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Prediction of Turbulent flow – Part 6

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DES: Detached-Eddy-Simulations

DES (Detached Eddy Simulation) is a mix of LES and URANS. The aim is to treat the boundary layer with RANS and capture the outer detached eddies with LES.

$$\frac{\partial \rho \tilde{\nu}_t}{\partial t} + \frac{\partial \rho \bar{v}_j \tilde{\nu}_t}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\mu + \mu_t}{\sigma_{\tilde{\nu}_t}} \frac{\partial \tilde{\nu}_t}{\partial x_j}\right) + \frac{C_{b2}\rho}{\sigma_{\tilde{\nu}_t}} \frac{\partial \tilde{\nu}_t}{\partial x_j} \frac{\partial \tilde{\nu}_t}{\partial x_j} + P - \Psi$$

$$\nu_t = \tilde{\nu}_t f_1$$

The production term P and the destruction term Ψ have the form

$$P = C_{b1}\rho\left(\bar{s} + \frac{\tilde{\nu}_t}{\kappa^2 d^2} f_2\right)\tilde{\nu}_t \qquad \bar{s} = \left(2\bar{s}_{ij}\bar{s}_{ij}\right)^{1/2}, \quad \Psi = C_{w1}\rho f_w \left(\frac{\tilde{\nu}_t}{d}\right)^2$$

d in the RANS SA model is equal to the distance to the nearest wall.

In DES $\quad \tilde{d} = \min(d, C_{des}\Delta) \qquad \Delta = \max(\Delta x_\xi, \Delta x_\eta, \Delta x_\zeta)$

constant C_{des} is usually set to 0.65

In the boundary layer $d < C_{des}\Delta$ and thus the model operates in RANS mode.

Outside the turbulent boundary layer $d > C_{des}\Delta$ so that the model operates in LES

DES based on two-equation models

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j k) = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P^k - \varepsilon_T$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j \varepsilon) = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + \frac{\varepsilon}{k}(C_1 P^k - C_2 \varepsilon)$$

$$P^k = 2\nu_t \bar{s}_{ij}\bar{s}_{ij}, \quad \nu_t = k^{1/2}\ell_t$$

The turbulent length scale, ℓ_t, and the turbulent dissipation, ε_T, are computed as

$$\ell_t = \min\left(C_\mu \frac{k^{3/2}}{\varepsilon}, C_{DES}\Delta\right) \qquad \varepsilon_T = \max\left(\varepsilon, C_\varepsilon \frac{k^{3/2}}{\Delta}\right)$$

When the grid is sufficiently fine, the length scale is taken as Δ. The result is that the dissipation in the k equation increases so that k decreases which gives a reduced ν_t. In regions where the turbulent length scales are taken from Δ (LES mode) the ε-equation is still solved, but ε is not used. However, ε is needed as soon as the model switches to RANS model again.

DES based on the $k - \omega$ SST model

$$\frac{\partial k}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j k) = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P^k - \beta^* k\omega$$

$$\frac{\partial \omega}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j \omega) = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + \alpha\frac{P^k}{\nu_t} - \beta\omega^2 + 2(1 - F_1)\sigma_{\omega 2}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}$$

$$F_1 = \tanh(\xi^4), \quad \xi = \min\left[\max\left\{\frac{\sqrt{k}}{\beta^*\omega d}, \frac{500\nu}{d^2\omega}\right\}\frac{4\sigma_{\omega_{k-\varepsilon}}k}{CD_\omega d^2}\right]$$

$$CD_\omega = \max\left\{2\sigma_{\omega_{k-\varepsilon}}\frac{1}{\omega}\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}, 10^{-10}\right\}$$

$$\nu_t = \frac{a_1 k}{\max(a_1\omega, |\bar{s}|F_2)}$$

$$F_2 = \tanh(\eta^2), \quad \eta = \max\left\{\frac{2k^{1/2}}{\beta^*\omega d}, \frac{500\nu}{d^2\omega}\right\}$$

