How can we predict the turbulent flow ?

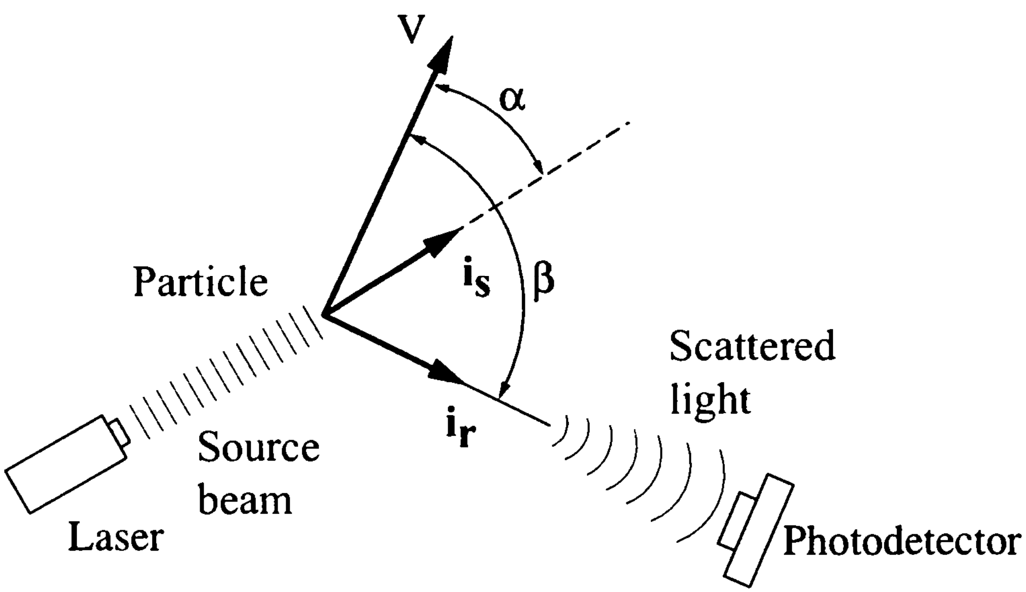
- Experimental Methods
- Numerical Methods

1. HOT-WIRE AND HOT-FILM ANEMOMETRY

The hot-wire method is based on the simple physical principle that the amount of cooling experienced by a heated wire can be related to the local flow velocity. A small-diameter metal wire sensor, usually tungsten, platinum, or a platinum alloy, is heated above the ambient temperature of the flow by an electrical current. Wire diameters typically vary in the range 0.5 to 5 /μm and the lengths from 0.15 to 1.5 mm, depending on the application and spatial resolution required.

2. LASER-DOPPLER VELOCIMETRY

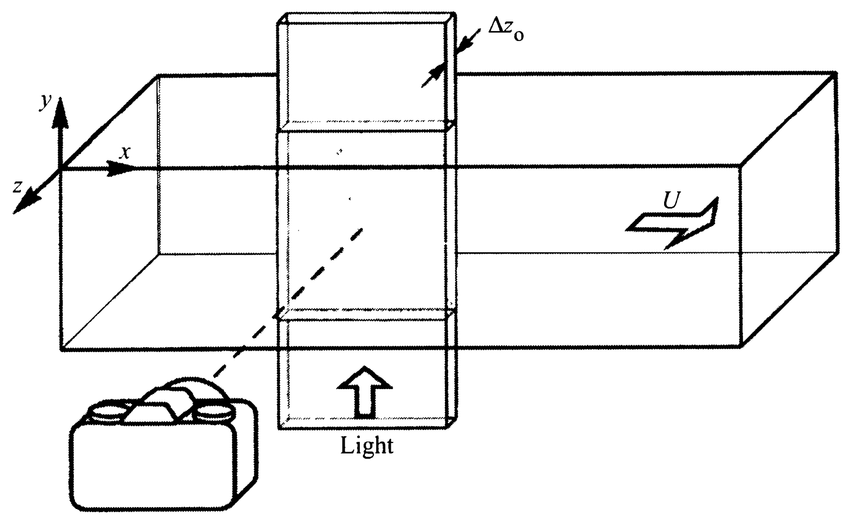
Laser-Doppler velocimetry (LDV) is an optical method for flow measurement utilizing the Doppler principle that coherent (laser) light reflected from a moving particle exhibits a frequency relative to a fixed observer that depends, not only on the known laser light source wavelength, but also on the velocity of the moving particle. Thus, if one can measure in a turbulent flow the frequency of the light scattered from tiny particles chosen to have sufficiently small diameter and appropriate density so that they follow the fluid motion faithfully, the local fluid velocity can be determined.



