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Prediction of Turbulent flow – Part 4

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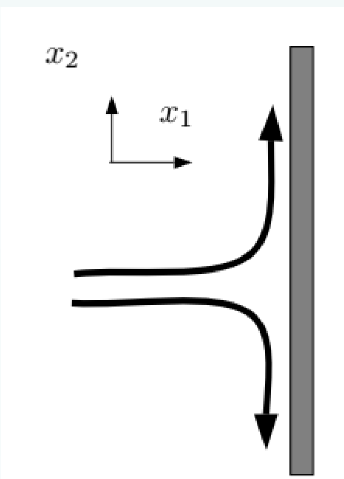
Realizability

There are a number of realizability constraints. The usual two ones are that all normal stresses should stay positive and that the correlation coefficient for the shear stress should not exceed one, i.e.

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$

$$\frac{|\overline{v_i' v_j'}|}{\left(\overline{v_i'^2}\;\overline{v_j'^2}\right)^{1/2}} \leq 1 \text{ no summation over } i \text{ and } j,\ i \neq j$$

These criteria are seldom used in RSMs. However, satisfying the first criteria is actually of importance for eddy-viscosity models in stagnation flow



$$\overline{v_i' v_j'} = -\nu_t \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) + \frac{1}{3}\delta_{ij}\overline{v_k' v_k'} = -2\nu_t \bar{s}_{ij} + \frac{2}{3}\delta_{ij} k$$

in the x_1 direction
$$\overline{v_1'^2} = \frac{2}{3}k - 2\nu_t \frac{\partial \bar{v}_1}{\partial x_1} = \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

It is seen that if $\bar{s}_{11}$ gets too large then $\overline{v_1'^2} < 0$ which is nonphysical, i.e. non-realizable

Let's now briefly repeat the concept "invariants". This means something that is independent of the coordinate system بدون تغییر

$$\lambda_{1,2} = \pm\left(-I_2^{2D}\right)^{1/2} = \pm\left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

The eigenvalues of $\bar{s}_{ij}$ correspond to the strains in the principal axis.

Hence, $\bar{s}_{11}$ in is replaced by the largest eigenvalue so that $\overline{v_1'^2} = \frac{2}{3}k - 2\nu_t\lambda_1$

The requirement $\overline{v_1'^2} \geq 0$ gives $\nu_t \leq \frac{k}{3|\lambda_1|} = \frac{k}{3}\left(\frac{2}{\bar{s}_{ij}\bar{s}_{ij}}\right)^{1/2}$ $\Rightarrow$ $|\lambda_k| = \left(\frac{2\bar{s}_{ij}\bar{s}_{ij}}{3}\right)^{1/2}$

This is a simple modification of an eddy-viscosity model, and it ensures that the normal stresses stay positive.

Another extension of the k-ε model was developed by Yakhot et al. With techniques from renormalization group theory they proposed the so-called $\boxed{\text{RNG } k\text{-}\varepsilon \text{ model}}$

$$u\frac{\partial k}{\partial x} + v\frac{\partial k}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\varepsilon_m}{\sigma_k}\frac{\partial k}{\partial y}\right) + \varepsilon_m\left(\frac{\partial u}{\partial y}\right)^2 - \varepsilon$$

$$u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\varepsilon_m}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) + c_{\varepsilon_1}\frac{\varepsilon}{k}\varepsilon_m\left(\frac{\partial u}{\partial y}\right)^2 - c_{\varepsilon_2}\frac{\varepsilon^2}{k}$$

$$c_{\varepsilon_1} = 1.42, \qquad c_{\varepsilon_2} = 1.68 + \frac{c_\mu\lambda^3(1-\lambda/\lambda_0)}{1+0.012\lambda^3}$$

$$\lambda = \frac{k}{\varepsilon}\sqrt{2s_{ij}s_{ji}}, \qquad \lambda_0 = 4.38, \qquad c_\mu = 0.085$$

$$\sigma_k = 0.72, \qquad \sigma_\varepsilon = 0.72, \qquad s_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

Cost and ease of use

In this discussion we take the k–ε model as a reference: the model is incorporated into most commercial CFD codes, and it is generally regarded as being easy to use and computationally inexpensive when it is used in conjunction with wall functions.

