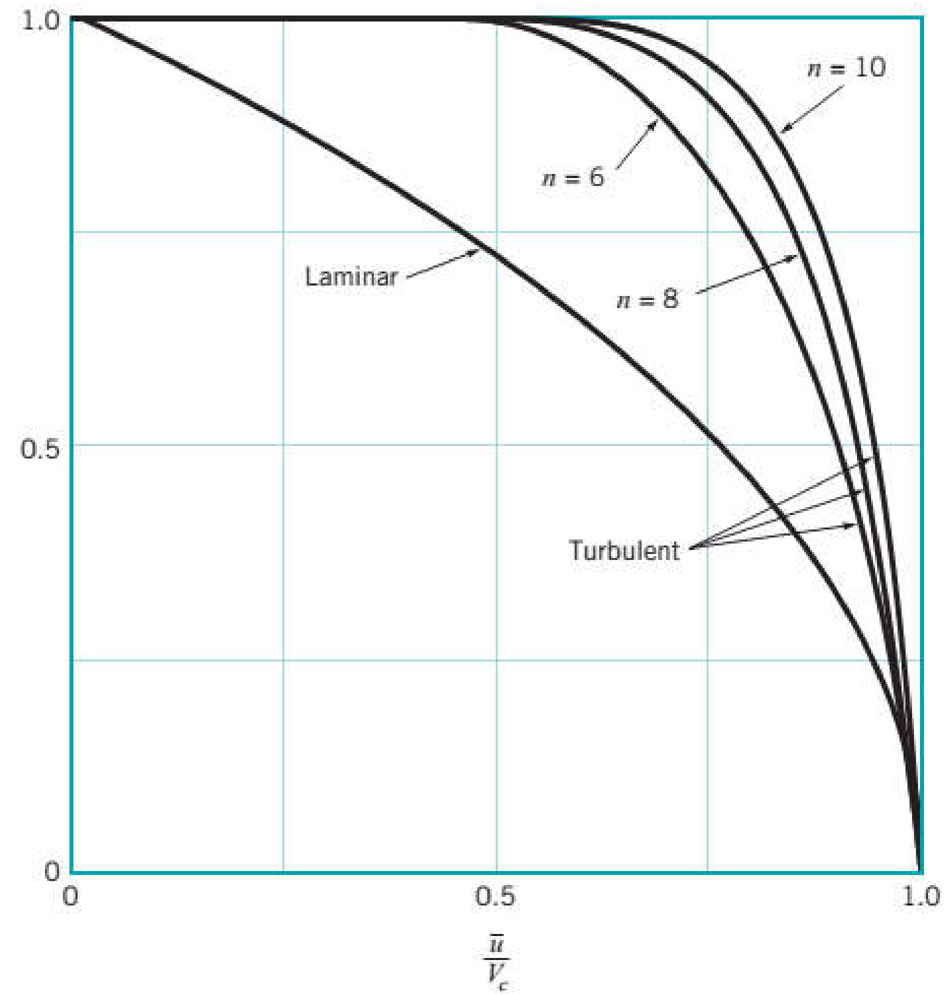
$$\frac{\bar{u}}{u^*} = 2.5 \ln\left(\frac{yu^*}{\nu}\right) + 5.0$$

$$\frac{\bar{u}}{V_c} = \left(1 - \frac{r}{R}\right)^{1/n}$$

correlation is the empirical *power-law velocity profile*

the value of n is a function of the Reynolds number. $(n = 7)$





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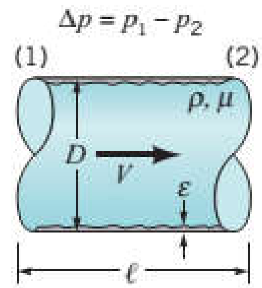
$$h_L = h_{L\,\text{major}} + h_{L\,\text{minor}}$$

major loss — **minor loss**



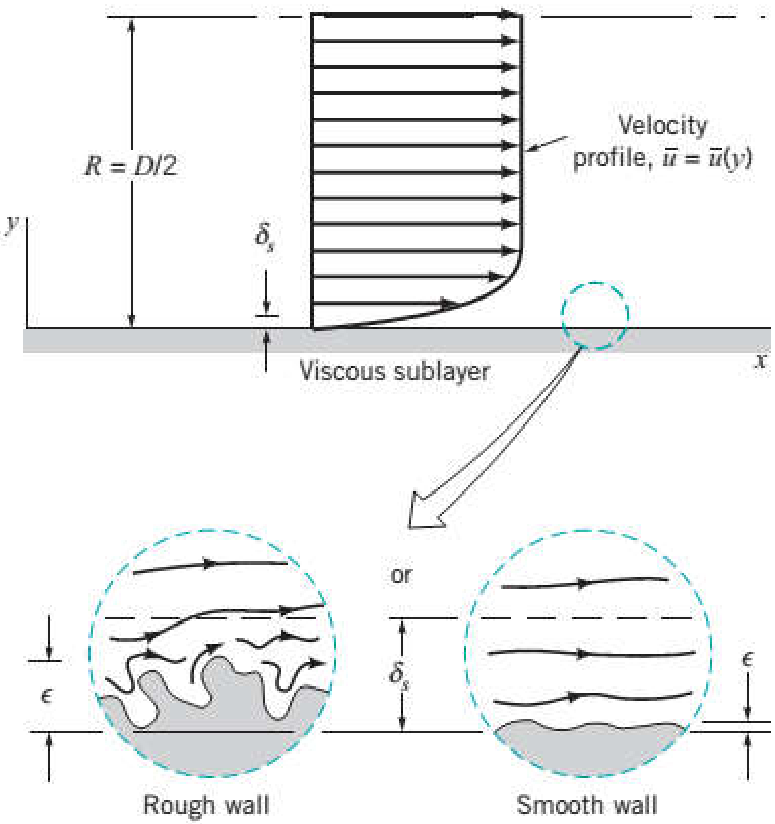
Major Losses

$$\Delta p = F(V, D, \ell, \varepsilon, \mu, \rho)$$

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho V D}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D}\right)$$

Turbulent pipe flow properties depend on the fluid density and the pipe roughness.

the ***relative roughness***, ε/D.



the pressure drop should be proportional to the pipe length.

