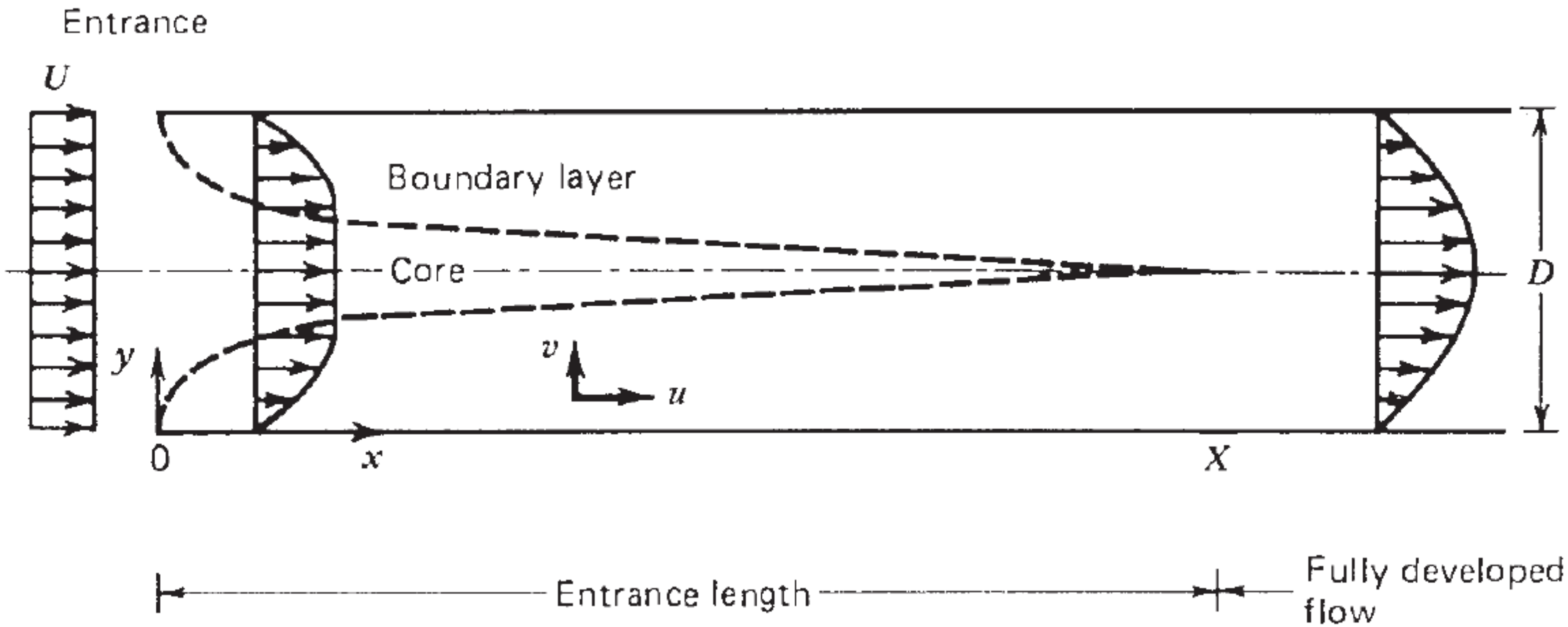
# LAMINAR DUCT FLOW

## HYDRODYNAMIC ENTRANCE LENGTH




Blasius's boundary layer thickness

the entrance length $X$ by writing $\delta(X) = D/2$

$$\frac{\delta}{x} = 4.92\mathrm{Re}_x^{-1/2}$$

$$\Rightarrow \frac{X/D}{\mathrm{Re}_D} = 0.01$$

$$U_\infty = U_c$$

$$Y = \delta(x)$$

$$\Rightarrow \frac{d}{dx}\int_0^Y u(U_\infty - u)\,dy = \frac{1}{\rho}Y\frac{dP_\infty}{dx} + \frac{dU_\infty}{dx}\int_0^Y u\,dy + \nu\left(\frac{\partial u}{\partial y}\right)_0 \Rightarrow U_c\frac{dU_c}{dx} + \frac{1}{\rho}\frac{dP}{dx} = 0$$



