بسم الله الرحمن الرحیم

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Turbulent Flow

By:

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What is Turbulent Fluid?

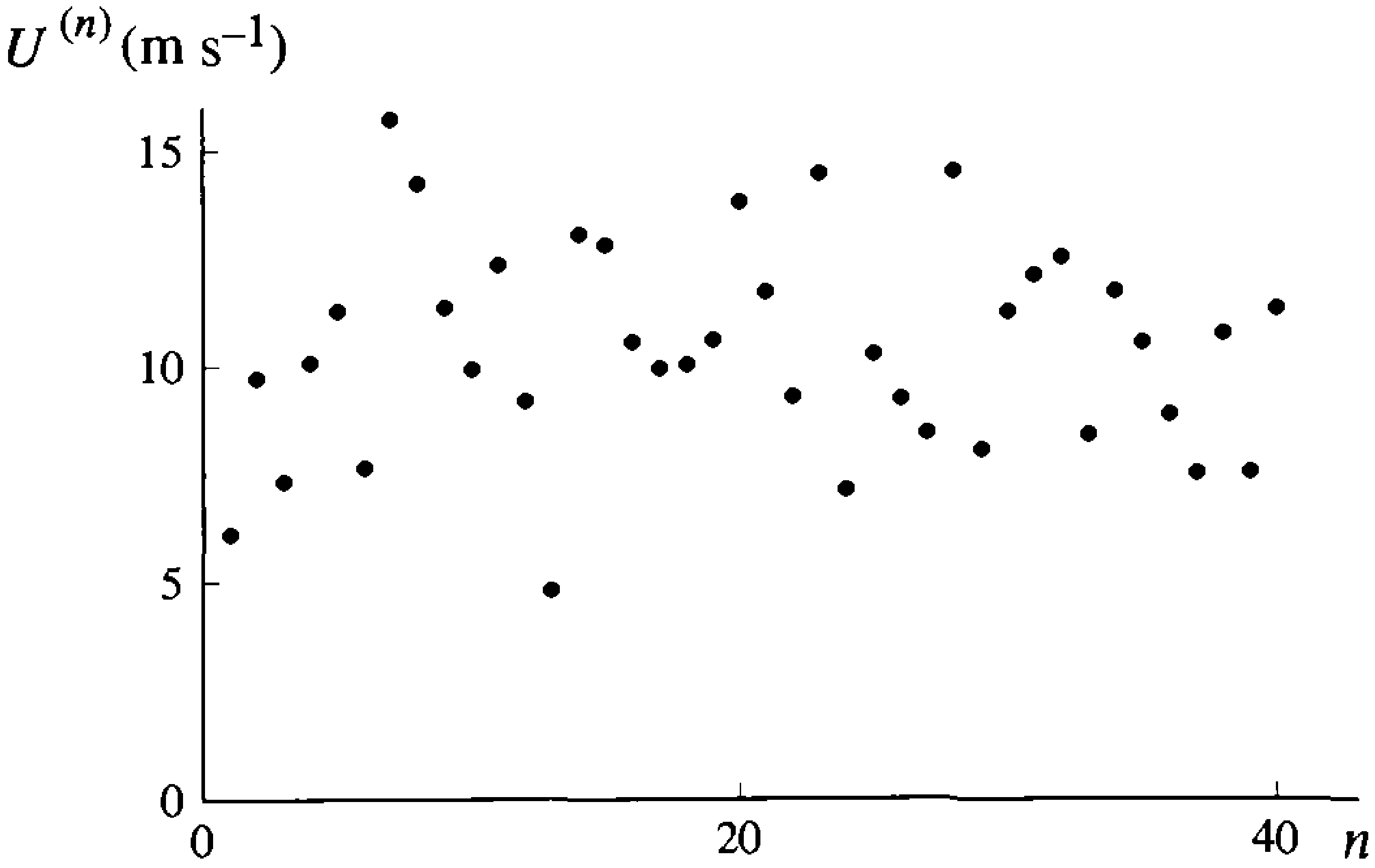
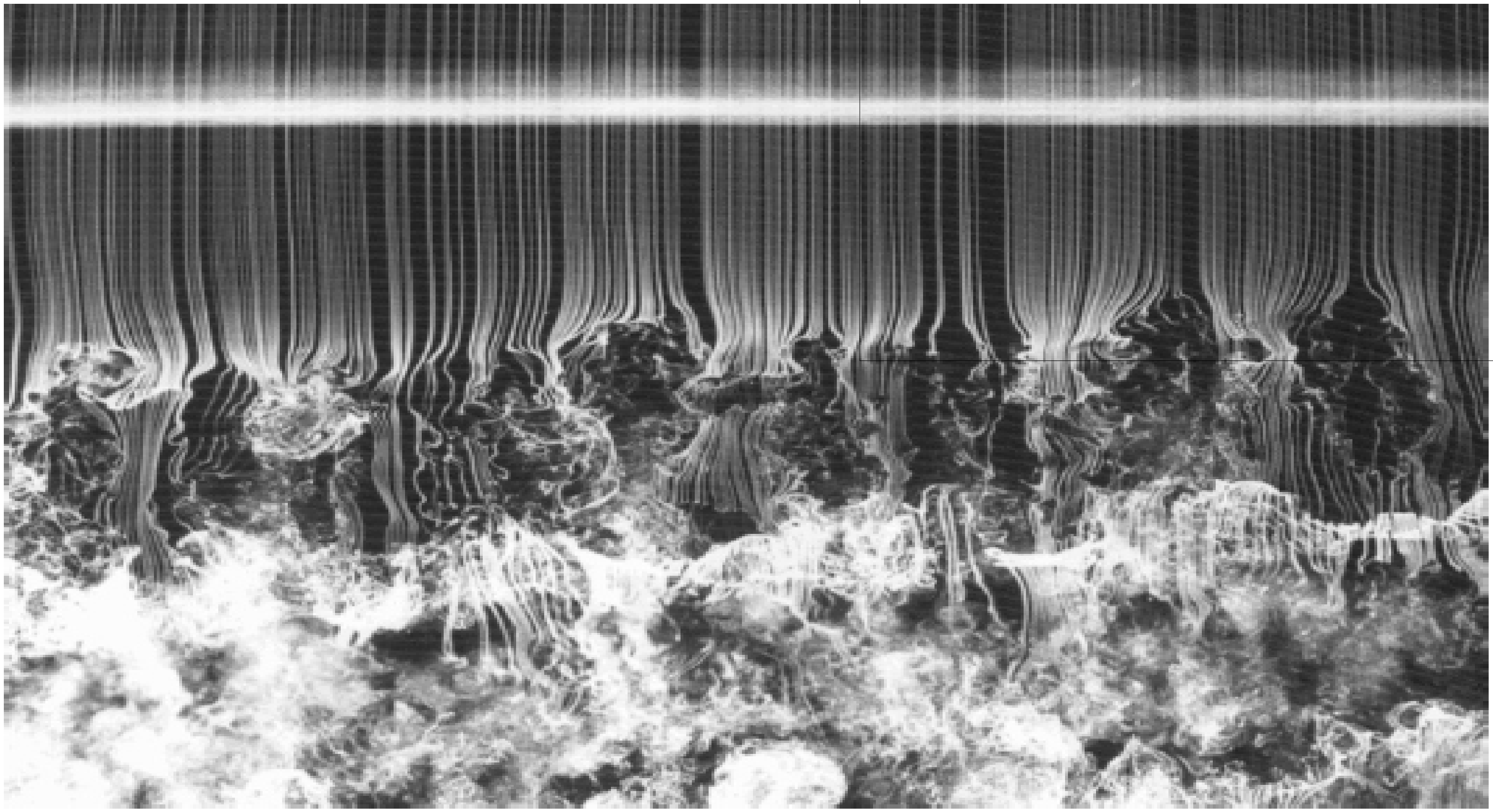