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\ell}{D}\phi\left(\text{Re}, \frac{\varepsilon}{D}\right)$$

$$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$$

$$f = \phi\left(\text{Re}, \frac{\varepsilon}{D}\right)$$

$$h_{L\,\text{major}} = f\frac{\ell}{D}\frac{V^2}{2g}$$

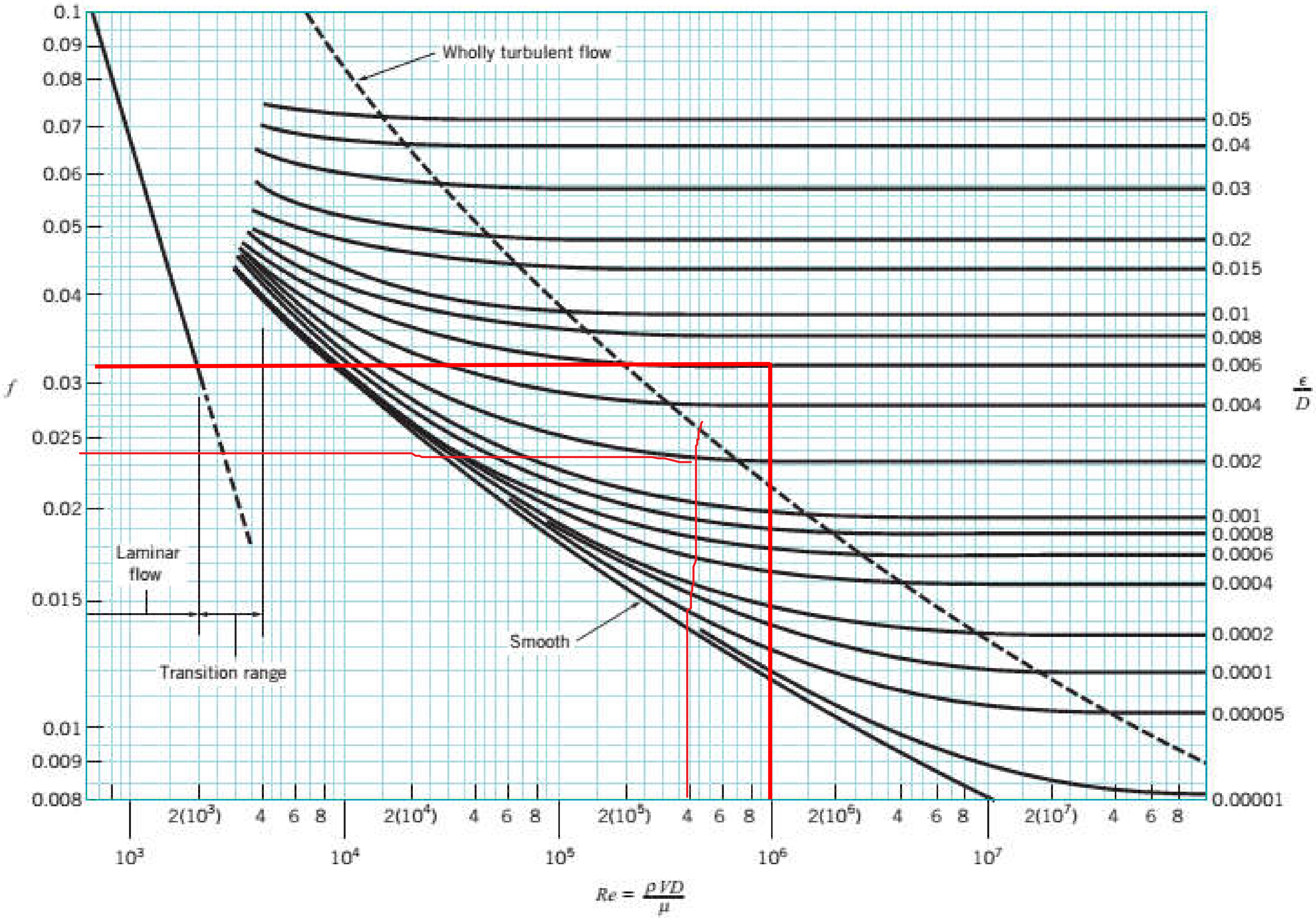


FIGURE 8.20 Friction factor as a function of Reynolds number and relative roughness for round pipes—the Moody chart. (Data from Ref. 7 with permission.)

TABLE 8.1

Equivalent Roughness for New Pipes [From Moody (Ref. 7) and Colebrook (Ref. 8)]

	Equivalent Roughness, ε	
Pipe	**Feet**	**Millimeters**
Riveted steel	0.003–0.03	0.9–9.0
Concrete	0.001–0.01	0.3–3.0
Wood stave	0.0006–0.003	0.18–0.9
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Commercial steel or wrought iron	0.00015	0.045
Drawn tubing	0.000005	0.0015
Plastic, glass	0.0 (smooth)	0.0 (smooth)

equation from Colebrook is valid for the entire nonlaminar range of the Moody

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\mathrm{Re}}\right]$$

GIVEN Air under standard conditions flows through a 4.0-mm-diameter drawn tubing with an average velocity of $V = 50$ m/s. For such conditions the flow would normally be turbulent. However, if precautions are taken to eliminate disturbances to the flow (the entrance to the tube is very smooth, the air is dust free, the tube does not vibrate, etc.), it may be possible to maintain laminar flow.

FIND **(a)** Determine the pressure drop in a 0.1-m section of the tube if the flow is laminar.

(b) Repeat the calculations if the flow is turbulent.

Under standard temperature and pressure conditions the density and viscosity are $\rho = 1.23 \text{ kg/m}^3$ and $\mu = 1.79 \times 10^{-5} \text{ N}\cdot\text{s/m}^2$. Thus, the Reynolds number is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(1.23 \text{ kg/m}^3)(50 \text{ m/s})(0.004 \text{ m})}{1.79 \times 10^{-5} \text{ N}\cdot\text{s/m}^2} = 13{,}700$$

which would normally indicate turbulent flow.

(a) If the flow were laminar, then $f = 64/\text{Re} = 64/13{,}700 = 0.00467$ and the pressure drop in a 0.1-m-long horizontal section of the pipe would be

$$\Delta p = f \frac{\ell}{D} \frac{1}{2} \rho V^2$$

$$= (0.00467) \frac{(0.1 \text{ m})}{(0.004 \text{ m})} \frac{1}{2} (1.23 \text{ kg/m}^3)(50 \text{ m/s})^2$$

$$\Delta p = 0.179 \text{ kPa}$$

(b) If the flow were turbulent, then $f = \phi(\text{Re}, \varepsilon/D)$, where from Table 8.1, $\varepsilon = 0.0015$ mm so that $\varepsilon/D = 0.0015 \text{ mm}/4.0 \text{ mm} = 0.000375$. From the Moody chart with $\text{Re} = 1.37 \times 10^4$ and $\varepsilon/D = 0.000375$ we obtain $f = 0.028$. Thus, the pressure drop in this case would be approximately

$$\Delta p = f \frac{\ell}{D} \frac{1}{2} \rho V^2 = (0.028) \frac{(0.1 \text{ m})}{(0.004 \text{ m})} \frac{1}{2} (1.23 \text{ kg/m}^3)(50 \text{ m/s})^2$$

$$\Delta p = 1.076 \text{ kPa}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) = -2.0 \log\left(\frac{0.000375}{3.7} + \frac{2.51}{1.37 \times 10^4 \sqrt{f}}\right)$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(1.01 \times 10^{-4} + \frac{1.83 \times 10^{-4}}{\sqrt{f}}\right) \rightarrow f = 0.0291$$

$$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\text{Re}}\right] = -1.8 \log\left[\left(\frac{0.000375}{3.7}\right)^{1.11} + \frac{6.9}{1.37 \times 10^4}\right]$$

$$= 0.0289$$

the Blasius formula, in smooth pipes ($\varepsilon/D = 0$) with $\text{Re} < 10^5$ is