3. PLANAR FLOW FIELD VELOCIMETRY

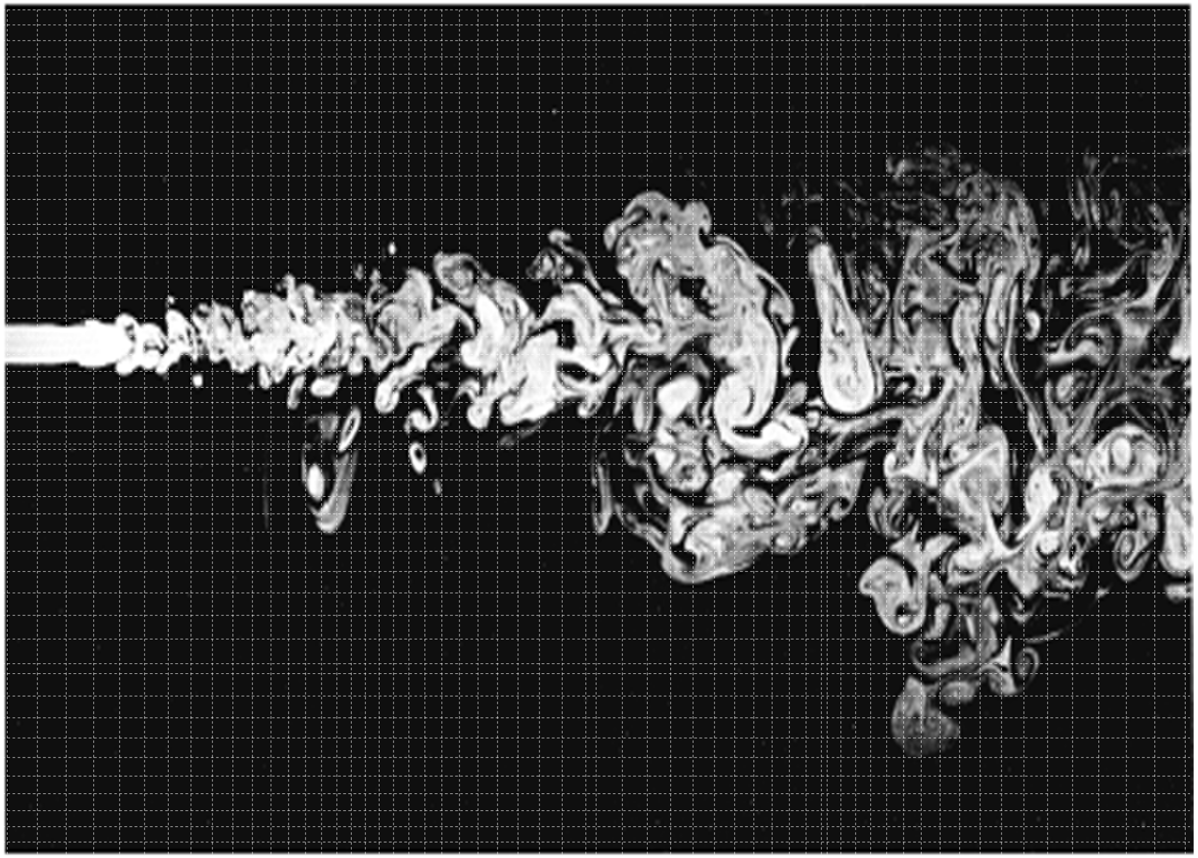
Hot-wire and laser-Doppler velocimetry are limited in practice to simultaneous measurements at only a relatively small number of locations in a flow field. Of more use in many applications are simultaneous planar measurements of the velocity components (i.e., measurement of the velocity on a fine grid of points covering a plane surface). The technique of planar flow field velocimetry has arisen in recent years and is now developed to a stage where commercial systems are available. Extensions of planar velocimetry to full three-dimensional flow field measurements have also been made in a few laboratories using holography, rapid planar scanning, and with limited spatial resolution, three-dimensional particle tracking. At the present time, such three-dimensional methods are difficult to use, quite expensive, and, generally speaking, are still in the development phase.

تکلیف شماره ۲: در ارتباط با تکنیکهای اندازه گیری گزارش کاملی تهیه نمایید٠ مرجع قابل بررسی شماره ۷- صفحه ۵۳



Numerical Methods

1. *Direct Numerical Simulation (DNS)*
2. *Reynolds Average Navier-Stoks Equation (RANS)*
3. *Large Eddy Simulation (LES)*



The Elements of Statistical Analysis

Much of the study of turbulence requires statistics and stochastic processes,

the instantaneous motions are too complicated to understand.

time average,

spatial average

ensemble average

Time averaging is appropriate for stationary turbulence. turbulent flow that, on the average, does not vary with time.

an instantaneous flow variable as $f(x, t)$. its time average, $F_T(x)$, is defined by :

$$F_T(\mathbf{x}) = \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} f(\mathbf{x}, t)\, dt$$

Spatial averaging is appropriate for **homogeneous turbulence**, **which is a turbulent flow that, on the average, is uniform in all directions**.

