یکی از مدلهای مورد استفاده جهت محاسبه لزجت متلاطم مدل یک معادله ای می باشد. در این روش معدله ای برای لزجت وجود دارد که در این قسمت به تعدادی از این روشها اشاره می گردد.

Baldwin and Barth (1990)

Kinematic Eddy Viscosity

$$\nu_T = C_\mu \nu \tilde{R}_T D_1 D_2$$

Turbulence Reynolds Number

$$\frac{\partial}{\partial t}\left(\nu\tilde{R}_T\right) + U_j \frac{\partial}{\partial x_j}\left(\nu\tilde{R}_T\right) = (C_{\epsilon 2} f_2 - C_{\epsilon 1})\sqrt{\nu\tilde{R}_T P}$$
$$+(\nu + \nu_T/\sigma_\epsilon)\frac{\partial^2(\nu\tilde{R}_T)}{\partial x_k \partial x_k} - \frac{1}{\sigma_\epsilon}\frac{\partial \nu_T}{\partial x_k}\frac{\partial(\nu\tilde{R}_T)}{\partial x_k}$$

Closure Coefficients

$$C_{\epsilon 1} = 1.2, \quad C_{\epsilon 2} = 2.0, \quad C_\mu = 0.09, \quad A_o^+ = 26, \quad A_2^+ = 10$$
$$\frac{1}{\sigma_\epsilon} = (C_{\epsilon 2} - C_{\epsilon 1})\sqrt{C_\mu}/\kappa^2, \qquad \kappa = 0.41$$

Auxiliary Relations

$$P = \nu_T\left[\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} - \frac{2}{3}\frac{\partial U_k}{\partial x_k}\frac{\partial U_k}{\partial x_k}\right]$$

$$D_1 = 1 - e^{-y^+/A_o^+} \quad \text{and} \quad D_2 = 1 - e^{-y^+/A_2^+}$$

$$f_2 = \frac{C_{\epsilon 1}}{C_{\epsilon 2}} + \left(1 - \frac{C_{\epsilon 1}}{C_{\epsilon 2}}\right)\left(\frac{1}{\kappa y^+} + D_1 D_2\right) \cdot \left[\sqrt{D_1 D_2} + \frac{y^+}{\sqrt{D_1 D_2}}\left(\frac{D_2}{A_o^+} e^{-y^+/A_o^+} + \frac{D_1}{A_2^+} e^{-y^+/A_2^+}\right)\right]$$

Spalart and Allmaras (1992)

Kinematic Eddy Viscosity

$$\nu_T = \tilde{\nu} f_{v1}$$

Eddy Viscosity Equation

$$\frac{\partial \tilde{\nu}}{\partial t} + U_j \frac{\partial \tilde{\nu}}{\partial x_j} = c_{b1} \tilde{S} \tilde{\nu} - c_{w1} f_w \left(\frac{\tilde{\nu}}{d}\right)^2 + \frac{1}{\sigma} \frac{\partial}{\partial x_k}\left[(\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial x_k}\right] + \frac{c_{b2}}{\sigma} \frac{\partial \tilde{\nu}}{\partial x_k} \frac{\partial \tilde{\nu}}{\partial x_k}$$

Closure Coefficients

$$c_{b1} = 0.1355, \quad c_{b2} = 0.622, \quad c_{v1} = 7.1, \quad \sigma = 2/3$$

$$c_{w1} = \frac{c_{b1}}{\kappa^2} + \frac{(1 + c_{b2})}{\sigma}, \quad c_{w2} = 0.3, \quad c_{w3} = 2, \quad \kappa = 0.41$$

Auxiliary Relations

$$f_{v1} = \frac{\chi^3}{\chi^3 + c_{v1}^3}, \qquad f_{v2} = 1 - \frac{\chi}{1 + \chi f_{v1}}, \qquad f_w = g\left[\frac{1 + c_{w3}^6}{g^6 + c_{w3}^6}\right]^{1/6}$$

$$\chi = \frac{\tilde{\nu}}{\nu}, \qquad g = r + c_{w2}(r^6 - r), \qquad r = \frac{\tilde{\nu}}{\tilde{S}\kappa^2 d^2}$$

$$\tilde{S} = S + \frac{\tilde{\nu}}{\kappa^2 d^2} f_{v2}, \qquad S = \sqrt{2\Omega_{ij}\Omega_{ij}}$$

The tensor $\Omega_{ij} = \frac{1}{2}(\partial U_i/\partial x_j - \partial U_j/\partial x_i)$ is the rotation tensor and d is distance from the closest surface.