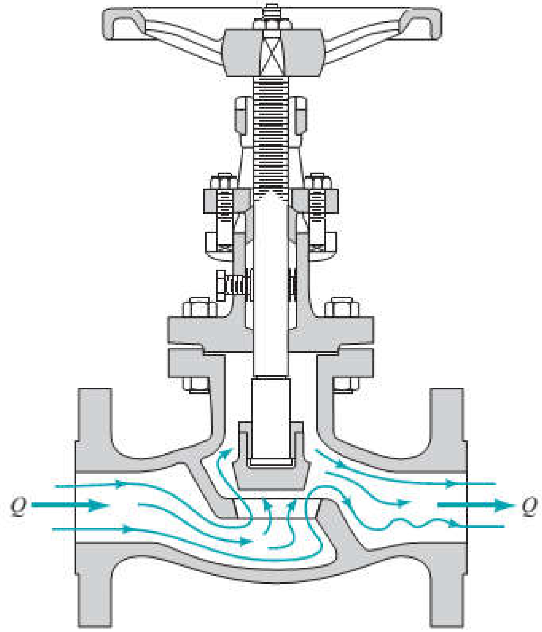
$$f = \frac{0.316}{\text{Re}^{1/4}} \rightarrow f = 0.316(13{,}700)^{-0.25} = 0.0292$$

Minor Losses

the various minor losses that commonly occur in pipe systems ***loss coefficient***, K_L,

$$K_L = \frac{h_{L\ \text{minor}}}{(V^2/2g)} = \frac{\Delta p}{\frac{1}{2}\rho V^2}$$

Losses due to pipe system components are given in terms of loss coefficients.



$$\Delta p = K_L \tfrac{1}{2}\rho V^2$$

$$h_{L\ \text{minor}} = K_L \frac{V^2}{2g}$$

K_L is strongly dependent on the geometry of the component considered.

$$K_L = \phi(\text{geometry}, \text{Re})$$

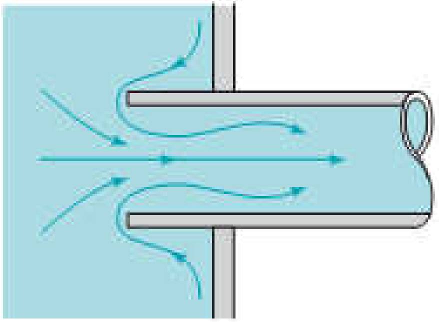
$$\text{Re} = \rho V D/\mu$$

For most flows the loss coefficient is independent of the Reynolds number.

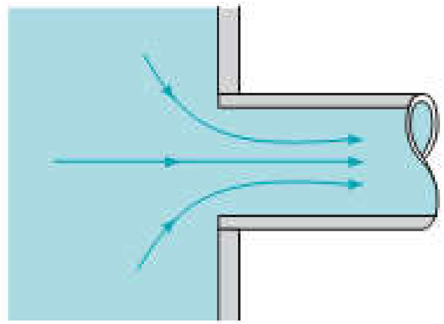
$$K_L = \phi(\text{geometry})$$

$$h_{L\ \text{minor}} = K_L \frac{V^2}{2g} = f \frac{\ell_{eq}}{D} \frac{V^2}{2g}$$

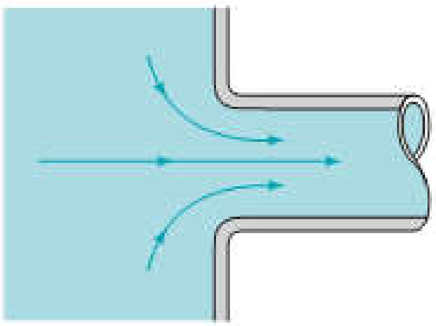
$$\ell_{eq} = \frac{K_L D}{f}$$ *equivalent length*, ℓ_{eq}.