$$\alpha = F_1\alpha_{k-\omega} + (1-F_1)\alpha_{k-\varepsilon}$$

where d is the distance to the closest wall node. The SST model behaves as a $k-\omega$ model near the wall where $F_1 = 1$ and a $k-\varepsilon$ model far from walls ($F_1 = 0$). All coefficients are blended between the $k-\omega$ and the $k-\varepsilon$ model using the function F_1, for α, for example,

$$\beta^* = 0.09, \quad a_1 = 0.3$$

$$\alpha_{k-\omega} = 5/9, \quad \beta_{k-\omega} = 3/40, \quad \sigma_{k,k-\omega} = 0.85, \quad \sigma_{\omega,k-\omega} = 0.5$$

$$\alpha_{k-\varepsilon} = 0.44, \quad \beta_{k-\varepsilon} = 0.0828, \quad \sigma_{k,k-\varepsilon} = 1, \quad \sigma_{\omega,k-\varepsilon} = 0.856.$$

In DES the dissipation term in the k equation is modified as

$$\beta^* k\omega \to \beta^* k\omega F_{DES}, \quad F_{DES} = \max\left\{\frac{L_t}{C_{DES}\Delta}, 1\right\} = \max\left\{\frac{k^{1/2}}{C_{DES}\beta^*\Delta\omega}, 1\right\}$$

$$\Delta = \max\{\Delta x_1, \Delta x_2, \Delta x_3\}, \quad L_t = \frac{k^{1/2}}{\beta^*\omega} \qquad C_{DES} = 0.61$$

$$\Delta = \max\{\Delta x_1, \Delta x_2, \Delta x_3\}$$

The larger the maximum cell size (usually Δx or Δz) is made, the further out from the wall does the switch take place.

DDES

Δx or Δz (Δx_1 or Δx_3) are too small (smaller than the boundary layer thickness, δ).

F_{DES} term switches to LES in the boundary layer

This means that the flow in the boundary layer is treated in LES mode with too a coarse mesh. This results in a poorly resolved LES and hence inaccurate predictions.

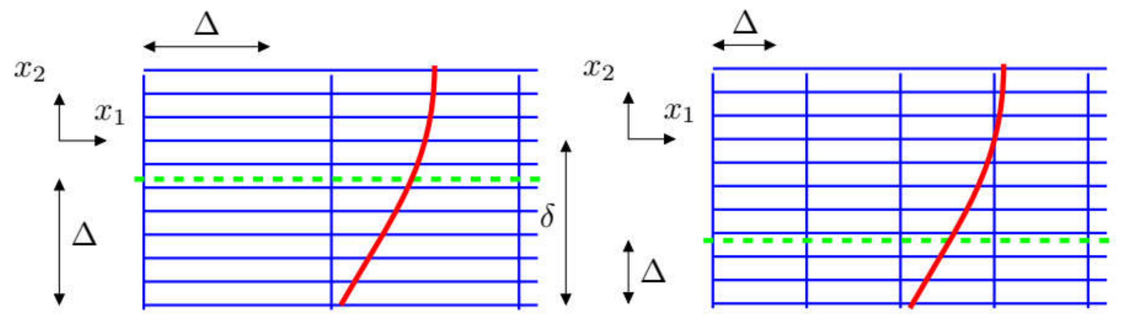


Figure 20.1: Grid (in blue) and a velocity profile (in red). RANS-LES interface is shown by the dashed-green line.