$$F_V(t) = \lim_{V\to\infty} \frac{1}{V} \iiint f(\mathbf{x},t)\, dV$$

Ensemble averaging is the most general type of averaging. As an idealized example, in terms of measurements from N identical experiments where ***$f(x,t)= f_n(x, t)$*** **in the *n^{th}* experiment**,

$$F_E(\mathbf{x},t) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f_n(\mathbf{x},t)$$

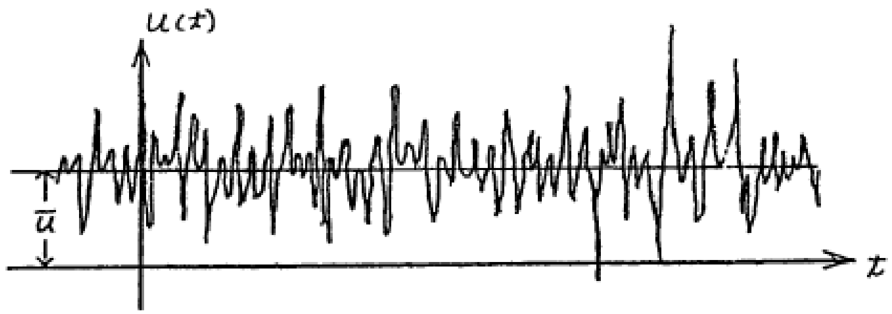
Because virtually all engineering problems involve **inhomogeneous turbulence**, time averaging is the most appropriate form of Reynolds averaging. The time-averaging process is most clearly **explained for stationary turbulence**.

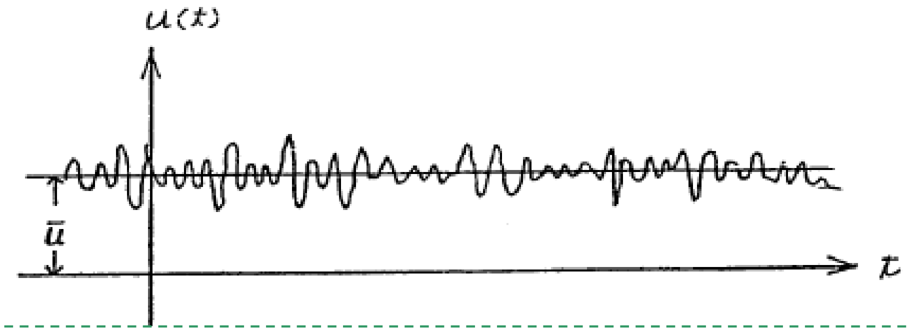
$$\overline{\mathbf{U}}(\mathbf{x},t) = \frac{1}{T} \int_{t-T/2}^{t+T/2} \mathbf{U}(\mathbf{x},s)\, ds$$

For either ensemble, time, or space averaging, a turbulent velocity field may be decomposed according to

$$\mathbf{U} = \overline{\mathbf{U}} + \mathbf{u}.$$

where u = U - $\overline{\text{U}}$ is referred to as the velocity fluctuation vector. $\overline{\mathbf{u}} \equiv 0$





توجه شود که بدلیل تصادفی بودن رفتار جریان متلاطم، بررسی متوسط زمانی و آماری برای این نوع از جریان مطرح می گردد. نکته مهم این است که با استفاده از متوسط زمانی به صورت فوق چگونه می توان از رفتار متوسط نوسانات اطلاع پیدا کرد. از این رو بحث استفاده از انحراف معیار مطرح می گردد.. $var[x] = \langle x^2 \rangle - X^2$

نکته موردتوجه این است که متوسط زمانی نوسانات با توان بیشتر از یک صفر نبوده و بیشتر مورد توجه می باشد

for example, $\overline{u_1^2}, \overline{u_2^2}, \overline{u_3^2}$, which are the variances of the components of **u**.

The sum of these yields the turbulent kinetic energy per unit volume, $K \equiv \rho \overline{u_i^2}/2$,