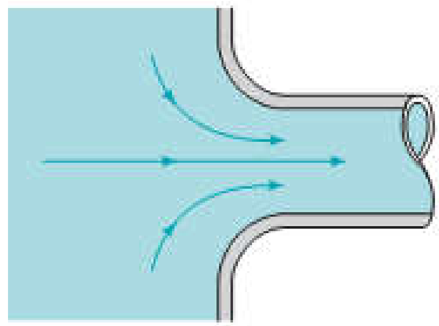
(a) $K_L = 0.8$



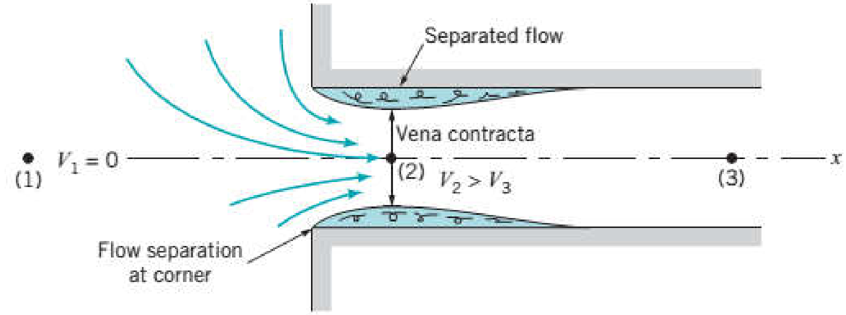
(b) $K_L = 0.5$



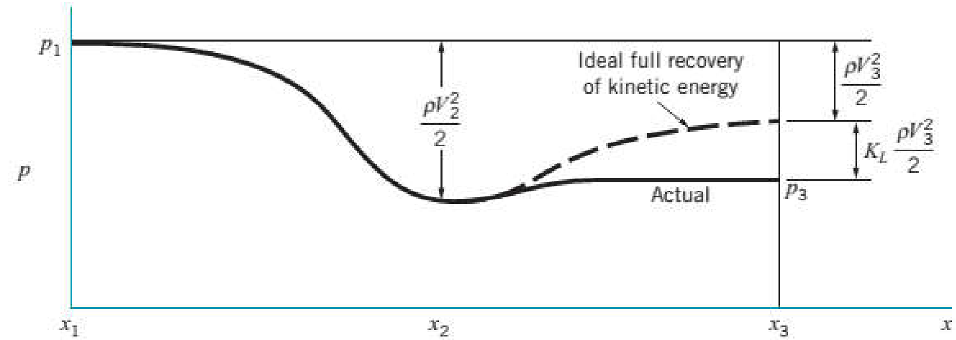
(c) $K_L = 0.2$

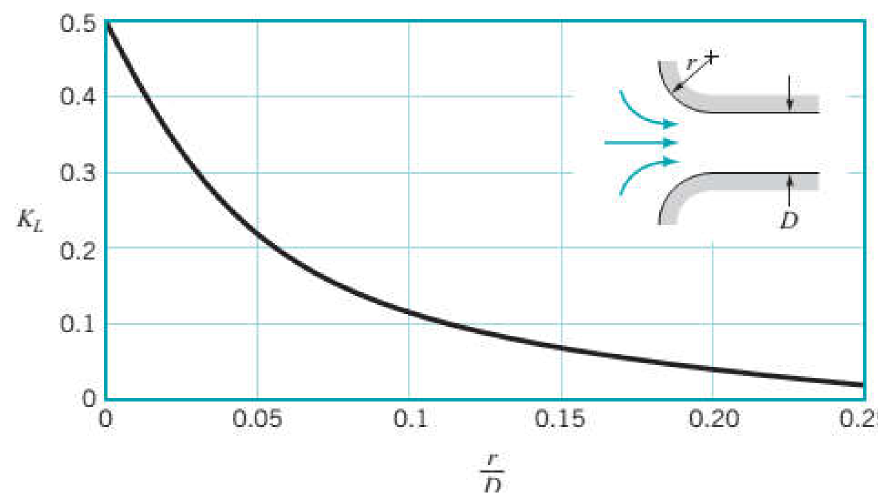


(d) $K_L = 0.04$



(a)





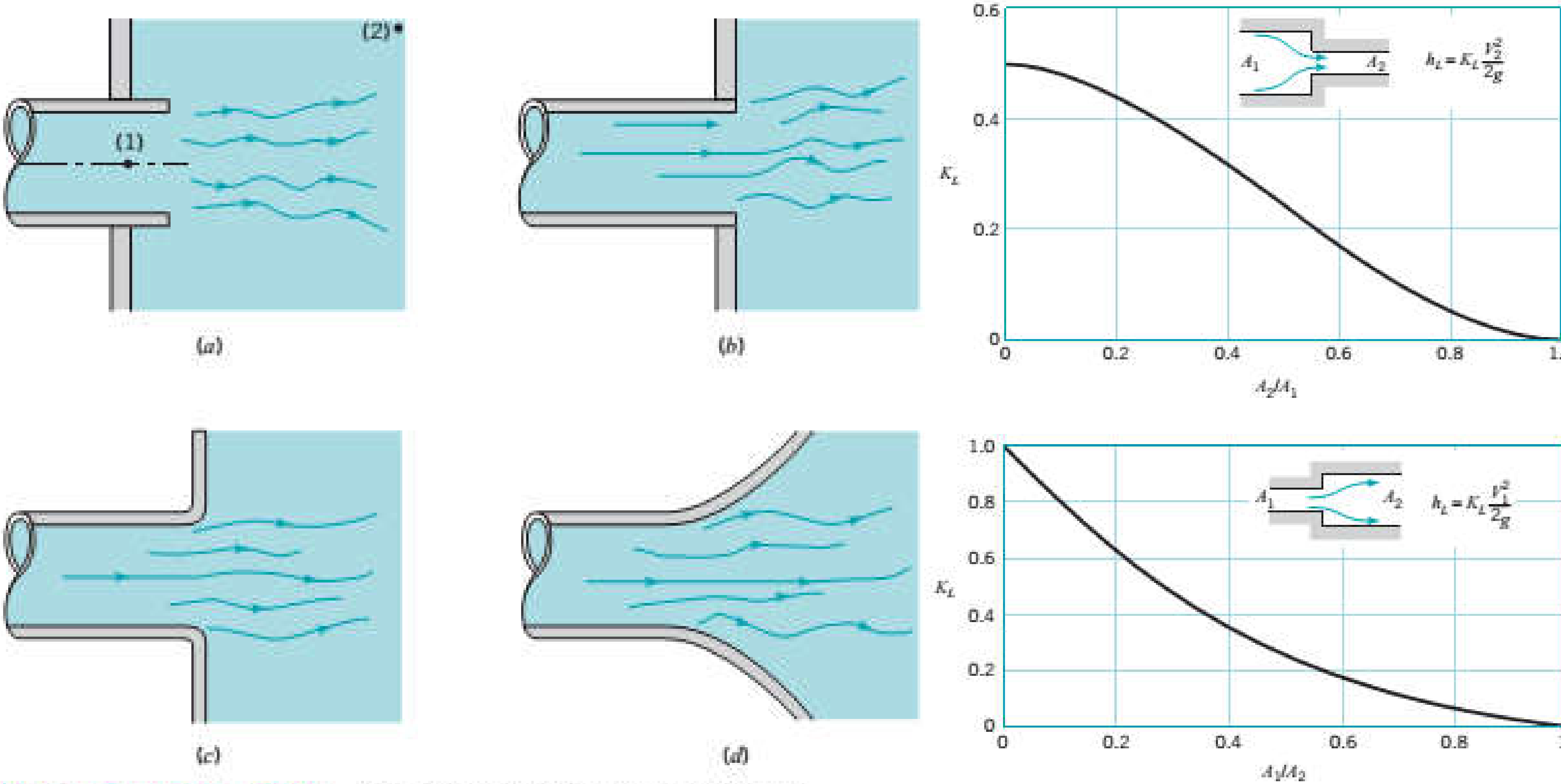
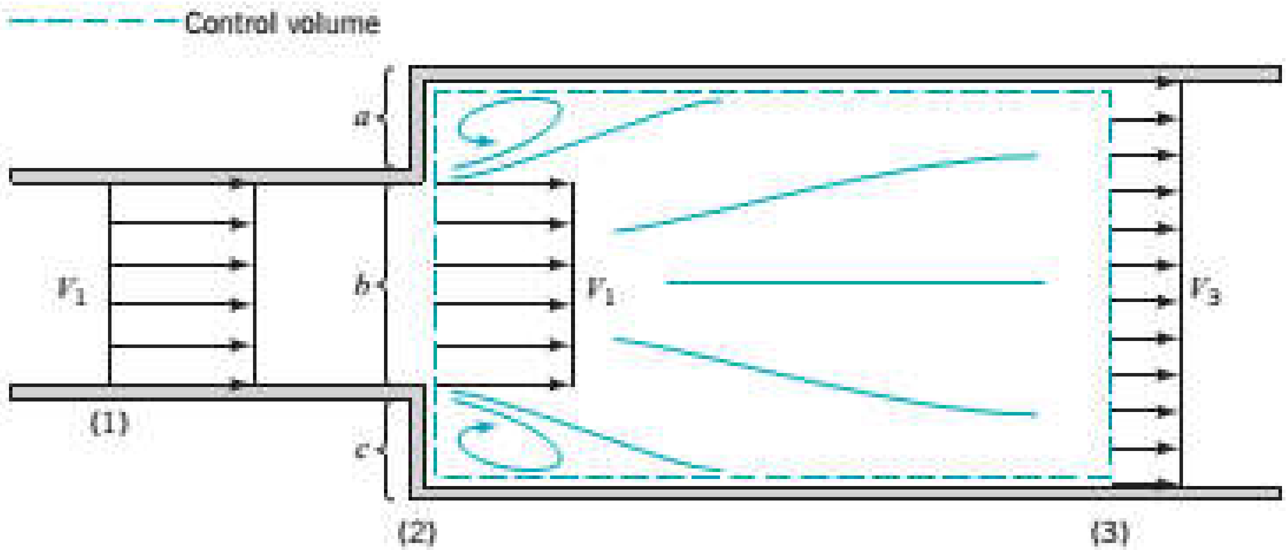