The grid on the left is a good DES mesh

Here $\Delta x = C_{DES}\partial x_1$ (assuming $\Delta x_3 < \Delta x_1$) is proportional to the boundary layer

The grid on the right $\Delta < \delta$ hence the outer part of the boundary layer is in LES mode

Different proposals have been made to *protect* the boundary layer from the LES mode

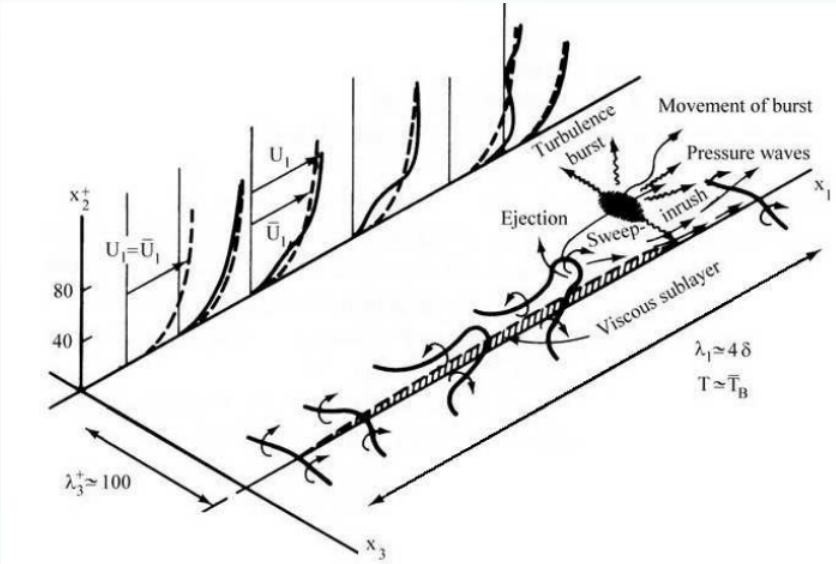
$$F_{DDES} = \max\left\{\frac{L_t}{C_{DES}\Delta}(1 - F_S), 1\right\}$$

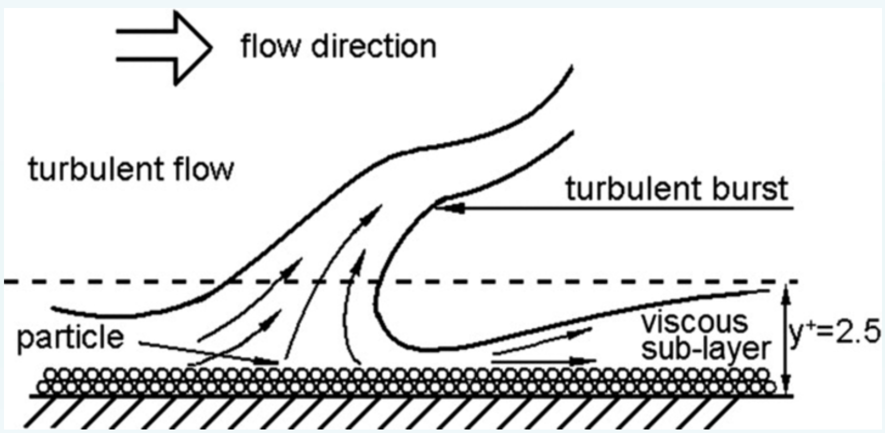
F_S is taken as F_1 or F_2 of the SST model