The k-ω Model

Eddy Viscosity

$$\mu_T = \rho k/\omega$$

Turbulence Kinetic Energy

$$\rho\frac{\partial k}{\partial t} + \rho U_j\frac{\partial k}{\partial x_j} = \tau_{ij}\frac{\partial U_i}{\partial x_j} - \beta^*\rho k\omega + \frac{\partial}{\partial x_j}\left[(\mu + \sigma^*\mu_T)\frac{\partial k}{\partial x_j}\right]$$

Specific Dissipation Rate

$$\rho\frac{\partial \omega}{\partial t} + \rho U_j\frac{\partial \omega}{\partial x_j} = \alpha\frac{\omega}{k}\tau_{ij}\frac{\partial U_i}{\partial x_j} - \beta\rho\omega^2 + \frac{\partial}{\partial x_j}\left[(\mu + \sigma\mu_T)\frac{\partial \omega}{\partial x_j}\right]$$

Closure Coefficients

$$\alpha = 5/9, \quad \beta = 3/40, \quad \beta^* = 9/100, \quad \sigma = 1/2, \quad \sigma^* = 1/2$$

Auxiliary Relations

$$\epsilon = \beta^*\omega k \quad \text{and} \quad \ell = k^{1/2}/\omega$$

مدلهای دو معادله دیگری نیزوجود دارند که از آن جمله می توان به مدلهای زیر اشاره نمود.