FIGURE 8.25 Exit flow conditions and loss coefficient. (*a*) Reentrant, $K_L = 1.0$, (*b*) sharp-edged, $K_L = 1.0$, (*c*) slightly rounded, $K_L = 1.0$, (*d*) well-rounded, $K_L = 1.0$.



$$A_1V_1 = A_3V_3$$

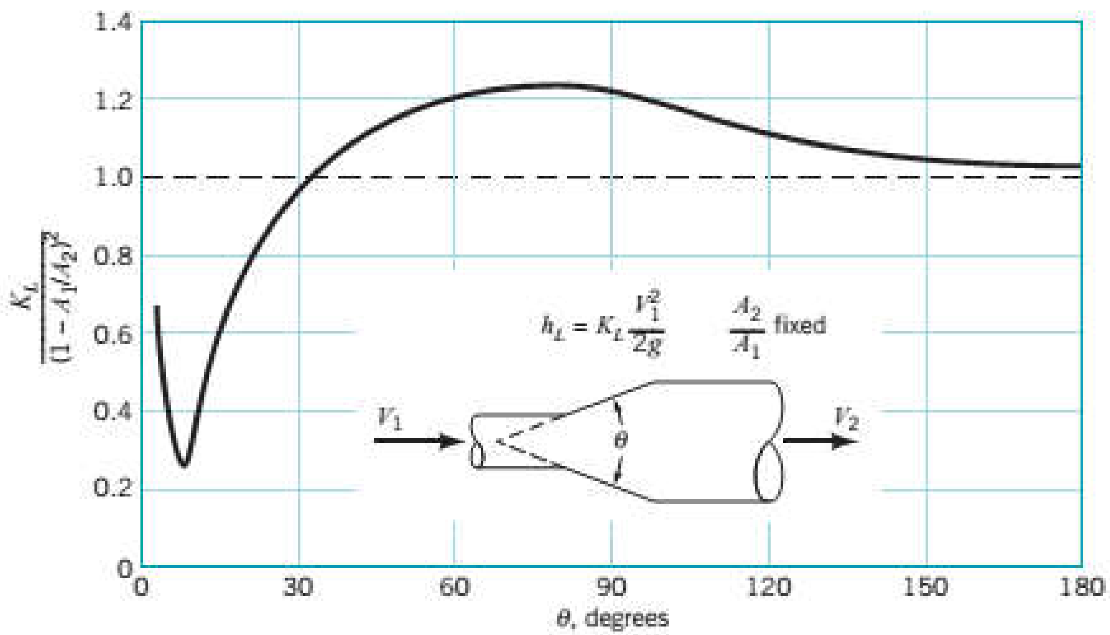
$$p_1A_3 - p_3A_3 = \rho A_3V_3(V_3 - V_1)$$

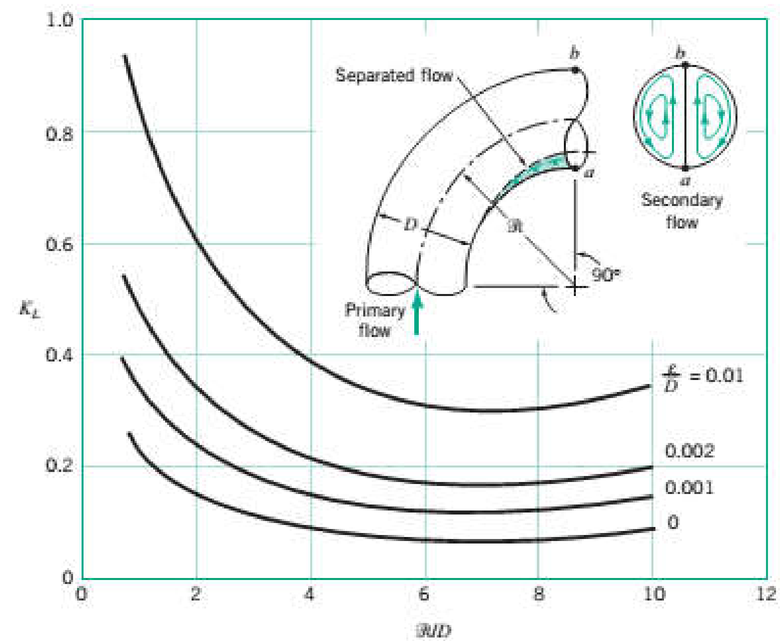
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + h_L$$

$$K_L = h_L/(V_1^2/2g)$$

$$K_L = \left(1 - \frac{A_1}{A_2}\right)^2$$

We assume that the flow is uniform at sections (1), (2), and (3) $(p_a = p_b = p_c = p_1)$





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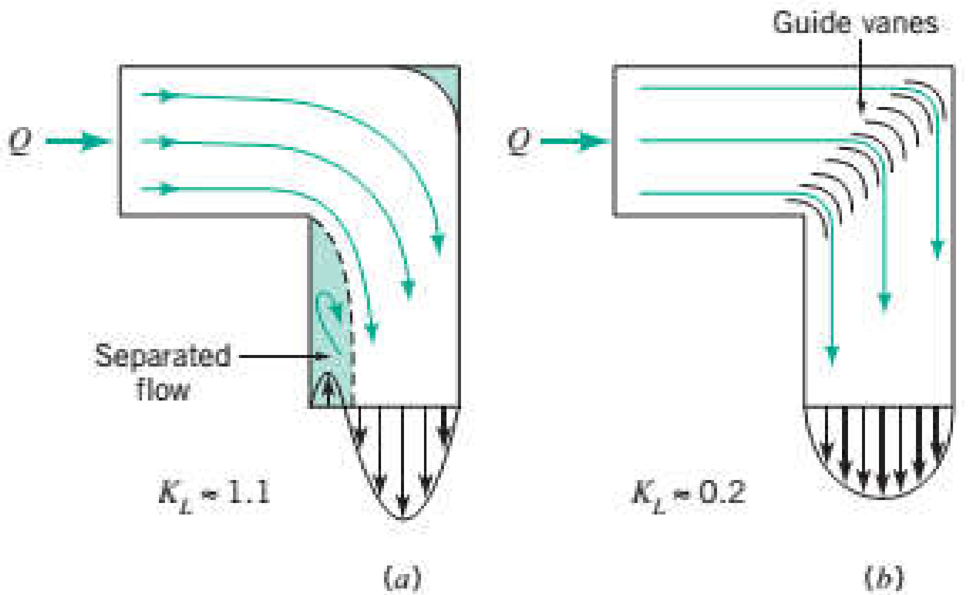
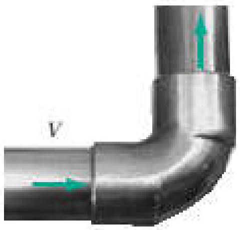


TABLE 8.2

Loss Coefficients for Pipe Components $h_L = K_L(V^2/2g)$. (Data from Refs. 4, 6, 11.)