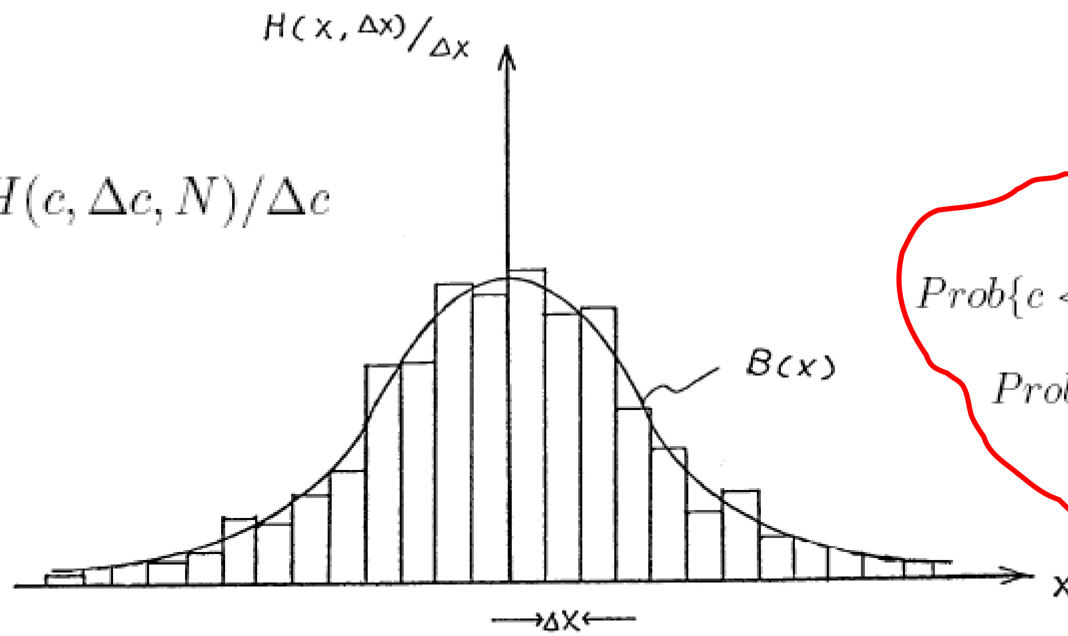
$R_{ij} \equiv \overline{u_i u_j}$

In many circumstances where averages are computed directly from the random field, our knowledge of the turbulent physics can be deepened by examining the associated ***probability Density Function (PDF).***

Thus, if $p(U_1)$ is the PDF of U_1, then by definition, $p(U_1)\, dU_1$ is the probability that U_1 takes on a value U such that $U_1 < U < U_1 + dU_1$.

$$\int p(U)\, dU = 1$$

$$B_x(c) \equiv \lim_{\substack{N \to \infty \\ \Delta c \to 0}} H(c, \Delta c, N)/\Delta c$$



$$B_x(c) > 0$$

$$Prob\{c < x < c + dc\} = B_x(c)dc$$

$$Prob\{x < c\} = \int_{-\infty}^{x} B_x(c)dc$$

$$\int_{-\infty}^{\infty} B_x(x)dx = 1$$

تکلیف شماره ۳: در ارتباط با PDF گزارش تهیه نمایید- مراجع ۱- فصل دوم درس توربولانس ویلیام جورج دانشگاه چالمرز ۲- مرجع ۷ ص ۸ و ۳- مرجع ۵ ص ۳۷

Reynolds-Averaged Navier–Stokes Equation

Navier–Stokes equation
$$\rho\left(\frac{\partial U_i}{\partial t}+U_j\frac{\partial U_i}{\partial x_j}\right)=\frac{\partial \sigma_{ij}}{\partial x_j}+\rho g_i$$

$\frac{\partial \sigma_{ij}}{\partial x_j}$: stress tensor.

g_i: gravitational acceleration vector

$$\sigma_{ij}=-P\delta_{ij}+d_{ij}$$

$$d_{ij}=2\mu e_{ij} \Rightarrow e_{ij}=\frac{1}{2}\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)$$

e_{ij}: rate-of-strain tensor

$$\rho\left(\frac{\partial U_i}{\partial t}+U_j\frac{\partial U_i}{\partial x_j}\right)=-\frac{\partial P}{\partial x_i}+\mu\nabla^2 U_i+\rho g_i$$

$$U_i = \bar{U}_i + u_i$$
$$P = \bar{P} + p.$$

$$\rho\left(\frac{\partial \overline{U}_i}{\partial t} + \overline{U}_j \frac{\partial \overline{U}_i}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left(\overline{\sigma}_{ij} - \rho\overline{u_i u_j}\right)$$

$$\overline{\sigma}_{ij} = -\overline{P}\delta_{ij} + 2\mu\overline{e}_{ij} \quad \Rightarrow \quad \overline{e}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial \overline{U}_j}{\partial x_i}\right)$$

the RANS equation

$$\rho\left(\frac{\partial \overline{U}_i}{\partial t} + \overline{U}_j \frac{\partial \overline{U}_i}{\partial x_j}\right) = -\frac{\partial \overline{P}}{\partial x_i} + \mu\nabla^2\overline{U}_i - \frac{\partial(\rho\overline{u_i u_j})}{\partial x_j}$$

Reynolds stress tensor

$$\sigma_{ij}^T \equiv -\rho\overline{u_i u_j} = -\rho R_{ij}$$

Non-linear and Unknown

Depend to flow field

the fluctuating motion

From continuity equation

$$\frac{\partial U_i}{\partial x_i} = 0 \begin{cases} \dfrac{\partial \overline{U}_i}{\partial x_i} = 0 \\ \dfrac{\partial u_i}{\partial x_i} = 0 \end{cases}$$

$$\rho\left(\frac{\partial U_i}{\partial t}+U_j\frac{\partial U_i}{\partial x_j}\right)=-\frac{\partial P}{\partial x_i}+\mu\nabla^2 U_i+\rho g_i$$