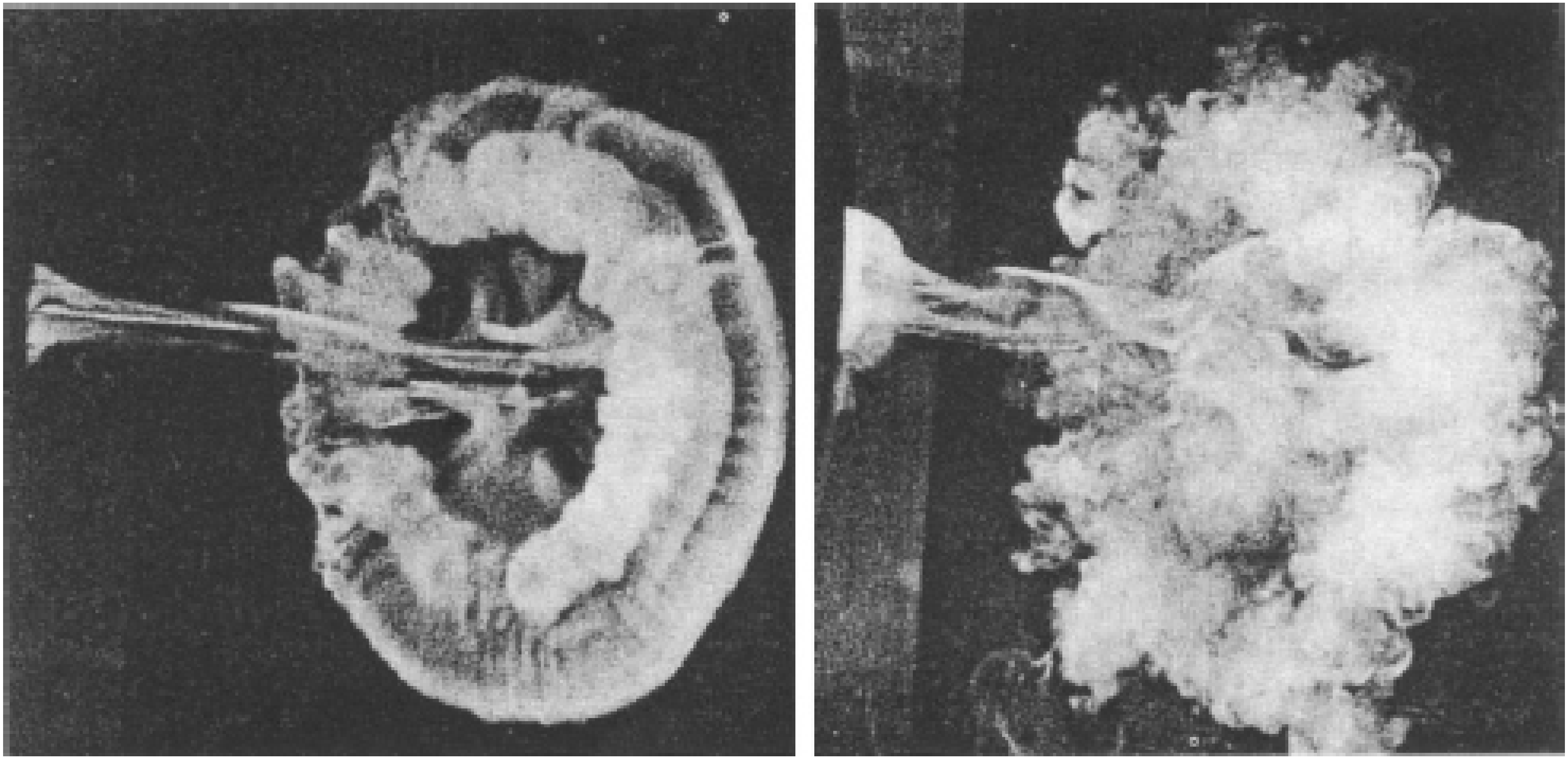
Fig. 3.1. A sketch of the value $U^{(n)}$ of the random velocity variable U on the nth repetition of a turbulent-flow experiment.