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5



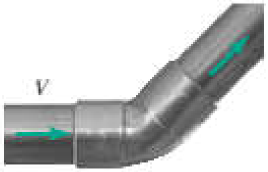
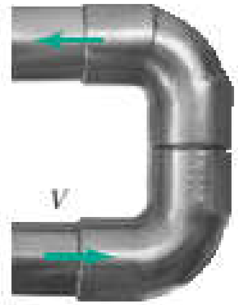
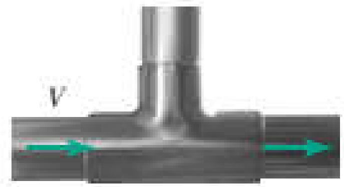


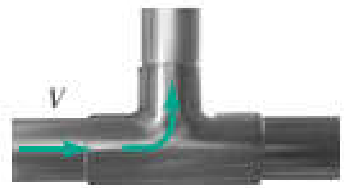
TABLE 8.2

Loss Coefficients for Pipe Components $h_L = K_L(V^2/2g)$. (Data from Refs. 4, 6, 11.)

Component	K_L
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
***e. Valves**	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210







*See Fig. 8.18 for typical valve geometry.

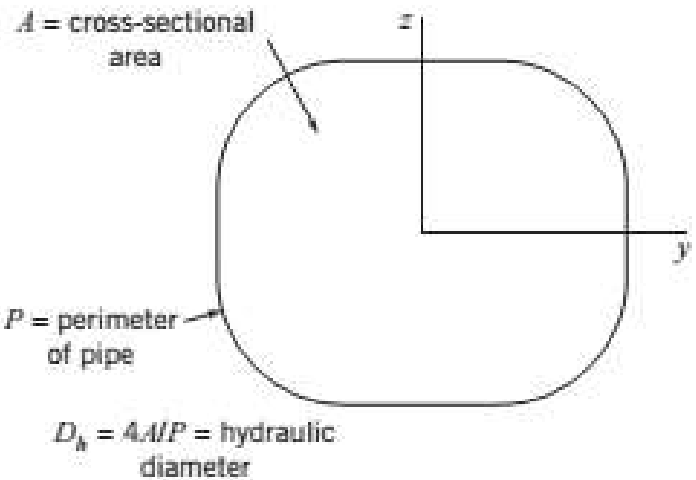
Noncircular Conduits

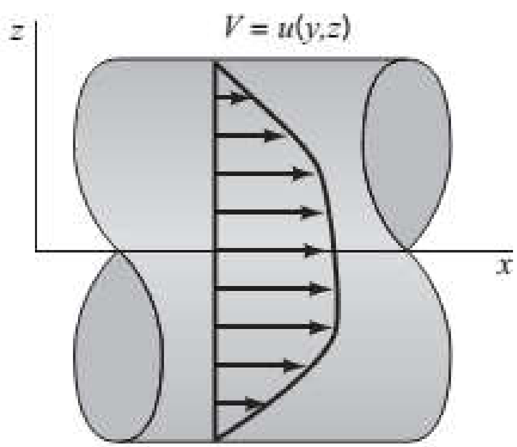
hydraulic diameter defined as $D_h = 4A/P$

for round pipes

$$[D_h = 4A/P = 4(\pi D^2/4)/(\pi D) = D]$$

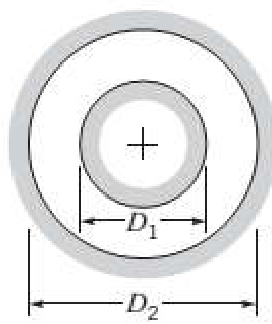
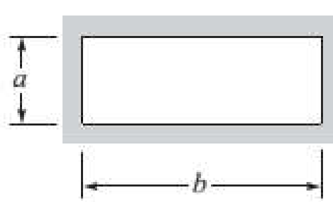
$$h_L = f(\ell/D_h)V^2/2g$$





■ **TABLE 8.3**

Friction Factors for Laminar Flow in Noncircular Ducts (Data from Ref. 18)

Shape	Parameter	$C = f\,\mathrm{Re}_h$
I. Concentric Annulus	D_1/D_2	
$D_h = D_2 - D_1$	0.0001	71.8
	0.01	80.1
	0.1	89.4
	0.6	95.6
	1.00	96.0
II. Rectangle	a/b	
$D_h = \dfrac{2ab}{a+b}$	0	96.0
	0.05	89.9
	0.10	84.7
	0.25	72.9
	0.50	62.2
	0.75	57.9
	1.00	56.9

Calculations for fully developed turbulent flow in ducts of noncircular cross section are usually carried out by using the Moody chart data for round pipes with the diameter replaced by the hydraulic diameter and the Reynolds number based on the hydraulic diameter. Such calculations are usually accurate to within about 15%. If greater accuracy is needed, a more detailed analysis based on the specific geometry of interest is needed.