If wall functions are not employed, the task of performing k–ε calculations for the viscous near-wall region is significantly more difficult and expensive. This is due to the need to resolve k and ε (which vary strongly in the near-wall region); and also to the fact that the source terms in these equations become very large close to the wall. (In the log-law region, the term $C_{\varepsilon 2}\varepsilon^2/k$ varies as u_τ^4/y^2.)

The Spalart–Allmaras model is – by design – much simpler and less expensive for near-wall aerodynamic flows. This is because, compared with k and ε, the turbulent viscosity ν_T behaves benignly in the near-wall region, and is more easily resolved.

In comparison with the k–ε model, Reynolds-stress models are somewhat more difficult and costly because

(i) in general there are seven turbulence equations to be solved (for $\langle u_i u_j \rangle$ and ε) instead of two (for k and ε);

(ii) the model Reynolds-stress equation is substantially more complicated than the k equation (and hence requires coding effort); and

(iii) in the mean-momentum equation, the term

$$-\frac{\partial}{\partial x_i}\langle u_i u_j \rangle$$

results in a less favorable numerical coupling between the flow and turbulence equations than does the corresponding term

$$\frac{\partial}{\partial x_i}\left[\nu_{\mathrm{T}}\left(\frac{\partial\langle U_i\rangle}{\partial x_j}+\frac{\partial\langle U_j\rangle}{\partial x_i}\right)\right]$$ in the k–ε model

Typically, the CPU time required for a Reynolds-stress-model calculation can be more than that for a k–ε calculation by a factor of two.

The primary motivation for the use of algebraic stress models is to avoid the cost and difficulty of solving the Reynolds-stress model equations. However, the general experience is that these benefits are not realized. The algebraic stress model equations are coupled nonlinear equations, often with multiple roots, which are non-trivial to solve economically. In addition, with respect to item (iii) above, algebraic stress models have the same disadvantage as Reynolds-stress models. As discussed in the previous section, algebraic stress models can be recast as nonlinear viscosity models. These add little cost and difficulty to k–ε-model calculations.

Range of applicability

The basic k–ε and Reynolds-stress models can be applied to any turbulent flow. They also provide lengthscale and timescale information that can be used in the modelling of additional processes. Consequently, they provide a basis for the modelling of turbulent reactive flows, multi-phase flows, etc. Model transport equations for the scalar flux can be solved in conjunction with a Reynolds-stress model to provide closure to the mean scalar equations. Such so-called *second-moment closures* have successfully been extended to atmospheric flows in which buoyancy effects are significant (e.g., Zeman and Lumley (1976)). Although it can, in principle, be applied to any turbulent flow (in the class considered), the Spalart–Allmaras model is intended only for aerodynamic applications.

Accuracy

(i) The k–ε model performs reasonably well for two-dimensional thin shear flows in which the mean streamline curvature and mean pressure gradient are small.

(ii) For boundary layers with strong pressure gradients the k–ε model performs poorly. However, the k–ω model performs satisfactorily, and indeed its performance is superior for many flows.

(iii) For flows far removed from simple shear (e.g., the impinging jet and three-dimensional flows), the k–ε model can fail profoundly.

(iv) The use of nonlinear viscosity models is beneficial and allows the calculation of secondary flows (which cannot be calculated using the isotropic viscosity hypothesis).

(v) Reynolds-stress models can be successful (whereas turbulent viscosity models are not) in calculating flows with significant mean streamline curvature, flows with strong swirl or mean rotation, secondary flows in ducts, and flows with rapid variations in the mean flow.

(vi) Reynolds-stress-model calculations are sensitive to the details of the modelling of the pressure–rate-of-strain tensor, including wall-reflection terms.

(vii) The elliptic relaxation models (both Reynolds-stress and k–ε–$\overline{v^2}$) have been quite successful in application to a number of challenging two-dimensional flows, including the impinging jet, and separated boundary layers.

(viii) The dissipation equation is frequently blamed for poor performance of a model. For many flows, much improved performance can be obtained by altering the model constants ($C_{\varepsilon 1}$ or $C_{\varepsilon 2}$) or by adding correction terms. No correction to the dissipation equation that is effective in all flows has been found.