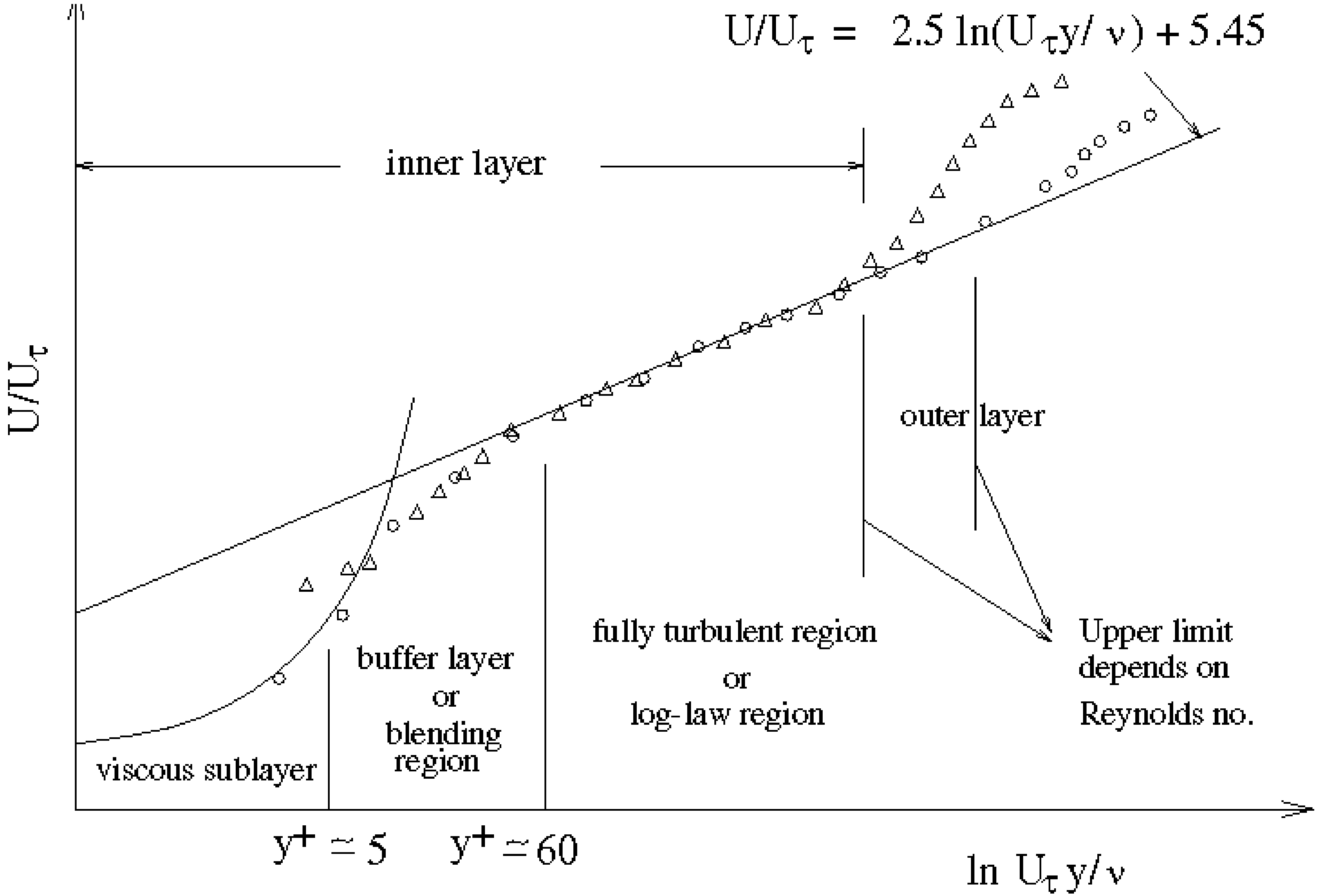
Rotta's (1968) k-$k\ell$ model

Zeierman-Wolfshtein k-$k\tau$ model

Speziale, Abid and Anderson (1990) k-τ model

برای اطلاعات بیشتر به مرجع [۹] ص ۹۰ مراجعه گردد

نکته مهم در استفاده از این مدلها عدم امکان استفاده از آنها در جریان آرام است. این جمله بدین معنی می باشد که مقدار لزجت متلاطم در محاسبات جریان آرام صفر نشده که موجب تولید خطا می گردد. از این رو در جریان کنار دیواره ها و مناطقی که جریان رفتار جریان آرام را دارد لازم است تمهیداتی در نظر گرفته شود. یکی از مهمترین مکانها، جریان در کنار دیواره ها می باشد که سرعت جریان کم شده و ناحیه زیر لایه آرام تشکیل می گردد. در این ناحیه مقدار لزجت متلاطم صفر است. در این راستا روشهای متعددی برای حل این مشکل ارایه شده است.


U/Uτ = 2.5 ln(Uτy/ ν) + 5.45
inner layer
U/Uτ
outer layer
fully turbulent region
or
log- law region
buffer layer
or
blending
region
viscous sublayer
Upper limit
depends on
Reynolds no.
y+ ≃ 5
y+ ≃ 60
ln Uτ y/ ν

two layer models Prandtl and Spalding

$$u^+ = y^+ \qquad 0. \le y^+ \le 11.5$$

$$u^+ = 2.5 \ln y^+ + 5.5 \qquad y^+ > 11.5$$

$$T^+ = Pr\, y^+ \qquad 0. \le y^+ \le 11.5$$

$$T^+ = Pr_t \left[2.5 \ln\left(9 y^+\right) + PFN \right] \qquad y^+ > 11.5$$

$$u_\tau = \sqrt{\frac{\tau_w}{\rho}}, \quad u^+ = \frac{u}{u_\tau}, \quad PFN = 9\left(\frac{Pr}{Pr_t} - 1\right)\left(\frac{Pr}{Pr_t}\right)^{1/4}$$

a single layer of Haroutunian and Engelman

$$u^+ = f_R(y^+) = \frac{1}{\kappa}\ln(1 + 0.4 y^+) + 7.8\left[1 - \exp\left(-\frac{y^+}{11}\right) - \frac{y^+}{11}\exp(-0.33 y^+)\right]$$

$$D_T = \exp\left(-\frac{y^+}{11} R_T^a\right): \quad a = \begin{cases} 1.1 \rightarrow R_T < 0.1 \\ 0.333 \rightarrow R_T \geqslant 0.1 \end{cases}$$

$$R_T = \frac{Pr}{Pr_t}$$

$$T^+ = f_T(y^+, Pr, Pr_t) = Pr\, D_T\, u^+ + Pr_t(1 - D_T)(u^+ + P_T)$$

Jayatilleke correlation

$$P_T = 9.24\left(R_T^{0.75} - 1\right)\left[1 + 0.28\exp(-0.007 R_T)\right]$$

$$U\frac{\partial k}{\partial x} + V\frac{\partial k}{\partial y} = \nu_T\left(\frac{\partial U}{\partial y}\right)^2 - \epsilon + \frac{\partial}{\partial y}\left[(\nu + \nu_T/\sigma_k)\frac{\partial k}{\partial y}\right] \qquad (4.181)$$

$$U\frac{\partial \tilde{\epsilon}}{\partial x} + V\frac{\partial \tilde{\epsilon}}{\partial y} = C_{\epsilon 1} f_1 \frac{\tilde{\epsilon}}{k}\nu_T\left(\frac{\partial U}{\partial y}\right)^2 - C_{\epsilon 2} f_2 \frac{\tilde{\epsilon}^2}{k} + E + \frac{\partial}{\partial y}\left[(\nu + \nu_T/\sigma_\epsilon)\frac{\partial \tilde{\epsilon}}{\partial y}\right]$$

where the dissipation, ϵ, is related to the quantity $\tilde{\epsilon}$ by

$$\epsilon = \epsilon_o + \tilde{\epsilon}$$

The quantity ϵ_o is the value of ϵ at $y = 0$, and is defined differently for each model. The eddy viscosity is defined as

$$\nu_T = C_\mu f_\mu k^2/\tilde{\epsilon}$$

f_1, f_2, f_μ, ϵ_o and E → **damping functions**

depend upon $\quad Re_T = \dfrac{k^2}{\tilde{\epsilon}\nu}, \quad R_y = \dfrac{k^{1/2}y}{\nu}, \quad y^+ = \dfrac{u_\tau y}{\nu}$