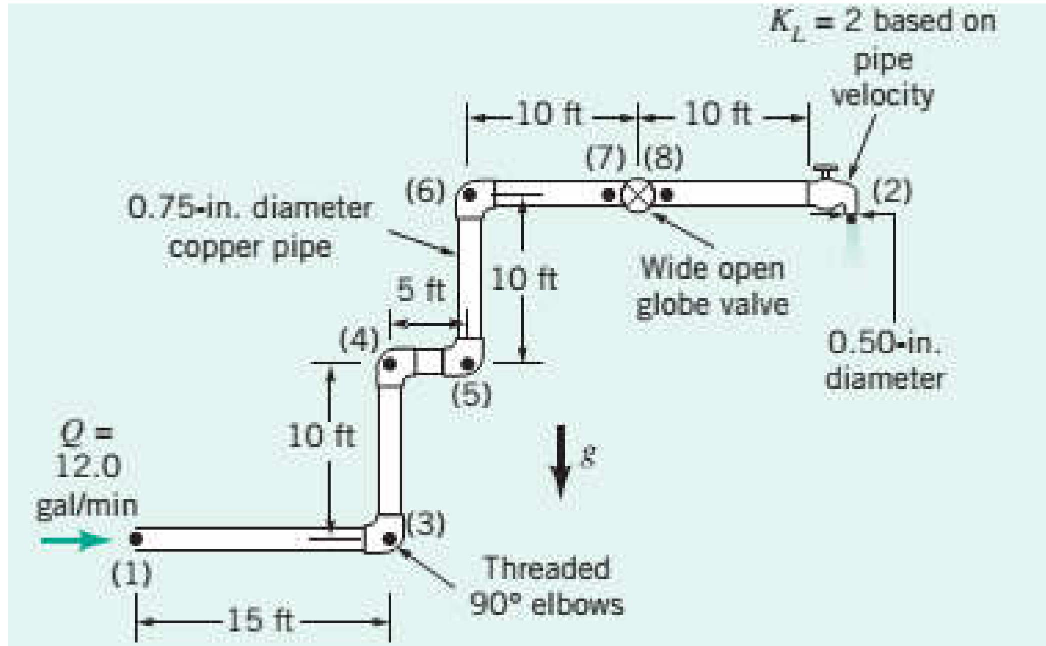
GIVEN Water at 60 °F flows from the basement to the second floor through the 0.75-in. (0.0625-ft)-diameter copper pipe (a drawn tubing) at a rate of $Q = 12.0$ gal/min $= 0.0267$ ft³/s and exits through a faucet of diameter 0.50 in. as shown in Fig. E8.6*a*.

FIND Determine the pressure at point (1) if

(a) all losses are neglected,

(b) the only losses included are major losses, or

(c) all losses are included.



Since the fluid velocity in the pipe is given by $V_1 = Q/A_1 = Q/(\pi D^2/4) = (0.0267 \text{ ft}^3/\text{s})/[\pi(0.0625 \text{ ft})^2/4] = 8.70$ ft/s, and the fluid properties are $\rho = 1.94$ slugs/ft³ and $\mu = 2.34 \times 10^{-5}$ lb · s/ft² (see Table B.1), it follows that $\text{Re} = \rho VD/\mu = (1.94 \text{ slugs/ft}^3)(8.70 \text{ ft/s})(0.0625 \text{ ft})/(2.34 \times 10^{-5} \text{ lb}\cdot\text{s/ft}^2) = 45{,}000$. Thus, the flow is turbulent. The governing equation for case **(a)**, **(b)**, or **(c)** is the energy equation as given by Eq. 5.59,

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

$V_2 = Q/A_2 = 19.6$ ft/s

α_1 and α_2 are unity.

where $z_1 = 0$, $z_2 = 20$ ft, $p_2 = 0$ (free jet), $\gamma = \rho g = 62.4$ lb/ft³.

$$p_1 = \gamma z_2 + \tfrac{1}{2}\rho(V_2^2 - V_1^2) + \gamma h_L$$

(a) If all losses are neglected ($h_L = 0$),

$$p_1 = (62.4 \text{ lb/ft}^3)(20 \text{ ft}) + \frac{1.94 \text{ slugs/ft}^3}{2}[(19.6 \text{ ft/s})^2 - (8.70 \text{ ft/s})^2]$$

$$= (1248 + 299) \text{ lb/ft}^2 = 1547 \text{ lb/ft}^2$$

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(b) If the only losses included are the major losses, the head loss is

$$h_L = f\frac{\ell}{D}\frac{V_1^2}{2g}$$

$$p_1 = \gamma z_2 + \frac{1}{2}\rho(V_2^2 - V_1^2) + \rho f\frac{\ell}{D}\frac{V_1^2}{2}$$

$$= (1248 + 299)\ \text{lb/ft}^2$$

$$+ (1.94\ \text{slugs/ft}^3)(0.0215)\left(\frac{60\ \text{ft}}{0.0625\ \text{ft}}\right)\frac{(8.70\ \text{ft/s})^2}{2}$$

$$= (1248 + 299 + 1515)\ \text{lb/ft}^2 = 3062\ \text{lb/ft}^2$$

(c) If major and minor losses are included,

$$p_1 = \gamma z_2 + \frac{1}{2}\rho(V_2^2 - V_1^2) + f\gamma\frac{\ell}{D}\frac{V_1^2}{2g} + \sum \rho K_L\frac{V^2}{2}$$

($K_L = 1.5$ for each elbow

$K_L = 10$ for the wide-open globe valve)

$$\sum \rho K_L\frac{V^2}{2} = (1.94\ \text{slugs/ft}^3)\frac{(8.70\ \text{ft/s})^2}{2}[10 + 4(1.5) + 2]$$

$$= 1321\ \text{lb/ft}^2$$

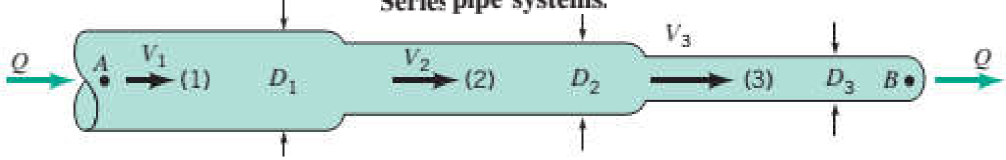
$$\sum \rho K_L\frac{V^2}{2} = 9.17\ \text{psi}$$

Multiple Pipe Systems

$$Q_1 = Q_2 = Q_3$$

$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$

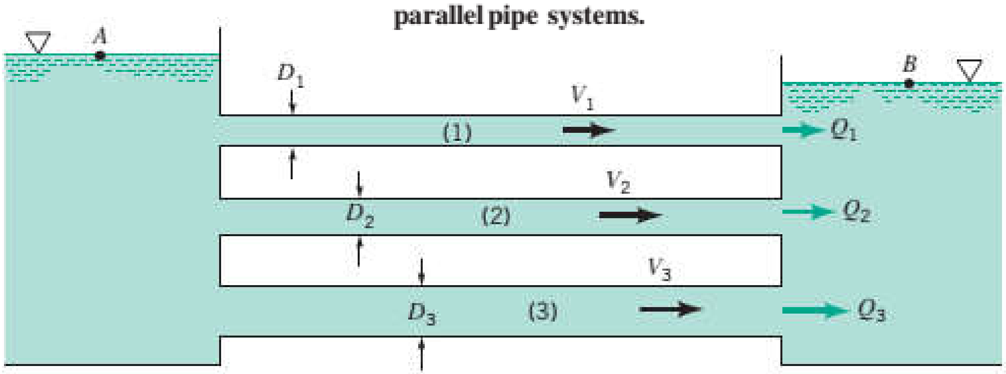
Series pipe systems.



parallel pipe systems.