The SST Model

The SST (Shear Stress Transport) model is an eddy-viscosity model which includes two main novelties:

1. It is combination of a $k-\omega$ model (in the inner boundary layer) and $k-\varepsilon$ model (in the outer region of the boundary layer as well as outside of it);
2. A limitation of the shear stress in adverse pressure gradient regions.

The $k-\varepsilon$ model has two main weaknesses: it over-predicts the shear stress in adverse pressure gradient flows because of too large length scale (due to too low dissipation) and it requires near-wall modification (i.e. low-Re number damping functions/terms).

The $k-\omega$ model is better than the $k-\varepsilon$ model at predicting adverse pressure gradient flow and the standard $k-\omega$ model does not use any damping functions.

However, the disadvantage of the standard $k-\omega$ model is that it is dependent on the free-stream value of ω

In order to improve both the $k-\varepsilon$ and the $k-\omega$ model, it was suggested to combine the two models.

it is convenient to transform the $k-\varepsilon$ model into a $k-\omega$ model using the relation $\omega=\varepsilon/(\beta^* k)$, where $\beta^*=c_\mu$.

$$\begin{aligned}\frac{d\omega}{dt} &= \frac{d}{dt}\left(\frac{\varepsilon}{\beta^* k}\right)=\frac{1}{\beta^* k}\frac{d\varepsilon}{dt}+\frac{\varepsilon}{\beta^*}\frac{d(1/k)}{dt}\\ &= \frac{1}{\beta^* k}\frac{d\varepsilon}{dt}-\frac{\varepsilon}{\beta^* k^2}\frac{dk}{dt}=\frac{1}{\beta^* k}\frac{d\varepsilon}{dt}-\frac{\omega}{k}\frac{dk}{dt}\end{aligned}$$

$$\frac{D\omega}{Dt}=\frac{\partial}{\partial x_k}\left[\left(\nu+\frac{\varepsilon_m}{\sigma_\omega}\right)\frac{\partial\omega}{\partial x_k}\right]+\alpha\frac{\omega}{k}R_{ik}\frac{\partial\bar{u}_i}{\partial x_k}-\beta\omega^2$$

به روش بالا بقیه قسمتها را نیز تبدیل می کنیم

$$\begin{aligned}\frac{D\omega}{Dt} =& \underbrace{\left[\frac{1}{\beta^* k}P_\varepsilon-\frac{\omega}{k}P^k\right]}_{\text{Production, } P_\omega}-\underbrace{\left[\frac{1}{\beta^* k}\Psi_\varepsilon-\frac{\omega}{k}\Psi_k\right]}_{\text{Destruction, } \Psi_\omega}+\\ & \underbrace{\left[\frac{1}{\beta^* k}D_\varepsilon^T-\frac{\omega}{k}D_k^T\right]}_{\text{Turbulent diffusion, } D_\omega^T}+\underbrace{\left[\frac{\nu}{\beta^* k}\frac{\partial^2\varepsilon}{\partial x_j^2}-\frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2}\right]}_{\text{Viscous diffusion, } D_\omega^\nu}\end{aligned}$$

- Production term

$$P_\omega = \frac{1}{\beta^* k} P_\varepsilon - \frac{\omega}{k} P^k = C_{\varepsilon 1} \frac{\varepsilon}{\beta^* k^2} P^k - \frac{\omega}{k} P^k$$
$$= (C_{\varepsilon 1} - 1) \frac{\omega}{k} P^k$$

- Destruction term

$$\Psi_\omega = \frac{1}{\beta^* k} \Psi_\varepsilon - \frac{\omega}{k} \Psi_k = C_{\varepsilon 2} \frac{\varepsilon^2}{k} - \frac{\omega}{k} \varepsilon$$
$$= (C_{\varepsilon 2} - 1) \beta^* \omega^2$$