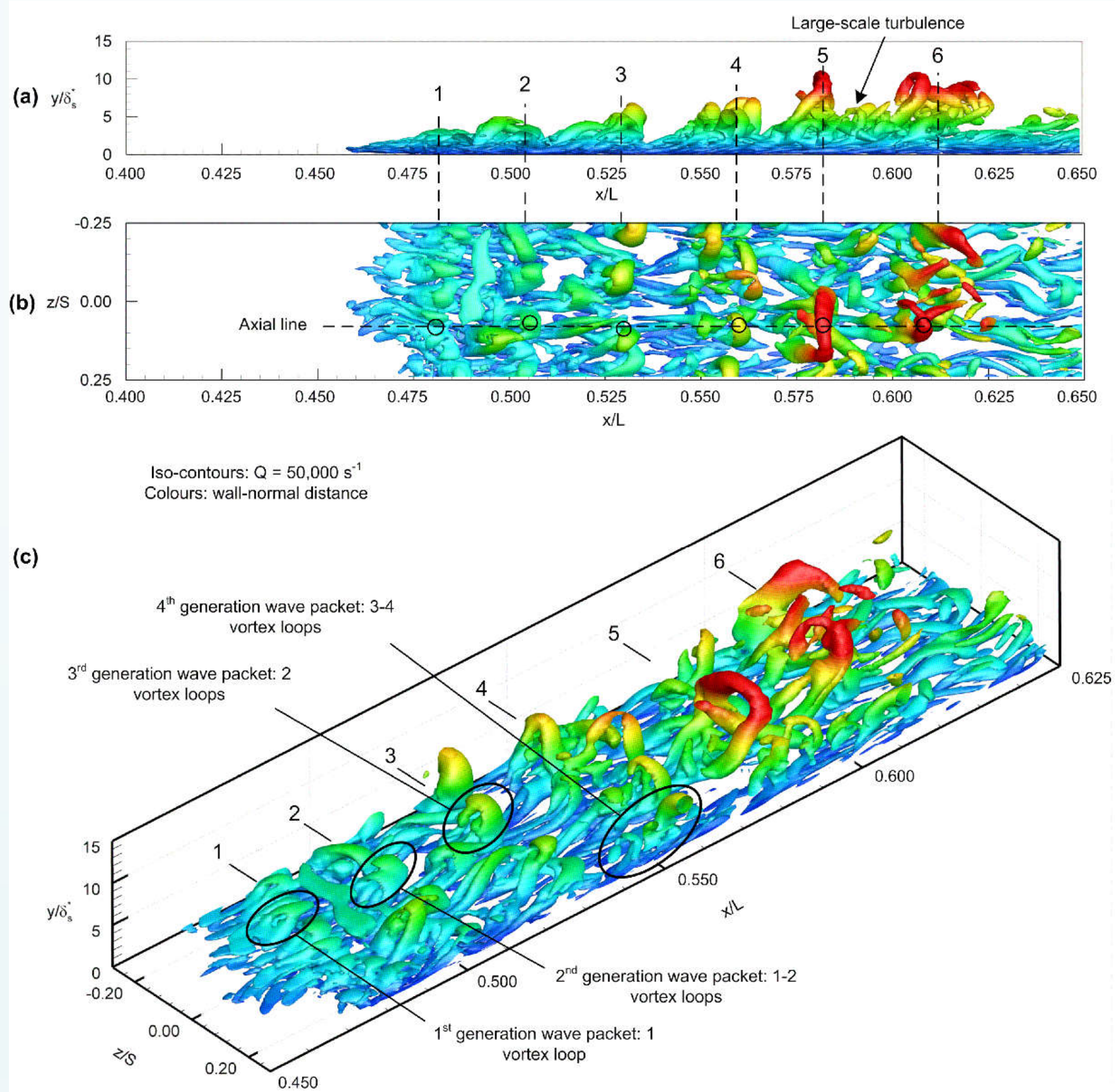
This is called DDES (Delayed DES)