- **2- Diffusivity,** *In turbulent flow the diffusivity increases. This means that the spreading rate of boundary layers, jets, etc. increases as the flow becomes turbulent. The turbulence increases the exchange of momentum in e.g. boundary layers and reduces or delays thereby separation at bluff bodies such as cylinders, airfoils and cars. The increased diffusivity also increases the resistance (wall friction) in internal ows such as in channels and pipes.*

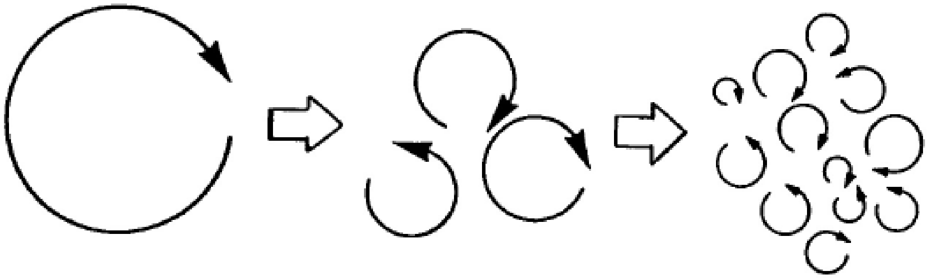


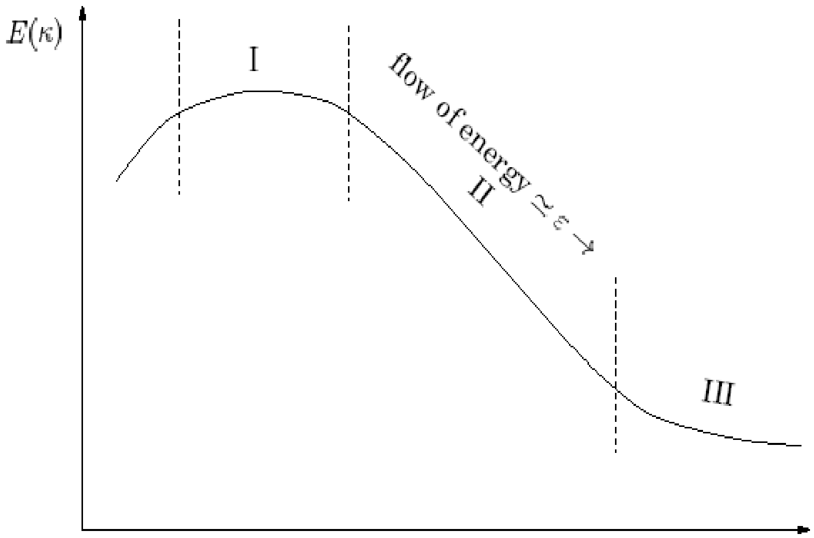
Flow over Cicular Cylinder Re=7×10^3

- **3- Three-dimensional Vorticity flactuations,** *Turbulent flow is always rotational and three-dimensional.However, when the equations are 2D time averaged, an important vorticity-maintenance mechanism known as vortex stretching is absent in two-dimensional flow.*



- **4- Dissipation,** *Turbulent flow is dissipative, which means that kinetic energy in the small (dissipative) eddies are transformed into internal energy. The small eddies receive the kinetic energy from slightly larger eddies. The slightly larger eddies receive their energy from even larger eddies and so on. The largest eddies extract their energy from the mean flow. This process of transferred energy from the largest turbulent scales (eddies) to the smallest is called cascade process.*





$$\left.\begin{array}{r}\text{eddies have energy of order } u_0^2 \text{ and timescale}\\ \tau_0 = \ell_0/u_0,\end{array}\right\} \left.\begin{array}{c}\text{rate of transfer of energy}\\ u_0^2/\tau_0 = u_0^3/\ell_0\end{array}\right\} \varepsilon \sim u_0^3/\ell_0.$$