Jones-Launder Model

$$f_\mu = e^{-2.5/(1+Re_T/50)}$$
$$f_1 = 1$$
$$f_2 = 1 - 0.3e^{-Re_T^2}$$
$$\epsilon_o = 2\nu\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2$$
$$E = 2\nu\nu_T\left(\frac{\partial^2 U}{\partial y^2}\right)^2$$
$$C_{\epsilon 1} = 1.45, \quad C_{\epsilon 2} = 2.00, \quad C_\mu = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\epsilon = 1.3$$

$$k = \tilde{\epsilon} = 0 \quad \text{at} \quad y = 0$$

Launder-Sharma Model

$$f_\mu = e^{-3.4/(1+Re_T/50)^2}$$
$$f_1 = 1$$
$$f_2 = 1 - 0.3e^{-Re_T^2}$$
$$\epsilon_o = 2\nu\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2$$
$$E = 2\nu\nu_T\left(\frac{\partial^2 U}{\partial y^2}\right)^2$$
$$C_{\epsilon 1} = 1.44, \quad C_{\epsilon 2} = 1.92, \quad C_\mu = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\epsilon = 1.3$$

$$k = \tilde{\epsilon} = 0 \quad \text{at} \quad y = 0$$

Lam-Bremhorst Model

$$\epsilon = \nu \frac{\partial^2 k}{\partial y^2} \quad \text{at} \quad y = 0$$

$$\frac{\partial \epsilon}{\partial y} = 0 \quad \text{at} \quad y = 0$$

$$f_\mu = \left(1 - e^{-0.0165 R_y}\right)^2 \left(1 + 20.5/Re_T\right)$$
$$f_1 = 1 + (0.05/f_\mu)^3$$
$$f_2 = 1 - e^{-Re_T^2}$$
$$\epsilon_o = 0$$
$$E = 0$$
$$C_{\epsilon 1} = 1.44, \quad C_{\epsilon 2} = 1.92, \quad C_\mu = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\epsilon = 1.3$$

Chien Model

$$f_\mu = 1 - e^{-0.0115 y^+}$$
$$f_1 = 1$$
$$f_2 = 1 - 0.22 e^{-(Re_T/6)^2}$$
$$\epsilon_o = 2\nu \frac{k}{y^2}$$
$$E = -2\nu \frac{\tilde{\epsilon}}{y^2} e^{-y^+/2}$$
$$C_{\epsilon 1} = 1.35, \quad C_{\epsilon 2} = 1.80, \quad C_\mu = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\epsilon = 1.3$$

$$k = \tilde{\epsilon} = 0 \quad \text{at} \quad y = 0$$

all four models guarantee

$$k \sim y^2 \quad \text{and} \quad \epsilon/k \rightarrow 2\nu/y^2 \quad \text{as} \quad y \rightarrow 0$$