0

$$\mathbf{U}=\overline{\mathbf{U}}+\mathbf{u}$$

$$\rho\left[\frac{\partial u_i}{\partial t}+\overline{U}_j\frac{\partial u_i}{\partial x_j}\right]=-\frac{\partial p}{\partial x_i}+\frac{\partial \tau_{ij}^{(v)}}{\partial x_j}-\rho\left[u_j\frac{\partial \overline{U}_i}{\partial x_j}\right]-\left\{u_j\frac{\partial u_i}{\partial x_j}-\rho\left\langle u_j\frac{\partial u_i}{\partial x_j}\right\rangle\right\}$$

fluctuating pressure gradient

fluctuating viscous stresses.

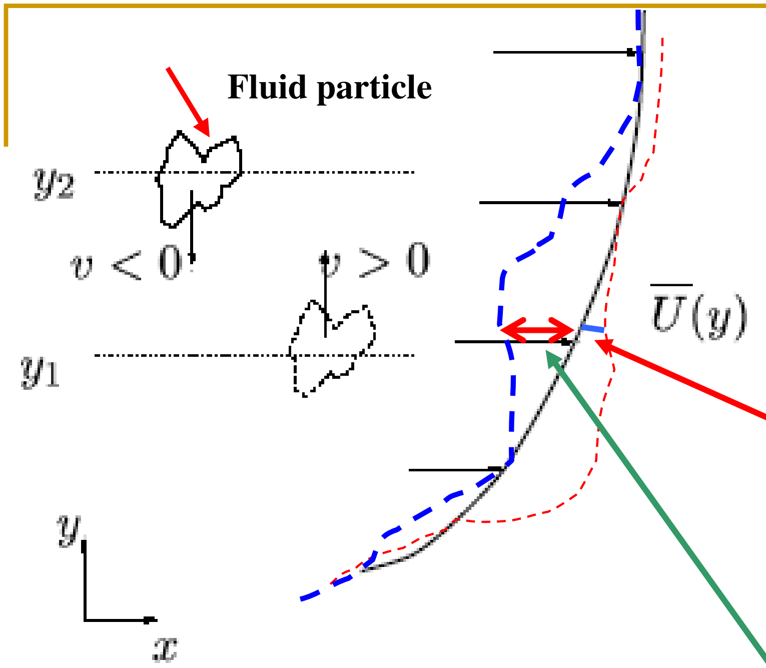
Reynolds Stress

extract energy from the mean flow, the so-called *production terms*

باید توجه نمود که با متوسط گیری از معادله
فوق خیلی از قسمتهای آن برابر صفر می گردد

$< u_1^2 >, < u_2^2 >, < u_3^2 >, < u_1 u_2 >$
$< u_1 u_3 >$, and $< u_2 u_3 >$

the Turbulence Closure Problem



$$-\rho < u_i u_j >= \mu_t \left[S_{ij} - \frac{1}{3} S_{kk} \delta_{ij} \right]$$

Negative

$u' > 0$

$u' < 0$

every time $u'v' < 0$

نکته مهم هم علامت بودن تنش رینولدز و تنش برشی ناشی از لزجت سیال در معادله ممنتوم می باشد. از این رو این دو عبارت در راستای یکدیگر بوده و بصورت یک تنش کلی مطرح می گردند. براین اساس رفتار تنش رینولدز نیز با استفاده از تشابه اثر لزجت بیان می گردد. در این راستا مفهوم لزجت ادی مطرح گردید.

$$\rho\left[\frac{\partial u_i}{\partial t}+U_j\frac{\partial u_i}{\partial x_j}\right]=-\frac{\partial p}{\partial x_i}+\frac{\partial \tau_{ij}^{(v)}}{\partial x_j}-\rho\left[u_j\frac{\partial U_i}{\partial x_j}\right]-\rho\left\{u_j\frac{\partial u_i}{\partial x_j}-\left\langle u_j\frac{\partial u_i}{\partial x_j}\right\rangle\right\} \quad (1)$$

$\times\ u_k$ and averaging yields:

$$\rho\left[\left\langle u_k\frac{\partial u_i}{\partial t}\right\rangle+U_j\left\langle u_k\frac{\partial u_i}{\partial x_j}\right\rangle\right]= -\left\langle u_k\frac{\partial p}{\partial x_i}\right\rangle+\left\langle u_k\frac{\partial \tau_{ij}^{(v)}}{\partial x_j}\right\rangle -\rho\left[< u_k u_j > \frac{\partial U_i}{\partial x_j}\right]-\rho\left\{\left\langle u_k u_j\frac{\partial u_i}{\partial x_j}\right\rangle\right\}$$