- **5-Continuum,** *Even though we have small turbulent scales in the flow they are much larger than the molecular scale and we can treat the flow as a continuum.*

هاینز:

جریان متلاطم یک جریان نامنظم بوده که مقدار متغیرها یک رفتار تصادفی وابسته به مکان و زمان را از خود نشان داده که از دیدگاه آماری می توان مقدار متوسطی را برای متغیرها مشخص نمود

Turbulent Scale

- در جریان متلاطم طیف وسیعی از اندازه ادیها وجود دارد. این اندازه می تواند به بزرگی ابعاد هندسه بوده و حتی در ابعاد بسیار کوچک (*Kolmogrov Scale*)باشد.
- بحث ابعاد اندازه ادیها بر اساس توزیع انرژی در جریان درهم مطرح گردید. در این دیدگاه انرژی از میدان سیال به ادیهای بزرگ منتقل شده و سپس این انرژی به ادیهای کوچکتر انتقال داده می شود. این انتقال انرژی تا کوچکترین مقدار ادی ادامه یافته و در نهایت انرژی منتقل شده توسط لزجت ازبین می رود. این موضوع برای اولین بار توسط آقای *Kolmogrov* در سال ۱۹۴۱ بیان گردید.

velocity $\mathcal{U}$ and lengthscale $\mathcal{L}$. → $\mathrm{Re} = \mathcal{U}\mathcal{L}/\nu$ is large;

Eddies of size ℓ have a characteristic velocity $u(\ell)$ → timescale $\tau(\ell) \equiv \ell/u(\ell)$.

dissipation is denoted by ε which is energy per unit time and unit mass

$$\varepsilon = [m^2/s^3] = \nu / \tau^2$$

لازم بذکر است که اصطکاک ناشی از لزجت سبب تلفات انرژی در همه ابعاد ادی می گردد
ولی اثر آن در ادیهای کوچک مشهود تر است

The Kolmogorov hypotheses

- **Kolmogorov's hypothesis of local isotropy.** At sufficiently high Reynolds number, the small-scale turbulent motions (($.l$ <<. l_0) are statistically isotropic.
- The first hypothesis concerns the isotropy of the small-scale motions. In general, the large eddies are anisotropic and are affected by the boundary conditions of the flow. Kolmogorov argued that the directional biases of the large scales are lost in the chaotic scale-reduction process, by which energy is transferred to successively smaller and smaller eddies
- **Kolmogorov's first similarity hypothesis**. In every turbulent flow at sufficiently high Reynolds number, the statistics of the small-scale motions have a universal form that is uniquely determined by ν and ε.

با استفاده از فرض بالا تقریبا کل انرژی منتقل شده به ادیها توسط نیروی اصطکاک حاصل از لزجت به تلفات تبدیل می گردد. بر این اساس انرژی انتقالی به ادیها = ε. در ادامه سعی می گردد تا کلیه مشخصات ادیهای کوچک با استفاده از دو کمیت ε و ν محاسبه گردند.

برای مثال محاسبه سرعت ادیهای کوچک بر اساس ε و ν

$$\begin{array}{cccc} v & = & \nu^a & \varepsilon^b \\ [m/s] & = & [m^2/s] & [m^2/s^3] \end{array}$$

باید ابعاد دو طرف معادله برابر باشد. یعنی برای متر ۱ و برای زمان (ثانیه) ۱-.

$$\left.\begin{aligned} 1 &= 2a+2b, \\ -1 &= -a-3b, \end{aligned}\right\} \Longrightarrow v = (\nu\varepsilon)^{1/4}$$

و به همین ترتیب خواهیم داشت

$$\left.\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}, \quad \tau = \left(\frac{\nu}{\varepsilon}\right)^{1/2}\right\} \Longrightarrow$$

مقیاس طول

مقیاس زمان

Reynolds number based on the Kolmogorov scales is unity, $v\eta/\nu=1$

Kolmogorov's second similarity hypothesis. In every turbulent flow at sufficiently high Reynolds number, the statistics of the motions of scale ℓ in the range $\ell_0 \gg \ell \gg \eta$ have a universal form that is uniquely determined by ε, independent of ν.