- Viscous diffusion term

$$D_\omega^\nu = \frac{\nu}{\beta^* k} \frac{\partial^2 \varepsilon}{\partial x_j^2} - \frac{\nu\omega}{k} \frac{\partial^2 k}{\partial x_j^2} = \frac{\nu}{k} \frac{\partial^2 \omega k}{\partial x_j^2} - \frac{\nu\omega}{k} \frac{\partial^2 k}{\partial x_j^2}$$
$$= \frac{\nu}{k} \left[\frac{\partial}{\partial x_j} \left(\omega \frac{\partial k}{\partial x_j} + k \frac{\partial \omega}{\partial x_j} \right) \right] - \nu \frac{\omega}{k} \frac{\partial^2 k}{\partial x_j^2}$$
$$= \frac{\nu}{k} \left[\frac{\partial \omega}{\partial x_j} \frac{\partial k}{\partial x_j} + \omega \frac{\partial^2 k}{\partial x_j^2} + \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_j} + k \frac{\partial^2 \omega}{\partial x_j^2} \right] - \nu \frac{\omega}{k} \frac{\partial^2 k}{\partial x_j^2}$$
$$= \frac{2\nu}{k} \frac{\partial \omega}{\partial x_j} \frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j} \left(\nu \frac{\partial \omega}{\partial x_j} \right)$$

$$D_\omega^T = \frac{2\nu_t}{\sigma_\varepsilon k}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j} + \frac{\omega}{k}\left(\frac{\nu_t}{\sigma_\varepsilon} - \frac{\nu_t}{\sigma_k}\right)\frac{\partial^2 k}{\partial x_j^2} + $$
$$+ \frac{\omega}{k}\left(\frac{1}{\sigma_\varepsilon} - \frac{1}{\sigma_k}\right)\frac{\partial \nu_t}{\partial x_j}\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \omega}{\partial x_j}\right)$$

In the standard $k-\varepsilon$ model we have $\sigma_k = 1$ and $\sigma_\varepsilon = 1.3$. If we assume that $\sigma_k = \sigma_\varepsilon$ in the second and third term of the right-hand side, we can considerably simplify the turbulence diffusion so that

$$D_\omega^T = \frac{2\nu_t}{\sigma_\varepsilon k}\frac{\partial k}{\partial x_j}\frac{\partial \omega}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \omega}{\partial x_j}\right)$$

We can now finally write the ε equation formulated as an equation for ω

$$\frac{\partial}{\partial x_j}(\bar{v}_j\omega) = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \omega}{\partial x_j}\right] + \alpha\frac{\omega}{k}P^k - \beta\omega^2$$
$$+ \frac{2}{k}\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}$$
$$\alpha = C_{\varepsilon 1} - 1 = 0.44, \beta = (C_{\varepsilon 2} - 1)\beta^* = 0.0828$$

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$$\frac{\partial}{\partial x_j}(\bar{v}_j\omega) = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \omega}{\partial x_j}\right] + \alpha\frac{\omega}{k}P^k - \beta\omega^2$$

cross-diffusion

$$+\boxed{\frac{2}{k}\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}}$$

$$\alpha = C_{\varepsilon 1} - 1 = 0.44, \beta = (C_{\varepsilon 2} - 1)\beta^* = 0.0828$$

Since the $k-\varepsilon$ model will be used for the outer part of the boundary layer, the viscous part of the cross-diffusion term (second line) is usually neglected (the viscous terms are negligible in the outer region). The turbulent viscosity is computed as (using dimensional analysis)

$$\nu_t = \frac{k}{\omega}$$

In the SST model the coefficients are smoothly switched from $k-\omega$ values in the inner region of the boundary layer to $k-\varepsilon$ values in the outer region. Functions of the form

$$F_1 = \tanh(\xi^4), \quad \xi = \min\left[\max\left\{\frac{\sqrt{k}}{\beta^*\omega y}, \frac{500\nu}{y^2\omega}\right\}, \frac{4\sigma_{\omega 2}k}{CD_\omega y^2}\right]$$

are used. $F_1 = 1$ in the near-wall region and $F_1 = 0$ in the outer region. The β-coefficient, for example, is computed as

$$\beta_{SST} = F_1\beta_{k-\omega} + (1 - F_1)\beta_{k-\varepsilon}$$

where $\beta_{k-\omega} = 0.075$ and $\beta_{k-\varepsilon} = 0.0828$. Since the standard $k - \omega$ model does not include any cross-diffusion term, the last term in the ω equation should only be active in the $k - \varepsilon$ region; hence it is multiplied by $(1 - F_1)$.