Hybrid LES-RANS

Illustration of near-wall turbulence







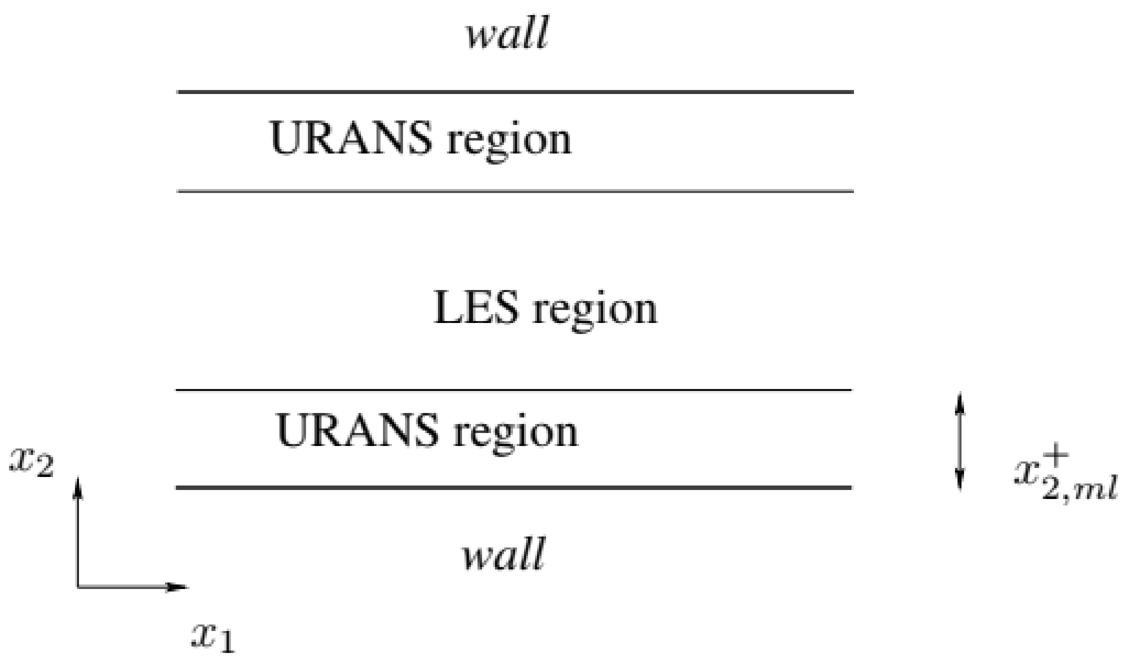


Figure 21.3: The LES and URANS region.

Hybrid LES-RANS is similar to DES (Detached Eddy Simulations) [137, 152, 153]. The main difference is that the original DES aims at covering the whole attached boundary layer with URANS, whereas hybrid LES-RANS aims at covering only the inner part of the boundary layer with URANS. In later work DES has been used as a wall model [148, 154] – called wall-modelled LES – and, in this form, DES is similar hybrid LES-RANS.

Momentum equations in hybrid LES-RANS

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left[(\nu + \nu_T)\frac{\partial \bar{v}_i}{\partial x_j}\right]$$

where $\nu_T = \nu_t$ (ν_t denotes the turbulent RANS viscosity) for $x_2 \leq x_{2,ml}$ (the URANS region, see Fig. 21.3) and, for $x_2 > x_{2,ml}$ (the LES region), $\nu_T = \nu_{sgs}$.

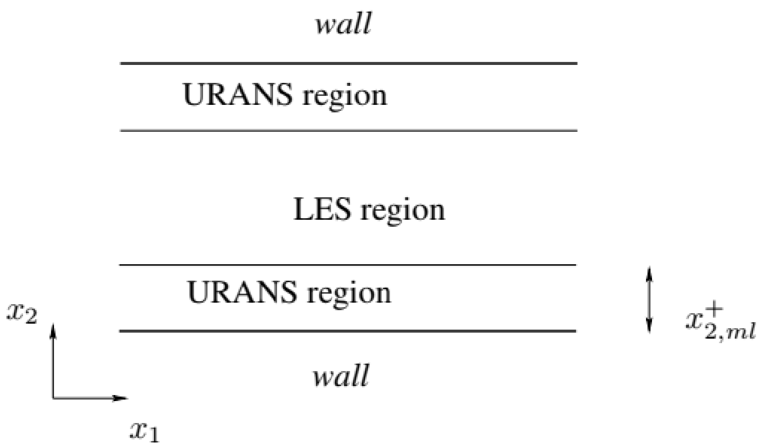


Figure 21.3: The LES and URANS region.