- The ratios of the smallest to largest scales are readily determined from the definitions of the Kolmogorov scales and from the scaling $\varepsilon \sim u_0^3/\ell_0$. The results are

$$\left.\begin{aligned} \eta &\equiv (\nu^3/\varepsilon)^{1/4}, \\ u_\eta &\equiv (\varepsilon\nu)^{1/4}, \\ \tau_\eta &\equiv (\nu/\varepsilon)^{1/2}. \\ \varepsilon &\sim u_0^3/\ell_0. \end{aligned}\right\} \Longrightarrow \begin{aligned} \eta/\ell_0 &\sim \mathrm{Re}^{-3/4}, \\ u_\eta/u_0 &\sim \mathrm{Re}^{-1/4}, \\ \tau_\eta/\tau_0 &\sim \mathrm{Re}^{-1/2}. \end{aligned}$$

velocity scale $v_d = (\nu\epsilon)^{1/4}$ d = dissipation

v_d, is related to the root-mean-square (rms) velocity components $\longrightarrow$ $u_{\rm rms} = \sqrt{\overline{u^2}}$

$$\left.\begin{array}{l} \varepsilon = [m^2/s^3] = \nu / \tau^2 \\ \tau = \dfrac{\lambda}{u_{rms}} \end{array}\right\} \epsilon \sim \nu \frac{u^2_{\rm rms}}{\lambda^2}$$

Micro scale

$$\left.\begin{array}{l} \varepsilon \approx \dfrac{u^2_{rms}}{\tau} \\ \text{a time scale} \sim l_e/u_{\rm rms} \end{array}\right\} \epsilon \sim \frac{u^2_{\rm rms}}{l_e/u_{\rm rms}} = \frac{u^3_{\rm rms}}{l_e}$$

e = eddy

$$\nu \frac{u^2_{\rm rms}}{\lambda^2} \sim \frac{u^3_{\rm rms}}{l_e}$$

$$\Big\downarrow$$

$$\frac{l_e}{\lambda} \sim u_{\rm rms}\lambda/\nu$$

$$\Big\downarrow$$

$$\underbrace{\qquad\qquad\qquad}_{}$$

R_λ turbulence Reynolds number.

$$\left.\begin{array}{l} \dfrac{l_e}{\lambda} \sim u_{\rm rms}\lambda/\nu \\ \text{Multiplying} \\ \text{by } u_{\rm rms}/\nu \text{ gives} \end{array}\right\} \left.\begin{array}{l} R_e \sim R_\lambda^2 \\ R_e = u_{\rm rms} l_e/\nu \end{array}\right\} \sqrt{R_e} \sim R_\lambda$$

a turbulence Reynolds number based on on the physical size of the flow domain

$$\left.\begin{aligned} \eta &\equiv \frac{\nu^{3/4}}{\epsilon^{1/4}} \\ \epsilon &\sim \nu \frac{u_{\mathrm{rms}}^2}{\lambda^2} \end{aligned}\right\} \quad \frac{\eta}{\lambda} \sim \frac{1}{\sqrt{R_\lambda}} \sim \frac{1}{R_e^{1/4}}$$

showing that η is generally smaller than λ but not so much so. In fact it can be seen that λ is a reasonable measure of the scales where most of the dissipation takes place.

$\frac{l_e}{\eta} \sim R_\lambda^{3/2} \sim R_e^{3/4}$ which is the ratio of the largest to smallest scales in the flow.

$$\frac{u_{\mathrm{rms}}}{v_d} \sim R_\lambda^{1/2} \sim R_e^{1/4}$$

a full numerical simulation on a mesh would generally require a mesh spacing $\sim \eta$ to resolve the flow details

On the other hand, the spatial extent of the flow domain is $\sim l_e$, so in any one direction, approximately l_e/η mesh points are required.