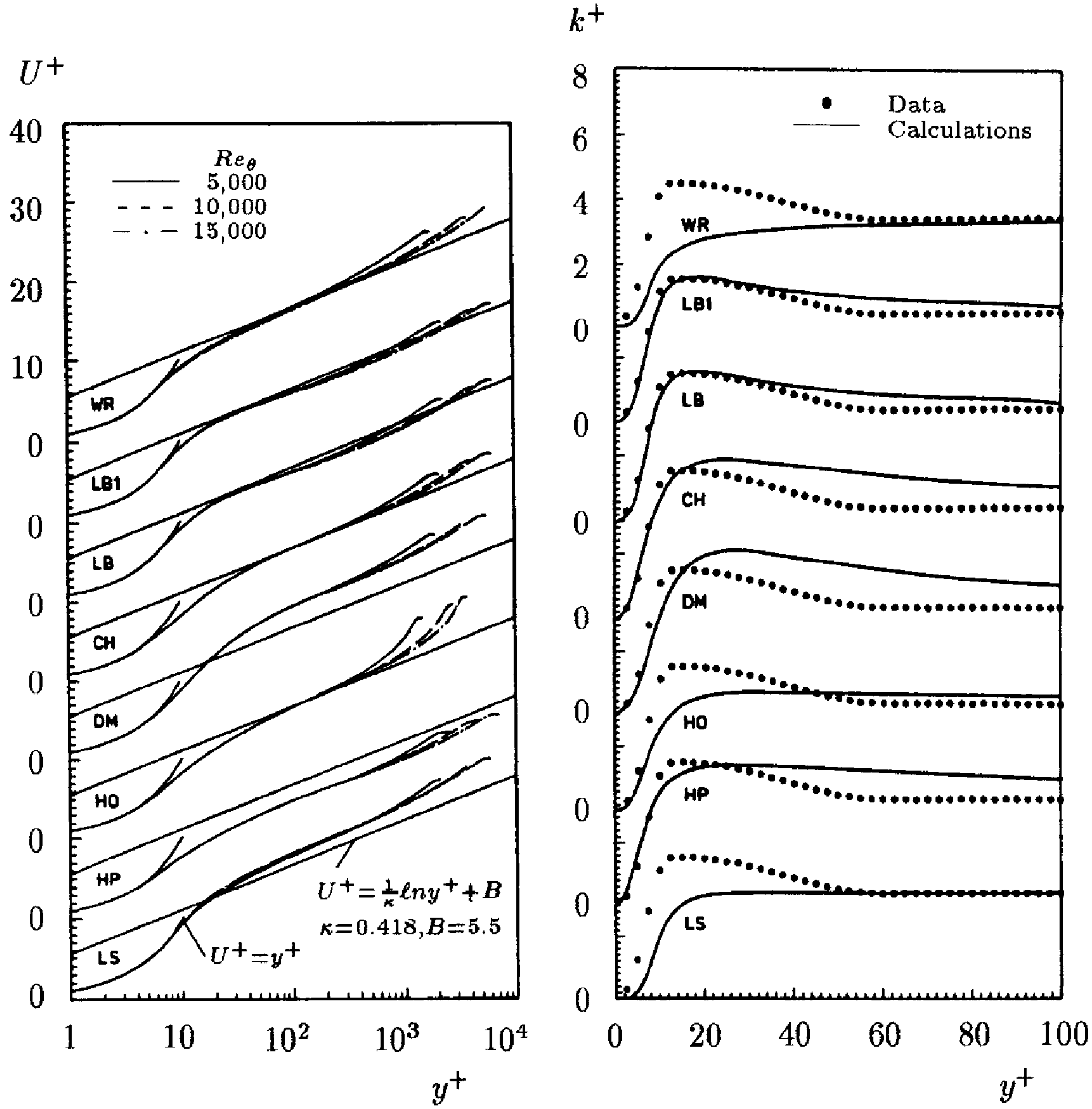
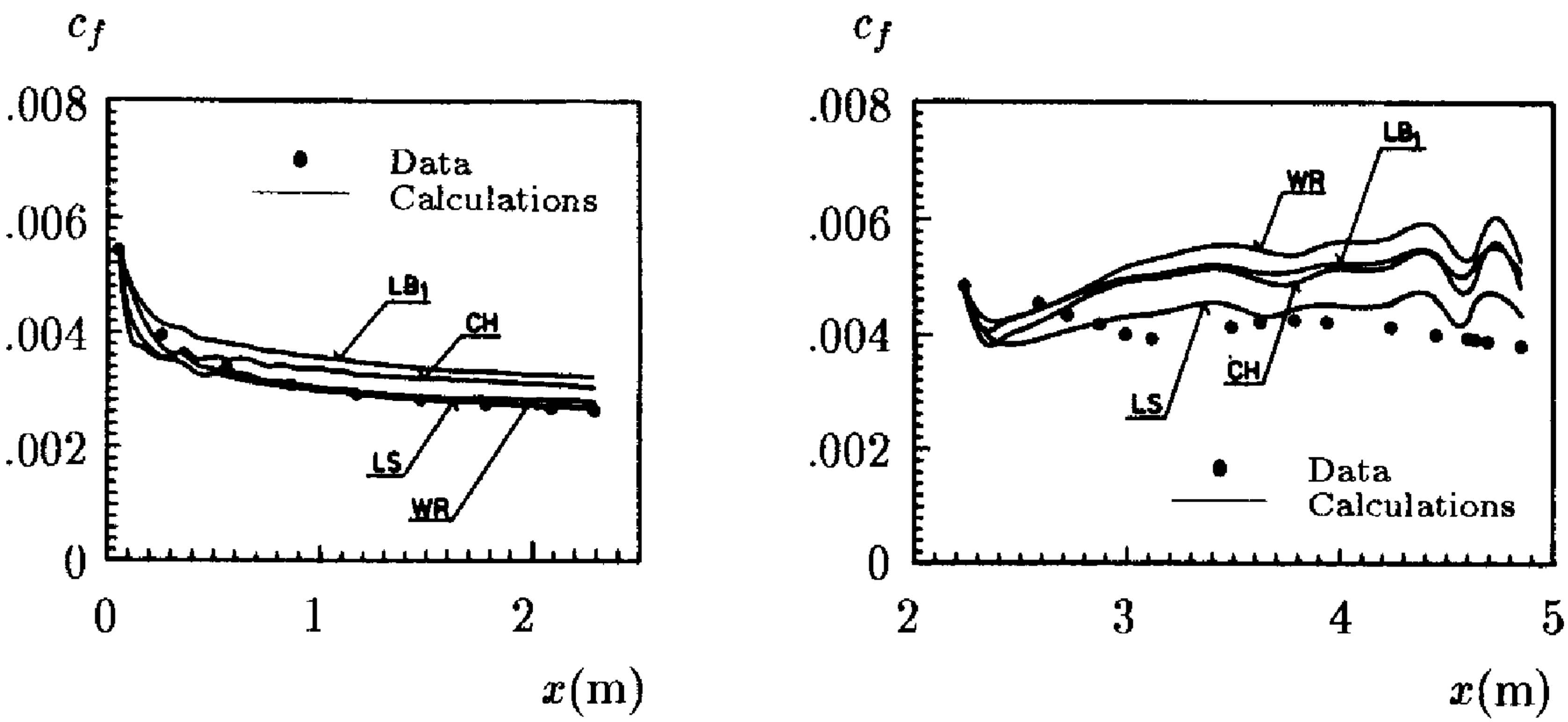