The V2F Model

In the V2F model two additional equations, apart from the k and ε-equations, are solved: the wall-normal stress $\overline{v_2'^2}$ and a function f. This is a model which aims at improving modeling of wall effects on the turbulence.

Walls affect the fluctuations in the wall-normal direction, $\overline{v_2'^2}$, in two ways. The wall damping of $\overline{v_2'^2}$ is felt by the turbulence fairly far from the wall ($x_2^+ \lesssim 200$) through the pressure field (i.e. the pressure-strain term) whereas the viscous damping takes place within the viscous and buffer layer ($x_2^+ \lesssim 10$). In usual eddy-viscosity models both these effects are accounted for through damping functions. The damping of $\overline{v_2'^2}$ is in the RSM accounted for through the modeled pressure-strain terms $\Phi_{22,1w}$ and $\Phi_{22,2w}$ (see Eqs. 11.95 and Eq. 11.96). They go to zero far away from the wall ($x_2^+ \gtrsim 400$).

In the V2F model the problem of accounting for the wall damping of $\overline{v_2'^2}$ is simply resolved by solving its transport equation. The $\overline{v_2'^2}$ equation in boundary-layer form reads (see Eq. 9.16 at p. 107)

$$\frac{\partial \rho \bar{v}_1 \overline{v_2'^2}}{\partial x_1} + \frac{\partial \rho \bar{v}_2 \overline{v_2'^2}}{\partial x_2} = \frac{\partial}{\partial x_2}\left[(\mu + \mu_t)\frac{\partial \overline{v_2'^2}}{\partial x_2}\right] - 2\overline{v_2' \frac{\partial p'}{\partial x_2}} - \rho \varepsilon_{22}$$

in boundary-layer

$\bar{v}_2 \ll \bar{v}_1$ and $\partial/\partial x_1 \ll \partial/\partial x_2$.

$$\varepsilon_{22}^{model} = \frac{\overline{v_2'^2}}{k}\varepsilon$$

This is a more elaborate model than in RSM

Add and subtract ε_{22}^{model}

$$\frac{\partial \rho \bar{v}_1 \overline{v_2'^2}}{\partial x_1} + \frac{\partial \rho \bar{v}_2 \overline{v_2'^2}}{\partial x_2} =$$

$$\frac{\partial}{\partial x_2}\left[(\mu+\mu_t)\frac{\partial \overline{v_2'^2}}{\partial x_2}\right] - 2\overline{v_2'\frac{\partial p'}{\partial x_2}} - \rho\varepsilon_{22} + \rho\frac{\overline{v_2'^2}}{k}\varepsilon - \rho\frac{\overline{v_2'^2}}{k}\varepsilon$$

In the V2F model $\mathcal{P}$ is now defined as

$$\mathcal{P} = -\frac{2}{\rho}\overline{v_2'\frac{\partial p'}{\partial x_2}} - \varepsilon_{22} + \frac{\overline{v_2'^2}}{k}\varepsilon$$

$$\frac{\partial \rho \bar{v}_1 \overline{v_2'^2}}{\partial x_1} + \frac{\partial \rho \bar{v}_2 \overline{v_2'^2}}{\partial x_2} = \frac{\partial}{\partial x_2}\left[(\mu+\mu_t)\frac{\partial \overline{v_2'^2}}{\partial x_2}\right] + \rho\mathcal{P} - \rho\frac{\overline{v_2'^2}}{k}\varepsilon$$

$\mathcal{P}$ is the source term in the $\overline{v_2'^2}$-equation above, and it includes the velocity-pressure gradient term and the difference between the exact and the modeled dissipation. Note that this term is commonly split into a diffusion term and the pressure-strain term as

$$\overline{v_2'\frac{\partial p'}{\partial x_2}} = \frac{\partial \overline{v_2' p'}}{\partial x_2} - \overline{p'\frac{\partial v_2'}{\partial x_2}}$$