Rewritten as:

$$\rho\left[\left\langle u_i\frac{\partial u_k}{\partial t}\right\rangle+U_j\left\langle u_i\frac{\partial u_k}{\partial x_j}\right\rangle\right]= -\left\langle u_i\frac{\partial p}{\partial x_k}\right\rangle+\left\langle u_i\frac{\partial \tau_{kj}^{(v)}}{\partial x_j}\right\rangle -\rho\left[< u_i u_j > \frac{\partial U_k}{\partial x_j}\right]-\rho\left\{\left\langle u_i u_j\frac{\partial u_k}{\partial x_j}\right\rangle\right\} \quad (2)$$

(1) ⊕ (2) →

$$\frac{\partial < u_i u_k >}{\partial t} + U_j \frac{\partial < u_i u_k >}{\partial x_j} = -\frac{1}{\rho}\left[\left\langle u_i \frac{\partial p}{\partial x_k}\right\rangle + \left\langle u_k \frac{\partial p}{\partial x_i}\right\rangle\right] - \left[\left\langle u_i u_j \frac{\partial u_k}{\partial x_j}\right\rangle + \left\langle u_k u_j \frac{\partial u_i}{\partial x_j}\right\rangle\right] + \frac{1}{\rho}\left[\left\langle u_i \frac{\partial \tau^{(v)}_{kj}}{\partial x_j}\right\rangle + \left\langle u_k \frac{\partial \tau^{(v)}_{ij}}{\partial x_j}\right\rangle\right] - \left[< u_i u_j > \frac{\partial U_k}{\partial x_j} + < u_k u_j > \frac{\partial U_i}{\partial x_j}\right]$$

$$\left[\left\langle u_i \frac{\partial p}{\partial x_k}\right\rangle + \left\langle u_k \frac{\partial p}{\partial x_i}\right\rangle\right] = \underbrace{\left\langle p\left[\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right]\right\rangle}_{pressure\ strain\text{-}rate} - \underbrace{\frac{\partial}{\partial x_j}\left[< p u_i > \delta_{kj} + < p u_k > \delta_{ij}\right]}_{pressure\ diffusion}$$

pressure strain-rate

pressure diffusion

Not create and not destroy anything

$$\frac{\partial < u_i u_k >}{\partial t} + U_j \frac{\partial < u_i u_k >}{\partial x_j} = -\frac{1}{\rho}\left[\left\langle u_i \frac{\partial p}{\partial x_k}\right\rangle + \left\langle u_k \frac{\partial p}{\partial x_i}\right\rangle\right]$$
$$- \left[\left\langle u_i u_j \frac{\partial u_k}{\partial x_j}\right\rangle + \left\langle u_k u_j \frac{\partial u_i}{\partial x_j}\right\rangle\right]$$
$$+ \frac{1}{\rho}\left[\left\langle u_i \frac{\partial \tau_{kj}^{(v)}}{\partial x_j}\right\rangle + \left\langle u_k \frac{\partial \tau_{ij}^{(v)}}{\partial x_j}\right\rangle\right]$$
$$- \left[< u_i u_j > \frac{\partial U_k}{\partial x_j} + < u_k u_j > \frac{\partial U_i}{\partial x_j}\right]$$

با استفاده از معادله پیوستگی

$$\frac{\partial u_j}{\partial x_j} = 0$$

$$- \left[\left\langle \tau_{ij}^{(v)} \frac{\partial u_k}{\partial x_j}\right\rangle + \left\langle \tau_{kj}^{(v)} \frac{\partial u_i}{\partial x_j}\right\rangle\right]$$

dissipation of Reynolds stress by the turbulence viscous stresses

$$2\nu \left[\left\langle s_{ij} \frac{\partial u_k}{\partial x_j}\right\rangle + \left\langle s_{kj} \frac{\partial u_i}{\partial x_j}\right\rangle\right]$$

$$+ \frac{\partial}{\partial x_j}\left[< u_i \tau_{kj}^{(v)} > + < u_k \tau_{ij}^{(v)} >\right]$$

$$\left[\left\langle u_i u_j \frac{\partial u_k}{\partial x_j}\right\rangle + \left\langle u_k u_j \frac{\partial u_i}{\partial x_j}\right\rangle\right] = \frac{\partial}{\partial x_j} < u_i u_k u_j >$$

the rate of change of Reynolds stress following the mean motion

pressure strain-rate

$$\frac{\partial}{\partial t} < u_i u_k > + U_j \frac{\partial}{\partial x_j} < u_i u_k > = -\left\langle \frac{p}{\rho}\left[\frac{\partial u_i}{\partial x_k} + \frac{\partial u_i}{\partial x_k}\right]\right\rangle$$

the turbulence transport (or divergence) term

$$+ \frac{\partial}{\partial x_j}\{-[< p u_k > \delta_{ij} + < p u_i > \delta_{kj}] - < u_i u_k u_j > + 2\nu[< s_{ij} u_k > + < s_{kj} u_i >]\}$$

the "production" term,

$$- \left[< u_i u_j > \frac{\partial U_k}{\partial x_j} + < u_k u_j > \frac{\partial U_i}{\partial x_j}\right]$$

$$- 2\nu\left[\left\langle s_{ij}\frac{\partial u_k}{\partial x_j}\right\rangle + \left\langle s_{kj}\frac{\partial u_i}{\partial x_j}\right\rangle\right]$$

the "dissipation" term

This is the so-called **Reynolds Stress Equation** which has been the primary vehicle for much of the turbulence modeling efforts of the past few decades.