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$$\frac{d}{dx}\left[\int_0^\delta (U_c - u)\, u\, dy\right] + \frac{dU_c}{dx}\int_0^\delta (U_c - u)\, dy = \nu\left(\frac{\partial u}{\partial y}\right)_0$$

mass conservation in the channel of half-width

$$\int_0^\delta \rho u\, dy \;+\; \int_\delta^{D/2} \rho U_c\, dy = \rho U \frac{D}{2}$$

first assuming a boundary layer profile shape ➡ $u/U_c = 2y/\delta - (y/\delta)^2$

$$\frac{x/D}{\mathrm{Re}_D} = \frac{3}{40}\left(9\frac{U_c}{U} - 2 - 7\frac{U}{U_c} - 16\ \ln\ \frac{U_c}{U}\right)$$

$$\frac{\delta(x)}{D/2} = 3\left[1 - \frac{U}{U_c(x)}\right]$$

At the location $X$

$$\delta(X) = D/2$$

$$U_c(X) = \tfrac{3}{2}U$$

⬇

$$\frac{X/D}{\mathrm{Re}_D} = 0.026$$

Schlichting [3] $\dfrac{X/D}{\mathrm{Re}_D} \cong 0.04$



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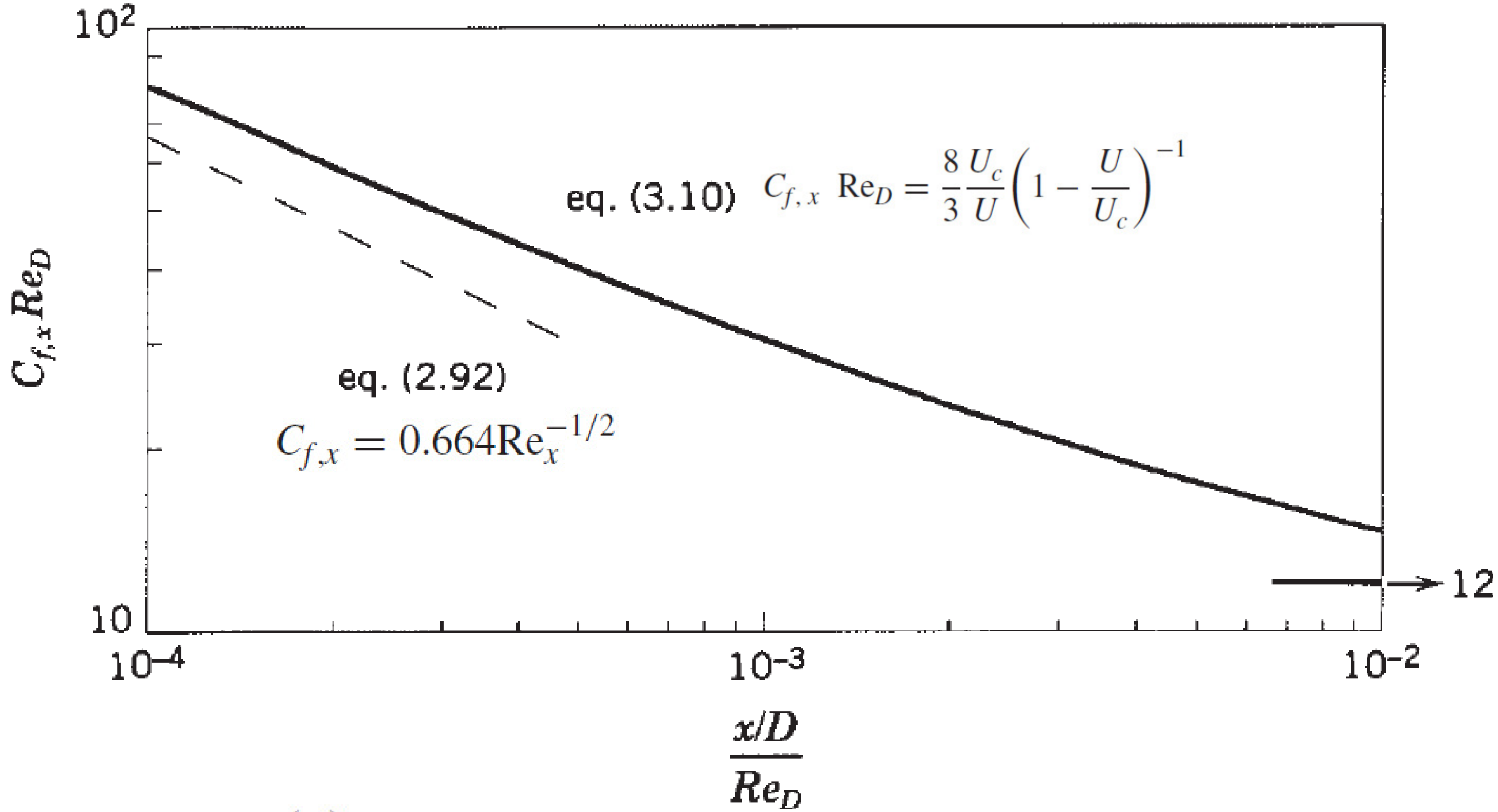