The one-equation hybrid LES-RANS model

$$\frac{\partial k_T}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j k_T) = \frac{\partial}{\partial x_j}\left[(\nu + \nu_T)\frac{\partial k_T}{\partial x_j}\right] + P_{k_T} - C_\varepsilon \frac{k_T^{3/2}}{\ell}$$

$$P_{k_T} = -\tau_{ij}\bar{s}_{ij}, \quad \tau_{ij} = -2\nu_T \bar{s}_{ij}$$

In the inner region ($x_2 \leq x_{2,ml}$) k_T corresponds to the RANS turbulent kinetic energy, k; in the outer region ($x_2 > x_{2,ml}$) it corresponds to the subgrid-scale kinetic turbulent energy (k_{sgs}).

At the walls, $k_T = 0$.

	URANS region	LES region
ℓ	$\kappa c_\mu^{-3/4} n[1 - \exp(-0.2k^{1/2}n/\nu)]$	$\ell = \Delta$
ν_T	$\kappa c_\mu^{1/4} k^{1/2} n[1 - \exp(-0.014k^{1/2}n/\nu)]$	$0.07k^{1/2}\ell$
C_ε	1.0	1.05

Table 21.1: Turbulent viscosity and turbulent length scales in the URANS and LES regions. n and κ denote the distance to the nearest wall and von Kármán constant ($= 0.41$), respectively. $\Delta = (\delta V)^{1/3}$

When prescribing the location of the RANS-LES interface, the velocity profile may show unphysical behaviour near the interface because of the rapid variation of the turbulence viscosity. Much work on forcing have been presented in order to alleviate this problem [99, 127, 148, 149].

The SAS model (Scale- Adaptive Simulation)

The von Kármán length scale

$$L_{vK,1D} = \kappa \left| \frac{\partial \langle \bar{v} \rangle / \partial x_2}{\partial^2 \langle \bar{v} \rangle / \partial x_2^2} \right|$$

In the SAS model

$$U'' = \left(\frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} \frac{\partial^2 \bar{v}_i}{\partial x_k \partial x_k} \right)^{0.5}$$

$$L_{vK,3D} = \kappa \frac{|\bar{s}|}{|U''|}$$

$$\begin{aligned}
U''^2 = & \left(\frac{\partial^2 \bar{v}_1}{\partial x_1^2}\right)^2 + 2\left(\frac{\partial^2 \bar{v}_1}{\partial x_1 \partial x_2}\right)^2 + 2\left(\frac{\partial^2 \bar{v}_1}{\partial x_1 \partial x_3}\right)^2 \\
& \left(\frac{\partial^2 \bar{v}_1}{\partial x_2^2}\right)^2 + 2\left(\frac{\partial^2 \bar{v}_1}{\partial x_2 \partial x_3}\right)^2 + \left(\frac{\partial^2 \bar{v}_1}{\partial x_3^2}\right)^2 \\
& \left(\frac{\partial^2 \bar{v}_2}{\partial x_1^2}\right)^2 + 2\left(\frac{\partial^2 \bar{v}_2}{\partial x_1 \partial x_2}\right)^2 + 2\left(\frac{\partial^2 \bar{v}_2}{\partial x_1 \partial x_3}\right)^2 \\
& \left(\frac{\partial^2 \bar{v}_2}{\partial x_2^2}\right)^2 + 2\left(\frac{\partial^2 \bar{v}_2}{\partial x_2 \partial x_3}\right)^2 + \left(\frac{\partial^2 \bar{v}_2}{\partial x_3^2}\right)^2 \\
& \left(\frac{\partial^2 \bar{v}_3}{\partial x_1^2}\right)^2 + 2\left(\frac{\partial^2 \bar{v}_3}{\partial x_1 \partial x_2}\right)^2 + 2\left(\frac{\partial^2 \bar{v}_3}{\partial x_1 \partial x_3}\right)^2 \\
& \left(\frac{\partial^2 \bar{v}_3}{\partial x_2^2}\right)^2 + 2\left(\frac{\partial^2 \bar{v}_3}{\partial x_2 \partial x_3}\right)^2 + \left(\frac{\partial^2 \bar{v}_3}{\partial x_3^2}\right)^2
\end{aligned}$$

PANS Model

THE PANS method uses the so-called "partial averaging" concept, which corresponds to a filtering operation for a portion of the fluctuating scales [160].