A three-dimensional mesh would then have to be $\sim (l_e/\eta)^3$ in size

$R_e^{9/4}$ dependence on Reynolds number.

In a numerical calculation the elapsed time for a single iteration can be taken to be $\sim \eta/u_{\mathrm{rms}}$, since this reflects the shortest time period needed to be resolved.

Ḡ

l_e/u_{rms} reflects time variations of the large-scale motions.

l_e/η, reflects roughly how many time steps must be computed

in a typical turbulent flow simulation in order to determine average quantities

this means that $O(R_e^{3/4})$ time steps are required

there are $O(R_e^{9/4})$ grid points

the total computational effort

$$O(R_e^3)$$

The Energy spectrum

It remains to be determined how the turbulent kinetic energy is distributed among the eddies of different sizes.

For homogenous turbulent flow**:**

motions of lengthscale ℓ correspond to wavenumber $\kappa = 2\pi/\ell$,

energy in the wavenumber range (κ_a, κ_b) is → $k_{(\kappa_a,\kappa_b)} = \int_{\kappa_a}^{\kappa_b} E(\kappa)\,d\kappa.$

The total turbulent kinetic energy is: $k = \int_0^\infty E(\kappa) d\kappa$

The kinetic energy is the sum of the kinetic energy of the three fluctuating velocity components, i.e.

$$\left.\begin{aligned} k &= \frac{1}{2}\left(\overline{u^2}+\overline{v^2}+\overline{w^2}\right) = \frac{1}{2}\overline{u_i u_i} \\ \varepsilon &= [m^2/s^3] = \nu / \tau^2 \end{aligned}\right\} \quad \varepsilon_{(\kappa_a,\kappa_b)} = \int_{\kappa_a}^{\kappa_b} 2\nu\kappa^2 E(\kappa)\,d\kappa.$$

From the second hypothesis it follows that,

A second far-reaching idea of Kolmogorov was that of an *inertial subrange* consisting of a section of wavenumber space between k_e and k_d where energy cascades toward small scales without significant dissipation or production. Such a cascade in this range of wavenumbers would depend on just ϵ and not ν. Kolmogorov argued that this has an important consequence for the form of the energy spectrum function $E(k, t)$.