Figure 4.18: Flat-plate boundary layer properties. CH = Chien; DM = Dutoya-Michard; HO = Hoffman; HP = Hassid-Poreh; LB = Lam-Bremhorst with $\epsilon = \nu\partial^2 k/\partial y^2$; LB1 = Lam-Bremhorst with $\partial\epsilon/\partial y = 0$; LS = Launder-Sharma; WR = Wilcox-Rubesin. [From Patel, Rodi and Scheuerer (1985) — Copyright © AIAA 1985 — Used with permission.]



(a) Adverse Pressure Gradient (b) Favorable Pressure Gradient

Figure 4.19: Comparison of computed and measured skin friction for low-Reynolds-number flows with pressure gradient. CH = Chien; LB1 = Lam-Bremhorst with $\partial\epsilon/\partial y = 0$; LS = Launder-Sharma; WR = Wilcox-Rubesin. [From Patel, Rodi and Scheuerer (1985) — Copyright © AIAA 1985 — Used with permission.]

The SST Model

The SST (Shear Stress Transport) model of Menter (1994) is an eddy-viscosity model which includes two main novelties:

1. It is combination of a $k-\omega$ model (in the inner boundary layer) and $k-\varepsilon$ model (in the outer region of and outside of the boundary layer);
2. A limitation of the shear stress in adverse pressure gradient regions is introduced.

The $k-\varepsilon$ model has two main weaknesses: it over-predicts the shear stress in adverse pressure gradient flows because of too large length scale (due to too low dissipation) and it requires near-wall modification (i.e. low-Re number damping functions/terms)

The $k-\omega$ model is better at predicting predicting adverse pressure gradient flow and the standard model of Wilcox (1988) does not use any damping functions. However, the disadvantage of the standard $k-\omega$ model is that it is dependent on the free-stream value of ω (Menter, 1992)

In order to improve both the $k-\varepsilon$ and the $k-\omega$ model, Menter (1994) suggested to combine the two models. Before doing this, it is convenient to transform the $k-\varepsilon$ model into a $k-\omega$ model using the relation $\omega = \varepsilon/(\beta^* k)$, where $\beta^* = c_\mu$.

$$\frac{D\omega}{Dt} = \frac{D\varepsilon/(\beta^* k)}{Dt} = \frac{1}{\beta^* k}\frac{D\varepsilon}{Dt} - \frac{\varepsilon}{\beta^* k^2}\frac{Dk}{Dt} = \frac{1}{\beta^* k}\frac{D\varepsilon}{Dt} - \frac{\omega}{k}\frac{Dk}{Dt}$$

$$\frac{D\omega}{Dt} = \underbrace{\left[\frac{1}{\beta^* k} P_\varepsilon - \frac{\omega}{k} P_k\right]}_{\text{Production, } P_\omega} - \underbrace{\left[\frac{1}{\beta^* k} \Psi_\varepsilon - \frac{\omega}{k} \Psi_k\right]}_{\text{Destruction, } \Psi_\omega} +$$

$$\underbrace{\left[\frac{1}{\beta^* k} D_\varepsilon^T - \frac{\omega}{k} D_k^T\right]}_{\text{Turbulent diffusion, } D_\omega^T} + \underbrace{\left[\frac{\nu}{\beta^* k} \frac{\partial^2 \varepsilon}{\partial x_j^2} + \frac{\nu\omega}{k} \frac{\partial^2 k}{\partial x_j^2}\right]}_{\text{Viscous diffusion, } D_\omega^\nu}$$

- Production term

$$P_\omega = \frac{1}{\beta^* k} P_\varepsilon - \frac{\omega}{k} P_k = C_{\varepsilon 1} \frac{\varepsilon}{\beta^* k^2} P_k - \frac{\omega}{k} P_k =$$
$$= (C_{\varepsilon 1} - 1) \frac{\omega}{k} P_k$$

- Destruction term

$$\Phi_\omega = \frac{1}{\beta^* k} \Psi_\varepsilon - \frac{\omega}{k} \Psi_k = C_{\varepsilon 2} \frac{\varepsilon^2}{k} - \frac{\omega}{k} \varepsilon =$$
$$= (C_{\varepsilon 2} - 1) \beta^* \omega^2$$