A new variable $f = \mathcal{P}/k$ is defined and a relaxation equation is formulated for f as

$$L^2 \frac{\partial^2 f}{\partial x_2^2} - f = -\frac{\Phi_{22}}{\rho k} - \frac{1}{T}\left(\frac{\overline{v_2'^2}}{k} - \frac{2}{3}\right)$$

$$T = \max\left\{\frac{k}{\varepsilon}, C_T\left(\frac{\nu}{\varepsilon}\right)^{1/2}\right\}$$

$$\frac{\Phi_{22}}{\rho k} = \frac{C_1}{T}\left(\frac{2}{3} - \frac{\overline{v_2'^2}}{k}\right) + C_2\frac{\nu_t}{k}\left(\frac{\partial \bar{v}_1}{\partial x_2}\right)^2$$

$$L = C_L \max\left\{\frac{k^{3/2}}{\varepsilon}, C_\eta\left(\frac{\nu^3}{\varepsilon}\right)^{1/4}\right\}$$

where Φ_{22} is the IP model of the pressure-strain term.

the following values: $c_\mu = 0.23,\ C_T = 6,\ c_{\varepsilon 1} = 1.44,\ c_{\varepsilon 2} = 1.9,\ \sigma_k = 0.9,\ \sigma_\varepsilon = 1.3,\ C_1 = 1.3,\ C_2 = 0.3,\ C_L = 0.2,\ C_\eta = 90.$

$k = 0,\ \overline{v_2'^2} = 0$ $\quad \varepsilon = 2\nu k/x_2^2$ $\quad f = -\dfrac{20\nu^2\overline{v_2'^2}}{\varepsilon x_2^4}$

The boundary condition for f makes the equation system numerically unstable.

One way to get around that problem is to solve both the k, ε and $\overline{v_2'^2}$, f equations coupled [64]. An alternative is to use the $\zeta - f$ model [65] which is more stable. In this model they solve for the ratio $\overline{v_2'^2}/k$ instead of for $\overline{v_2'^2}$ which gives a simpler wall boundary condition for f, namely $f = 0$.

Modified V2F model

In [66] they proposed a modification of the V2F model allowing the simple explicit boundary condition $f = 0$ at walls. They introduced a new variable

$$f^* = f - 5\varepsilon v^2/k^2$$

and they neglected the term

$$-5L^2 \frac{\partial^2}{\partial x_j \partial x_j}\left(\frac{\varepsilon v^2}{k^2}\right)$$

The resulting $\overline{v_2'^2}$ and f^*-equation read [66]

$$\frac{\partial \bar{v}_j v^2}{\partial x_j} = \frac{\partial}{\partial x_j}\left[(\nu + \nu_t)\frac{\partial v^2}{\partial x_j}\right] + kf^* - 6\frac{v^2}{k}\varepsilon$$

$$-L^2\frac{\partial^2 f^*}{\partial x_j \partial x_j} + f^* = -\frac{1}{T}\left[(C_1 - 6)\frac{v^2}{k} - \frac{2}{3}(C_1 - 1)\right] + C_2\frac{P^k}{k}$$

$$P^k = \nu_t\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right)\frac{\partial \bar{v}_i}{\partial x_j}$$

$$T = \max\left\{\frac{k}{\varepsilon}, 6\left(\frac{\nu}{\varepsilon}\right)^{1/2}\right\}$$

$$L = C_L \max\left\{\frac{k^{3/2}}{\varepsilon}, C_\eta\left(\frac{\nu^3}{\varepsilon}\right)^{1/4}\right\}$$

Boundary conditions at the walls are

$$k = 0,\ v^2 = 0$$
$$\varepsilon = 2\nu k/x_2^2$$
$$f^* = 0$$

This modified model is numerically much more stable. Note that the modified model is identical to the original model far from the wall.

Realizable V2F model

The realizable condition for stagnation flow (see p. 161) is used also for the V2F model, and they read [66]

$$T = \min\left[\frac{k}{\varepsilon}, \frac{0.6k}{\sqrt{6}C_\mu v^2 \left(\bar{s}_{ij}\bar{s}_{ij}\right)^{1/2}}\right]$$

$$L = \min\left[\frac{k^{3/2}}{\varepsilon}, \frac{k^{3/2}}{\sqrt{6}C_\mu v^2 \left(2\bar{s}_{ij}\bar{s}_{ij}\right)^{1/2}}\right]$$