For an instantaneous flow variable, F, we use $\bar{f}$ to denote the partially-averaged part, namely $\bar{f} = \mathcal{P}(F)$, where $\mathcal{P}$ denotes the partial-averaging operator. We consider incompressible flows. Applying the partial averaging to the governing equations gives

$$\frac{\partial \bar{v}_i}{\partial x_i} = 0$$

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial (\bar{v}_i \bar{v}_j)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial \bar{v}_i}{\partial x_j} - \tau_{ij}\right)$$

$$\tau_{ij} = \mathcal{P}(v_i v_j) - \bar{v}_i \bar{v}_j$$

In order to formulate the PANS eddy viscosity, they defined in [160] another two quantities, the partially-averaged turbulent kinetic energy, k_u and its dissipation rate ε_u,

$\tau_{ij} = -2\nu_u \bar{s}_{ij}$, where $\bar{s}_{ij}$ is the strain-rate tensor

ν_u is the PANS eddy viscosity

$$\nu_u = C_\mu k_u^2 / \varepsilon_u$$

$$k = \frac{k_u}{f_k} \text{ and } \varepsilon = \frac{\varepsilon_u}{f_\varepsilon}$$

Usually $f_\varepsilon = 1; f_\varepsilon < 1$

[160] S. S. Girimaji. Partially-averaged Navier-Stokes model for turbulence: A Reynolds-averaged Navier-Stokes to direct numerical simulation bridging method. *ASME Journal of Applied Mechanics*, 73(2):413–421, 2006.

The k_u equation is derived by multiplying the RANS k equation $k-\varepsilon$ model by f_k, i.e. (for simplicity we omit the buoyancy term)

$$f_k\left\{\frac{\partial k}{\partial t}+\bar{V}_j\frac{\partial k}{\partial x_j}\right\}=f_k\left\{P^k-\varepsilon+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]\right\}$$

V_i denotes the RANS velocity

$$f_k\left\{\frac{\partial k}{\partial t}+\bar{V}_j\frac{\partial k}{\partial x_j}\right\}=\frac{\partial k_u}{\partial t}+\bar{V}_j\frac{\partial k_u}{\partial x_j}=\frac{\partial k_u}{\partial t}+\bar{v}_j\frac{\partial k_u}{\partial x_j}+(\bar{V}_j-\bar{v}_j)\frac{\partial k_u}{\partial x_j}$$

$$\begin{aligned}f_k\left\{\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]\right\}&=\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k_u}{\partial x_j}\right]\\&=\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_u}{\sigma_{ku}}\right)\frac{\partial k_u}{\partial x_j}\right]\end{aligned}$$

$$\sigma_{ku}=\sigma_k\frac{f_k^2}{f_\varepsilon}\qquad \nu_u=c_\mu\frac{k_u^2}{\varepsilon_u}$$

$$f_k\left(P^k-\varepsilon\right)=P_u-\varepsilon_u$$

$$P^k=\frac{1}{f_k}(P_u-\varepsilon_u)+\frac{\varepsilon_u}{f_\varepsilon}$$

$$\frac{\partial k_u}{\partial t} + \frac{\partial (k_u \bar{v}_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{ku}}\right)\frac{\partial k_u}{\partial x_j}\right] + P_u - \varepsilon_u$$

$$P_u = \nu_u \left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right)\frac{\partial \bar{v}_i}{\partial x_j}$$

The ε_u equation is derived by multiplying the RANS ε equation by f_ε, i.e.