$$
\begin{aligned}
<pu_i> &\; - \; 3 \quad unknowns \\
<u_i s_{jk}> &\; - \; 27 \\
<s_{ij} s_{jk}> &\; - \; 9 \\
<u_i u_k u_j> &\; - \; 27 \\
<p\frac{\partial u_i}{\partial x_j}> &\; - \; 9 \\
TOTAL &\; - \; 75
\end{aligned}
$$

•The $\overline{u_i u_j}$-equation (Reynolds Stress equation) has the form:

if $t \to \infty$:

$$
\underbrace{\bar{U}_k \frac{\partial \overline{u_i u_j}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{u_i u_k}\frac{\partial \bar{U}_j}{\partial x_k} - \overline{u_j u_k}\frac{\partial \bar{U}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p}{\rho}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}}_{\Phi_{ij}}
$$

Production P_{ij} ; Pressure-strain rate Φ_{ij}

$$
\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{u_i u_j u_k} + \frac{\overline{pu_j}}{\rho}\delta_{ik} + \frac{\overline{pu_i}}{\rho}\delta_{jk} - \nu\frac{\partial \overline{u_i u_j}}{\partial x_k}\right]}_{D_{ij}} \qquad (1)
$$

Diffusion

$$
\underbrace{-g_i\beta\overline{u_j t} - g_j\beta\overline{u_i t}}_{G_{ij}} - \underbrace{2\nu\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}}_{\varepsilon_{ij}} \qquad (2)
$$

Production terms of Buoyancy

Dissipation

which symbolically can be written:

$$C_{ij} = P_{ij} + \Phi_{ij} + D_{ij} + G_{ij} - \varepsilon_{ij}$$

•The turbulent kinetic energy is the sum of all normal Reynolds stresses, i.e.

$$k = \frac{1}{2}\left(\overline{u_1^2} + \overline{u_2^2} + \overline{u_3^2}\right) \equiv \frac{1}{2}\overline{u_i u_i}$$

If in Reynolds stress equation i=j and equation dividing by 2 (/2)

turbulent kinetic energy:

$$\underbrace{\bar{U}_j \frac{\partial k}{\partial x_j}}_{C_k} = \underbrace{-\overline{u_i u_j}\frac{\partial \bar{U}_i}{\partial x_j}}_{P_k} \underbrace{-g_i \beta \overline{u_i t}}_{G_k} - \underbrace{\nu \overline{\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}}}_{\varepsilon}$$

Production — Dissipation

$$\underbrace{-g_i \beta \overline{u_i t}}_{G_k} \underbrace{- \frac{\partial}{\partial x_j}\left\{\overline{u_j\left(\frac{p}{\rho} + \frac{1}{2}u_i u_i\right)} - \nu \frac{\partial k}{\partial x_j}\right\}}_{D_k}$$

Diffusion

$$C_k = P_k + D_k + G_k - \varepsilon$$

MODELLING ASSUMPTIONS

Reynolds Stress Model (RSM) Algebraic Stress Model (ASM)

- Production term, RSM, ASM:

$$P_{ij} = -\rho\overline{u_i u_k}\frac{\partial \bar{U}_j}{\partial x_k} - \rho\overline{u_j u_k}\frac{\partial \bar{U}_i}{\partial x_k}$$

- Production term, $k-\varepsilon$:

$$-\overline{u_i u_j} = \nu_t\left(\frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}k$$

$$P_k = \mu_t\left(\frac{\partial \bar{U}_i}{\partial x_j} + \frac{\partial \bar{U}_j}{\partial x_i}\right)\frac{\partial \bar{U}_i}{\partial x_j} = 2\mu_t\bar{S}_{ij}\bar{S}_{ij}$$

•Diffusion term in the $\overline{u_i u_j}$, k & ε-equations, RSM, ASM:

$$D_k = \frac{\partial}{\partial x_j}\left[\left(\nu + c_k\,\overline{u_j u_m}\,\frac{k}{\varepsilon}\right)\frac{\partial k}{\partial x_m}\right]$$
$$D_\varepsilon = \frac{\partial}{\partial x_j}\left[\left(\nu + c_\varepsilon\,\overline{u_j u_m}\,\frac{k}{\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_m}\right] \qquad (3)$$
$$D_{ij} = \frac{\partial}{\partial x_m}\left[\left(\nu + c_k\,\overline{u_k u_m}\,\frac{k}{\varepsilon}\right)\frac{\partial \overline{u_i u_j}}{\partial x_m}\right]$$