- Viscous diffusion term

$$D_\omega^\nu = \frac{\nu}{\beta^* k}\frac{\partial^2 \varepsilon}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{\nu}{k}\frac{\partial^2 \omega k}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} =$$

$$= \frac{\nu}{k}\left[\frac{\partial}{\partial x_j}\left(\omega\frac{\partial k}{\partial x_j} + k\frac{\partial \omega}{\partial x_j}\right)\right] - \nu\frac{\omega}{k}\frac{\partial^2 k}{\partial x_j^2} =$$

$$= \frac{\nu}{k}\left[\frac{\partial \omega}{\partial x_j}\frac{\partial k}{\partial x_j} + \omega\frac{\partial^2 k}{\partial x_j^2} + \frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j} + k\frac{\partial^2 \omega}{\partial x_j^2}\right] - \nu\frac{\omega}{k}\frac{\partial^2 k}{\partial x_j^2} =$$

$$= \frac{2\nu}{k}\frac{\partial \omega}{\partial x_j}\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\nu\frac{\partial \omega}{\partial x_j}\right)$$

The turbulent diffusion term is obtained as

$$D_\omega^T = \frac{2\nu_t}{\sigma_\varepsilon k}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j} + \frac{\omega}{k}\left(\frac{\nu_t}{\sigma_\varepsilon} - \frac{\nu_t}{\sigma_k}\right)\frac{\partial^2 k}{\partial x_j^2} +$$

$$+ \frac{\omega}{k}\left(\frac{1}{\sigma_\varepsilon} - \frac{1}{\sigma_k}\right)\frac{\partial \nu_t}{\partial x_j}\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \omega}{\partial x_j}\right)$$

$\sigma_k = 1$ and $\sigma_\varepsilon = 1.3$

standard $k-\varepsilon$ model

If we assume that $\sigma_k = \sigma_\varepsilon$

$$D_\omega^T = \frac{2\nu_t}{\sigma_\varepsilon k}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \omega}{\partial x_j}\right)$$

ε equation formulated as an equation for ω

$$\frac{\partial}{\partial x_j}(\bar{U}_j\omega) = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + \alpha\frac{\omega}{k}P_k - \beta\omega^2$$
$$+ \frac{2}{k}\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}$$
$$\alpha = C_{\varepsilon 1} - 1 = 0.44,\ \beta = (C_{\varepsilon 2} - 1)\beta^* = 0.0828$$

Since this equation will be used for the outer part of the boundary layer, the viscous part in the last term is omitted.

In the SST model the coefficients are smoothly switched from $k-\omega$ values in the inner region of the boundary layer and $k-\varepsilon$ values in the outer region. Functions of the form

$$F_1 = \tanh(\xi^4), \qquad \xi = \min\left[\max\left\{\frac{\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{y^2\omega}\right\}, \frac{4\sigma_{\omega 2} k}{CD_\omega y^2}\right]$$

$F_1 = 1$ in the near-wall region and $F = 0$ in the outer region

The standard $k-\omega$ SST model reads (Menter, 1994; Menter *et al.*, 2003*b*)

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial x_j}(\bar{U}_j k) = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \beta^* k\omega$$

$$\frac{\partial \omega}{\partial t} + \frac{\partial}{\partial x_j}(\bar{U}_j \omega) = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + P_\omega - \beta\omega^2 + 2(1-F_1)\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}$$

$$F = \tanh(\xi^4)$$

$$\xi = \min\left[\max\left\{\frac{\sqrt{k}}{\beta^* \omega y}, \frac{500\nu}{y^2\omega}\right\}, \frac{4\sigma_{\omega 2} k}{CD_\omega y^2}\right]$$

$$\nu_t = \frac{a_1 k}{\max(a_1\omega, SF_2)}, \qquad P_\omega = \alpha\frac{P_k}{\nu_t}$$

$$F_2 = \tanh(\eta^2), \qquad \eta = \max\left\{\frac{2k^{1/2}}{\beta^* \omega y}, \frac{500\nu}{y^2\omega}\right\}$$

The $v^2 - f$ Model

In the $v^2 - f$ model of Durbin (1991, 1993, 1996) two additional equations, apart from the k and ε-equations, are solved: the wall-normal stress $\overline{v^2}$ and a function f. This is a model which is aimed at improving modeling of wall effects on the turbulence.

$$\frac{\partial \rho \bar{U} \overline{v^2}}{\partial x} + \frac{\partial \rho \bar{V} \overline{v^2}}{\partial y} = \frac{\partial}{\partial y}\left[(\mu + \mu_t)\frac{\partial \overline{v^2}}{\partial y}\right] - 2\overline{p \partial v / \partial y} - \rho \varepsilon_{22}$$

$\bar{V} \ll \bar{U}$ and $\partial/\partial x \ll \partial/\partial y$ → $P_{22} = 0$

$$\varepsilon_{22}^{model} = \frac{\overline{v^2}}{k}\varepsilon$$

The damping of $\overline{v^2}$ → $\Phi'_{22,1}$ and $\Phi'_{22,2}$ go to zero far away from the wall

$$\frac{\partial \rho \bar{U} \overline{v^2}}{\partial x} + \frac{\partial \rho \bar{V} \overline{v^2}}{\partial y} = \frac{\partial}{\partial y}\left[(\mu + \mu_t)\frac{\partial \overline{v^2}}{\partial y}\right] - 2\overline{p \partial v/\partial y} - \rho \varepsilon_{22}$$