$$Q = Q_1 + Q_2 + Q_3$$

$$h_{L_1} = h_{L_2} = h_{L_3}$$



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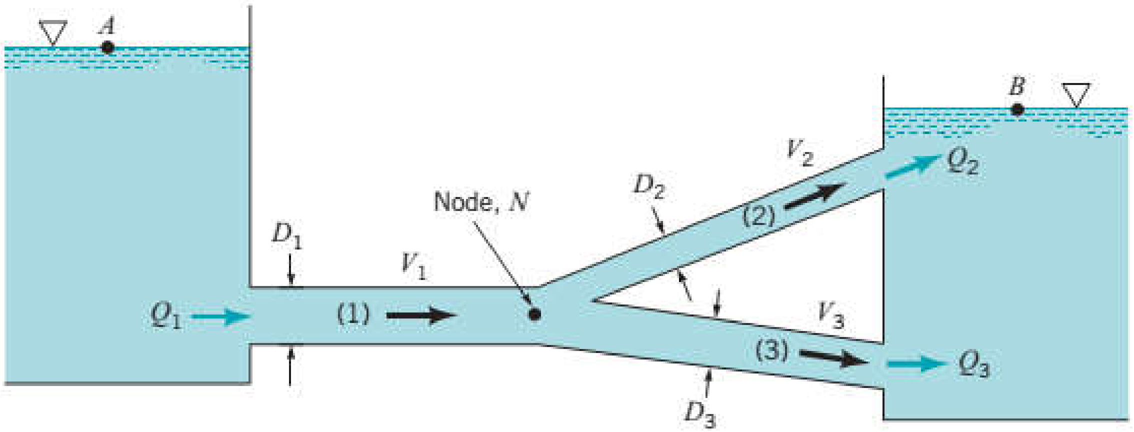
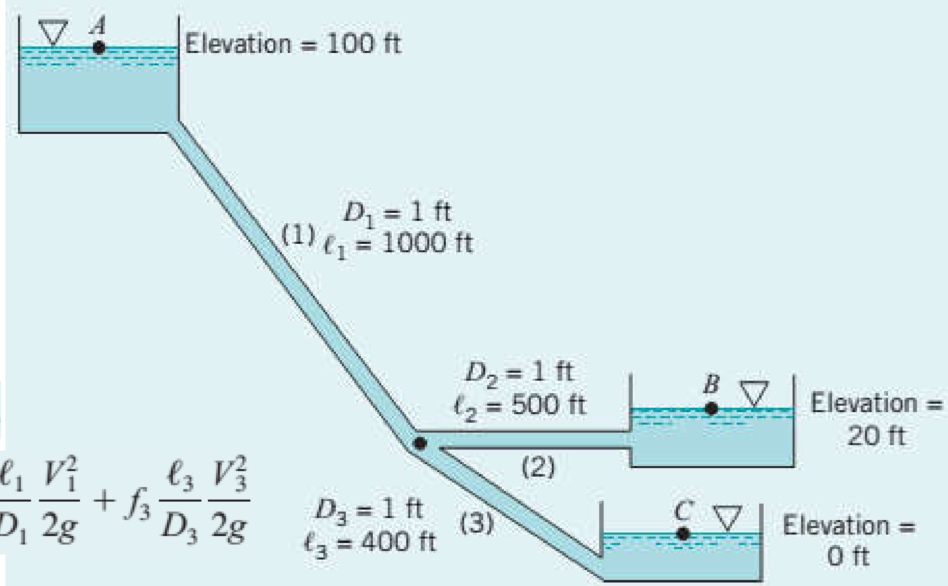


$$Q_1 = Q_2 + Q_3$$

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_2}$$

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_3}$$

GIVEN Three reservoirs are connected by three pipes as are shown in Fig. E8.14. For simplicity we assume that the diameter of each pipe is 1 ft, the friction factor for each is 0.02, and because of the large length-to-diameter ratio, minor losses are negligible.



FIND Determine the flowrate into or out of each reservoir.

$Q_1 + Q_2 = Q_3$, the diameters are the same for each pipe

$V_1 + V_2 = V_3$

from A to C

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

$p_A = p_C = V_A = V_C = z_C = 0,$ $\Rightarrow$

$$z_A = f_1 \frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

$$100 \text{ ft} = \frac{0.02}{2(32.2 \text{ ft/s}^2)} \frac{1}{(1 \text{ ft})} [(1000 \text{ ft})V_1^2 + (400 \text{ ft})V_3^2]$$

$$322 = V_1^2 + 0.4V_3^2$$

from B and C is

$$\frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + f_2 \frac{\ell_2}{D_2}\frac{V_2^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

$$z_B = f_2 \frac{\ell_2}{D_2}\frac{V_2^2}{2g} + f_3 \frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$ $\Rightarrow$

$$64.4 = 0.5V_2^2 + 0.4V_3^2$$

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a trial-and-error solution

$V_2 = 2.88 \text{ ft/s}$

$V_1 = 15.9 \text{ ft/s}$

$$Q_1 = A_1V_1 = \frac{\pi}{4}D_1^2V_1 = \frac{\pi}{4}(1 \text{ ft})^2(15.9 \text{ ft/s}) = 12.5 \text{ ft}^3\text{/s from } A$$

$$Q_2 = A_2V_2 = \frac{\pi}{4}D_2^2V_2 = \frac{\pi}{4}(1 \text{ ft})^2(2.88 \text{ ft/s}) = 2.26 \text{ ft}^3\text{/s into } B$$

$$Q_3 = Q_1 - Q_2 = (12.5 - 2.26) \text{ ft}^3\text{/s} = 10.2 \text{ ft}^3\text{/s into } C$$

Entrance length	$\frac{\ell_e}{D} = 0.06 \text{ Re for laminar flow}$ $\frac{\ell_e}{D} = 4.4 (\text{Re})^{1/6} \text{ for turbulent flow}$
Pressure drop for fully developed laminar pipe flow	$\Delta p = \frac{4\ell\tau_w}{D}$
Velocity profile for fully developed laminar pipe flow	$u(r) = \left(\frac{\Delta p D^2}{16\mu\ell}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right] = V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$
Volume flowrate for fully developed laminar pipe flow	$Q = \frac{\pi D^4 \Delta p}{128\mu\ell}$
Friction factor for fully developed laminar pipe flow	$f = \frac{64}{\text{Re}}$
Pressure drop for a horizontal pipe	$\Delta p = f\frac{\ell}{D}\frac{\rho V^2}{2}$
Head loss due to major losses	$h_{L\text{ major}} = f\frac{\ell}{D}\frac{V^2}{2g}$

Colebrook formula	$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$
Explicit alternative to Colebrook formula	$\frac{1}{\sqrt{f}} = -1.8 \log\left[\left(\frac{\varepsilon/D}{3.7}\right)^{1.11} + \frac{6.9}{\text{Re}}\right]$
Head loss due to minor losses	$h_{L\text{ minor}} = K_L\frac{V^2}{2g}$