توجه شود که عبارت فوق در مدل ساده K-ε بصورت زیر می باشد

•Diffusion term in the k & ε-equations, $k-\varepsilon$ model:

$$D_k = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]$$

(Davidson, 1995)

used from k-ε model

$$D_k = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]$$

$$D_\varepsilon = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]$$

$$D_{ij} = \frac{\partial}{\partial x_m}\left[\left(\nu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial \overline{u_i u_j}}{\partial x_m}\right]$$

- The dissipation term is modelled as isotropic:

$$\varepsilon_{ij} = \frac{2}{3}\varepsilon\delta_{ij}$$

- Pressure-Strain Redistribution term:

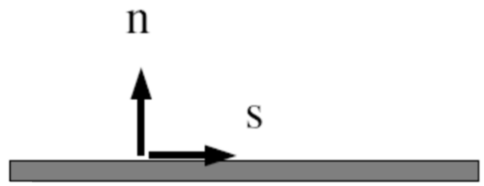
$$\Phi_{ij} = \Phi_{ij,1} + \Phi_{ij,2} + \Phi'_{ij,1} + \Phi'_{ij,2}$$

where

$$\Phi_{ij,1} = -c_1\frac{\varepsilon}{k}\left(\overline{u_i u_j} - \frac{2}{3}\delta_{ij}k\right)$$

$$\Phi_{ij,2} = -c_2\left(P_{ij} - \frac{2}{3}\delta_{ij}P^k\right),\ P^k = \frac{1}{2}P_{\ell\ell}$$

WALL CORRECTION



در کنار دیواره اثر نوسانات بواسطه لزجت از بین رفته و شرایط حاکم بر اساس قانون دیواره می باشد.

- Wall-corrections for $\overline{u_n^2}$:

$$\Phi'_{nn,1} = -2c'_1 \frac{\varepsilon}{k} \overline{u_n^2} f$$

$$f = \frac{k^{\frac{3}{2}}}{2.55 x_n \varepsilon}$$

Close to the wall $f \to 0$ because $k = \mathcal{O}(y^2)$, $\varepsilon = \mathcal{O}(y^0)$ and $1/x_n = \mathcal{O}(y^{-1})$.

- Wall-corrections for $\overline{u_s^2}$:

$$\Phi'_{ss,1} = c'_1 \frac{\varepsilon}{k} \overline{u_n^2} f$$

the third coordinate direction by t is tangential to the wall and hence $\Phi'_{tt,1} = \Phi'_{ss,1}$

the sum of the pressure strain term

$$\Phi'_{tt,1} + \Phi'_{ss,1} + \Phi'_{nn,1} = 0.$$

- Wall-corrections for $\overline{u_s u_n}$:

$$\Phi'_{sn,1} = -\frac{3}{2} c'_1 \frac{\varepsilon}{k} \overline{u_s u_n} f$$

The modeled $\overline{u_i u_j}$ equation

$$\bar{U}_k \frac{\partial \overline{u_i u_j}}{\partial x_k} = \quad \text{(convection)}$$

$$-\overline{u_i u_k}\frac{\partial \bar{U}_j}{\partial x_k} - \overline{u_j u_k}\frac{\partial \bar{U}_i}{\partial x_k} \quad \text{(production)}$$

$$-c_1 \frac{\varepsilon}{k}\left(\overline{u_i u_j} - \frac{2}{3}\delta_{ij} k\right) \quad \text{(slow part)}$$

$$-c_2 \left(P_{ij} - \frac{2}{3}\delta_{ij} P^k\right) \quad \text{(rapid part)}$$

$$+c_1' \rho \frac{\varepsilon}{k}\left[\,\overline{u_k u_m} n_k n_m \delta_{ij} - \frac{3}{2}\overline{u_i u_k} n_k n_j\right.$$

$$\left. - \frac{3}{2}\overline{u_j u_k} n_k n_i\,\right] f\left[\frac{\ell_t}{x_n}\right] \quad \text{(wall, slow part)}$$

$$+c_2' \left[\,\phi_{km,2} n_k n_m \delta_{ij} - \frac{3}{2}\phi_{ik,2} n_k n_j\right.$$

$$\left. - \frac{3}{2}\phi_{jk,2} n_k n_i\,\right] f\left[\frac{\ell_t}{x_n}\right] \quad \text{(wall, rapid part)}$$

$$+\nu \frac{\partial^2 \overline{u_i u_j}}{\partial x_k \partial x_k} \quad \text{(viscous diffusion)}$$

$$+\frac{\partial}{\partial x_k}\left[\left(\nu + c_k\, \overline{u_k u_m}\, \frac{k}{\varepsilon}\right)\frac{\partial \overline{u_i u_j}}{\partial x_m}\right] \quad \text{(turbulent diffusion)}$$

$$-\frac{2}{3}\varepsilon \delta_{ij} \quad \text{(dissipation)}$$