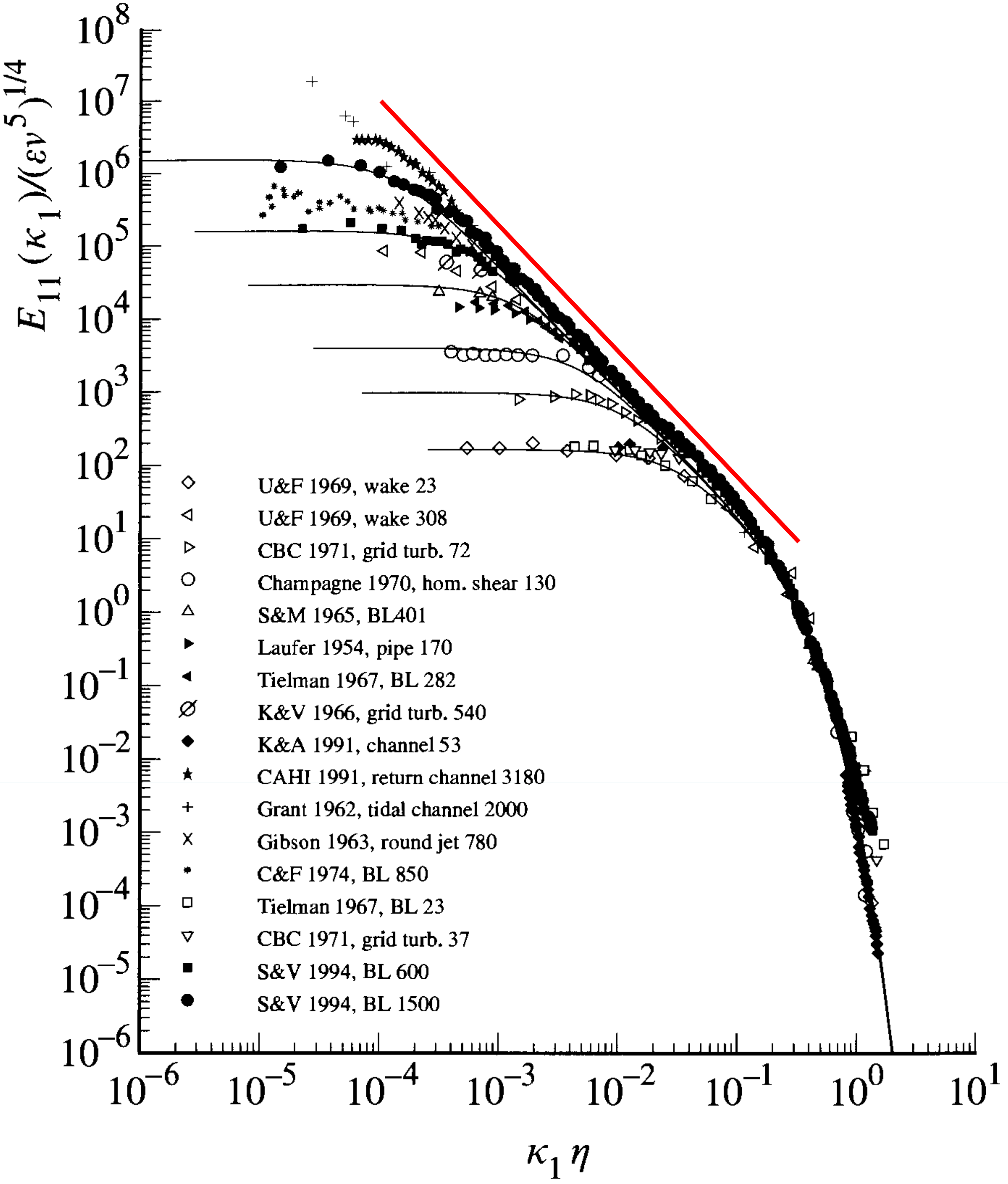
$$E(k, t) \sim k^{-5/3} \epsilon^{2/3} \longrightarrow E(\kappa) = C\varepsilon^{2/3}\kappa^{-5/3}$$

Kolmogorov constant

$C_K = 1.4$

has been observed to occur in a wide range of turbulent flows at high Reynolds

For more detail see Ref.[5] section 6.1.3



Measurements of one dimensional longitudinal velocity spectra (symbols ,(and model spectra (Eq. F.246)) for R_λ =30,70,130,300,600, and 1,500 (lines). The experimental data are taken from Saddoughi and Veeravalli 1994) where references to the various experiments are given. For each experiment, the final number in the key is the value of R_λ.

Vorticity/Velocity Gradient Interaction

Why are vorticity and vortex stretching so important to the study of turbulence?

We shall discover that energy is transferred to small scales by vortex stretching and that the dissipation rate of energy is proportional to the mean-square vorticity fluctuations if the Reynolds number is large enough.

vortex stretching and vortex tilting.

$$\boldsymbol{\Omega} \equiv \nabla \times \mathbf{u} \quad \Rightarrow \quad \frac{\partial \Omega_i}{\partial t} + U_j \frac{\partial \Omega_i}{\partial x_j} = \underbrace{\Omega_j \frac{\partial U_i}{\partial x_j}}_{} + \nu \nabla^2 \Omega_i$$

$$(\Omega_i = \bar{\Omega}_i + \omega_i)$$

$$\Omega_i = \epsilon_{ijk} U_{k,j}$$

vortex stretching

Vortex reorientation

rotation/tilting

تکلیف شماره ۱: در ارتباط بااثر کشیدگی گردابه و چرخش آن یک گزارش کامل ارائه دهید. مرجع۱ و ۷ و.....