$$C_{f,x} = \frac{\tau_{\mathrm{wall}}(x)}{\frac{1}{2}\rho U^2}$$

$$\tau_{\mathrm{wall}} = \mu(\partial u/\partial y)_0$$

$$C_{f,x}\ \mathrm{Re}_D = \frac{8}{3}\frac{U_c}{U}\left(1-\frac{U}{U_c}\right)^{-1}$$

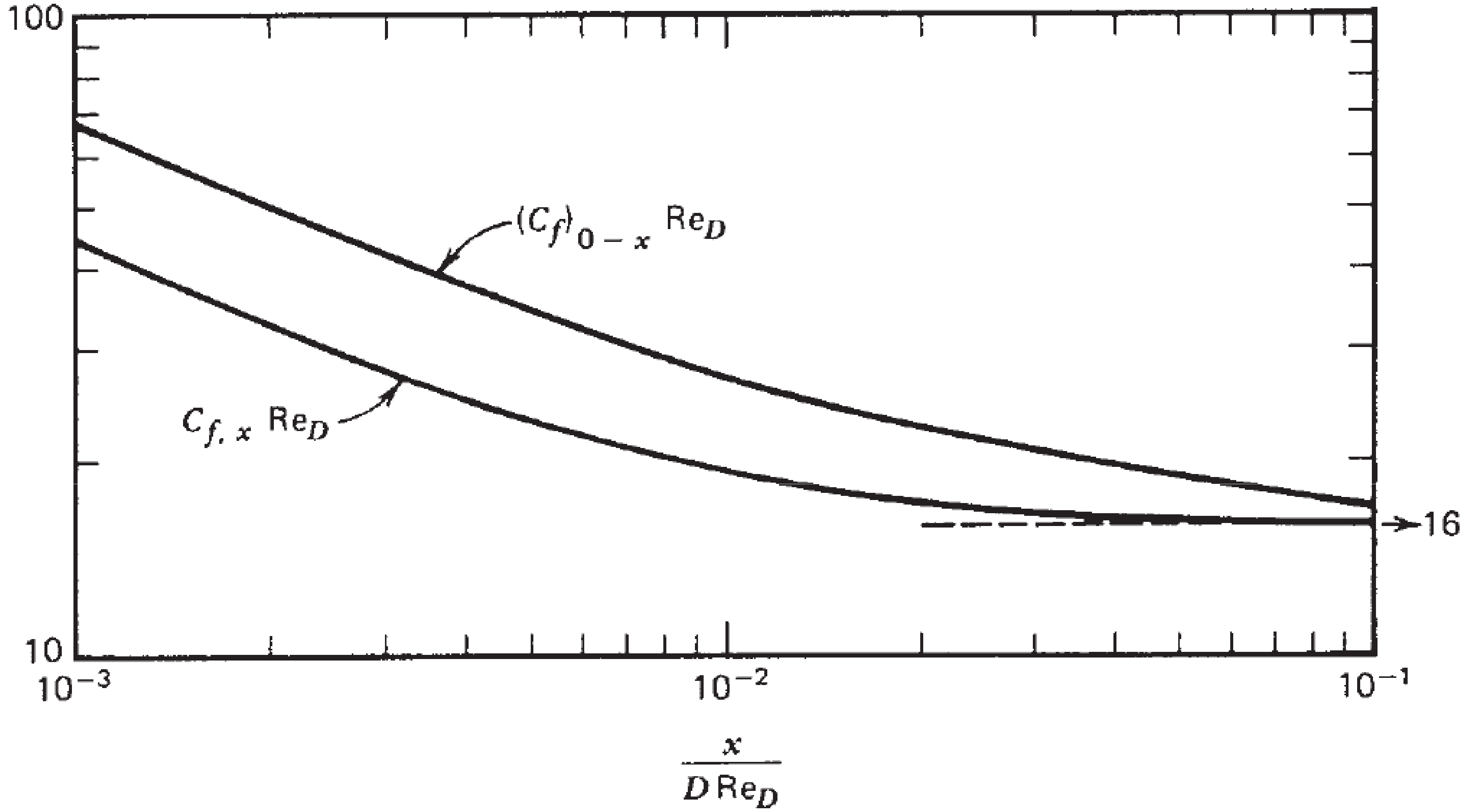


**Figure 3.3** Local and average skin friction coefficients in the entrance region of a round tube.

## FULLY DEVELOPED FLOW

$x \sim L \qquad y \sim D \qquad u \sim U$ → ?