Adding ε_{22}^{model} on both sides

$$\frac{\partial \rho \bar{U} \overline{v^2}}{\partial x} + \frac{\partial \rho \bar{V} \overline{v^2}}{\partial y} + \rho \frac{\overline{v^2}}{k}\varepsilon =$$

$$\frac{\partial}{\partial y}\left[(\mu + \mu_t)\frac{\partial \overline{v^2}}{\partial y}\right] - 2\overline{v \partial p/\partial y} - \rho \varepsilon_{22} + \rho \frac{\overline{v^2}}{k}\varepsilon$$

$$\mathcal{P} = -\frac{2}{\rho}\overline{v \partial p/\partial y} - \varepsilon_{22} + \frac{\overline{v^2}}{k}\varepsilon$$

$\mathcal{P}$ is the source term in the $\overline{v^2}$-equation

pressure-strain and the difference between exact and modelled dissipation

$$\frac{\partial \rho \bar{U}\overline{v^2}}{\partial x} + \frac{\partial \rho \bar{V}\overline{v^2}}{\partial y} = \frac{\partial}{\partial y}\left[\left(\mu + \mu_t\right)\frac{\partial \overline{v^2}}{\partial y}\right] + \rho \mathcal{P} - \rho \frac{\overline{v^2}}{k}\varepsilon$$

A new variable $f = \mathcal{P}/k$ is defined

$$L^2 \frac{\partial^2 f}{\partial y^2} - f = -\frac{\Phi_{22}}{k} - \frac{1}{T}\left(\frac{\overline{v^2}}{k} - \frac{2}{3}\right)$$

slow term

rapid term

$$T = \max\left\{\frac{k}{\varepsilon}, C_T\left(\frac{\nu}{\varepsilon}\right)\right\}$$

$$\frac{\Phi_{22}}{k} = \frac{C_1}{T}\left(\frac{2}{3} - \frac{\overline{v^2}}{k}\right) + C_2 \frac{\nu_t}{k}\left(\frac{\partial \bar{U}}{\partial y}\right)^2$$

$$L = C_L \max\left\{\frac{k^{3/2}}{\varepsilon}, C_\eta\left(\frac{\nu^3}{\varepsilon}\right)^{1/4}\right\}$$

$C_\mu = 0.23$

$C_T = 6,\ C_{e1} = 1.44,\ c_{\varepsilon 2} = 1.9,$

$\sigma_k = 0.9,\ \sigma_\varepsilon = 1.3,$

$C_2 = 0.3,\ C_L = 0.2,\ C_\eta = 90.$

$C_1 = 1.3$

Near the wall the fluctuating pressure $\partial p/\partial y = \mathcal{O}(y^0)$ so when $y \longrightarrow 0$ we find from Eq.

$$\mathcal{P} = -\frac{2}{\rho}\overline{v\partial p/\partial y} - \varepsilon_{22} + \frac{\overline{v^2}}{k}\varepsilon \quad \Longrightarrow \quad \mathcal{P} = \mathcal{O}(y^2)$$

Near the wall f goes towards zero due to its boundary condition. This means that the source term $\mathcal{P}$ in the $\overline{v^2}$-equation also goes to zero making $\overline{v^2} \to 0$. Furthermore, as $y \to 0$ we get $f = \mathcal{O}(y^0)$, and thus the near-wall behavior of $\mathcal{P}$ is enforced.

far from the wall when $\partial^2 f/\partial y^2 \simeq 0$ → $(T = k/\varepsilon)$

$$kf \equiv \mathcal{P} \to \Phi_{22} + \varepsilon(\overline{v^2}/k - 2/3)$$

When this expression is inserted in $\overline{v^2}$-equation

$$\frac{\partial \rho \bar{U} \overline{v^2}}{\partial x} + \frac{\partial \rho \bar{V} \overline{v^2}}{\partial y} = \frac{\partial}{\partial y}\left[(\mu + \mu_t)\frac{\partial \overline{v^2}}{\partial y}\right] + \rho\Phi_{22} - \frac{2}{3}\rho\varepsilon$$

In the $\overline{v^2} - f$ model the turbulent viscosity is computed from

$$\nu_t = C_\mu \overline{v^2} T$$

Boundary conditions at the walls are

$$k = 0,\ \overline{v^2} = 0$$

$$\varepsilon = 2\nu k / y^2$$

$$f = -\frac{20\nu^2 \overline{v^2}}{\varepsilon y^4}$$

The boundary condition for f makes the equation system numerically unstable. One way to get around that problem is to solve both the k, ε and $\overline{v^2}$, f equations coupled

مدلهای بهبود یافته برای این روش وجود دارد که به جزوه داویدسون مراجعه شود.