$$\begin{aligned}\frac{\partial \varepsilon_u}{\partial t} + \frac{\partial (\varepsilon_u \bar{v}_j)}{\partial x_j} &= f_\varepsilon \left[\frac{\partial \varepsilon}{\partial t} + \frac{\partial (\varepsilon \bar{V}_j)}{\partial x_j}\right] \\ &= f_\varepsilon \left\{\frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{\varepsilon 1} P^k \frac{\varepsilon}{k} - C_{\varepsilon 2}\frac{\varepsilon^2}{k}\right\}\end{aligned}$$

$$\begin{aligned}f_\varepsilon \left\{\frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]\right\} &= \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon_u}{\partial x_j}\right] \\ &= \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon_u}{\partial x_j}\right]\end{aligned}$$

$$\sigma_{\varepsilon u} = \sigma_\varepsilon \frac{f_k^2}{f_\varepsilon}$$

$$f_\varepsilon \left\{ C_{\varepsilon 1} P^k \frac{\varepsilon}{k} - C_{\varepsilon 2} \frac{\varepsilon^2}{k} \right\} = C_{\varepsilon 1} \frac{\varepsilon_u f_k}{k_u} \left(\frac{1}{f_k}(P_u - \varepsilon_u) + \frac{\varepsilon_u}{f_\varepsilon} \right) - C_{\varepsilon 2} \frac{\varepsilon_u^2 f_k}{f_\varepsilon k_u}$$

$$= C_{\varepsilon 1} \frac{\varepsilon_u}{k_u} P_u - C_{\varepsilon 1} \frac{\varepsilon_u^2}{k_u} + C_{\varepsilon 1} \frac{\varepsilon_u^2 f_k}{k_u f_\varepsilon} - C_{\varepsilon 2} \frac{\varepsilon_u^2 f_k}{f_\varepsilon k_u}$$

$$= C_{\varepsilon 1} \frac{\varepsilon_u}{k_u} P_u - C^*_{\varepsilon 2} \frac{\varepsilon_u^2}{k_u}$$

$$C^*_{\varepsilon 2} = C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon}(C_{\varepsilon 2} - C_{\varepsilon 1})$$

The ε_u equation in the PANS model now takes the following form

$$\frac{\partial \varepsilon_u}{\partial t} + \frac{\partial(\varepsilon_u \bar{v}_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_u}{\sigma_{\varepsilon u}} \right) \frac{\partial \varepsilon_u}{\partial x_j} \right] + C_{\varepsilon 1} P_u \frac{\varepsilon_u}{k_u} - C^*_{\varepsilon 2} \frac{\varepsilon_u^2}{k_u}$$

A Low Reynolds number PANS model

$$\frac{\partial k_u}{\partial t} + \frac{\partial (k_u \bar{v}_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{ku}}\right)\frac{\partial k_u}{\partial x_j}\right] + (P_u - \varepsilon_u)$$

$$\frac{\partial \varepsilon_u}{\partial t} + \frac{\partial (\varepsilon_u \bar{v}_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon_u}{\partial x_j}\right] + C_{\varepsilon 1} P_u \frac{\varepsilon_u}{k_u} - C^*_{\varepsilon 2}\frac{\varepsilon_u^2}{k_u}$$

$$\nu_u = C_\mu f_\mu \frac{k_u^2}{\varepsilon_u}, C^*_{\varepsilon 2} = C_{\varepsilon 1} + \boxed{\frac{f_k}{f_\varepsilon}}(C_{\varepsilon 2} f_2 - C_{\varepsilon 1})$$

$$\sigma_{ku} \equiv \sigma_k \boxed{\frac{f_k^2}{f_\varepsilon}}, \sigma_{\varepsilon u} \equiv \sigma_\varepsilon \boxed{\frac{f_k^2}{f_\varepsilon}}$$

$$C_{\varepsilon 1} = 1.5, C_{\varepsilon 2} = 1.9, \sigma_k = 1.4, \sigma_\varepsilon = 1.4, C_\mu = 0.09$$