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow v \sim \frac{DU}{L}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

*fully developed* region → $v$ is negligible

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

$$v = 0 \quad \text{and} \quad \frac{\partial u}{\partial x} = 0$$

$$\frac{\partial P}{\partial y} = 0 \rightarrow \frac{dP}{dx} = \mu\frac{d^2u}{dy^2} = \text{constant}$$

$P$ is a function of $x$

$u$ is a function of $y$

$$u = \frac{3}{2}U\left[1 - \left(\frac{y}{D/2}\right)^2\right] \qquad U = \frac{D^2}{12\mu}\left(-\frac{dP}{dx}\right)$$

$u = 0$ at $y = \pm D/2$

the fully developed laminar flow in a round tube of radius $r_0$ → $\frac{dP}{dx} = \mu\left(\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr}\right)$

$$u = 2U\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

$$U = \frac{r_0^2}{8\mu}\left(-\frac{dP}{dx}\right) \qquad \dot{m} = \frac{\pi r_0^4}{8\nu}\left(-\frac{dP}{dx}\right)$$

it is not true

$\mathrm{Re}_D = UD/\nu$

is the ratio of two forces,

the inertia divided by the friction force ?



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Hagen–Poiseuille flows are flows in which the fluid inertia is *zero* everywhere

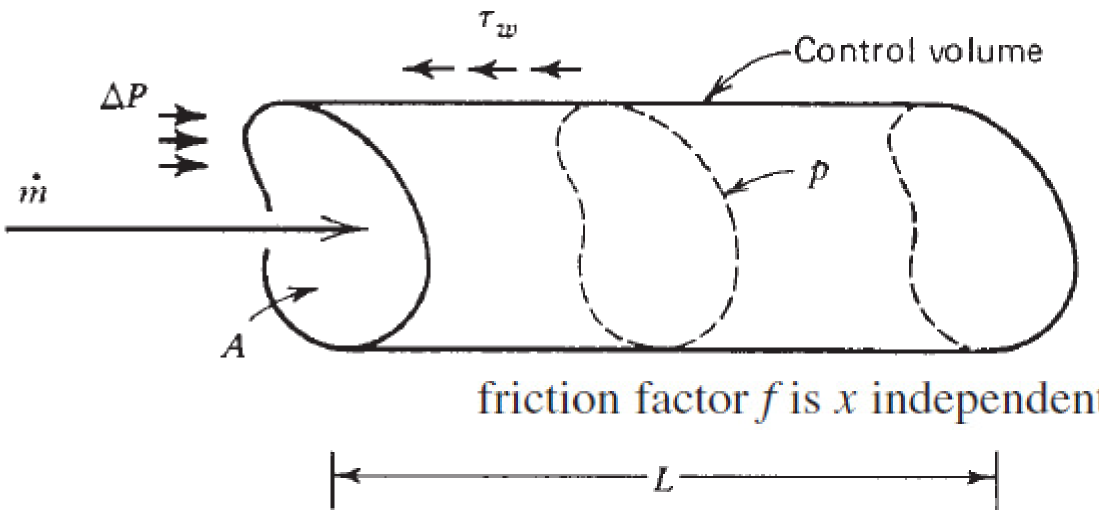
$$\frac{\text{longitudinal pressure force}}{\text{friction force}} \rightarrow \frac{-dP/dx}{\mu \partial^2 u/\partial r^2} \sim \frac{\Delta P/L}{\mu U/D^2} = O(1)$$

$Re_D$ has no meaning in fully developed laminar duct flow

is discussed later

The dimensionless group $f\,Re_{D_h}$

## HYDRAULIC DIAMETER AND PRESSURE DROP




$$A\,\Delta P = \tau_w p L$$

the *friction factor* $f = \dfrac{\tau_w}{\frac{1}{2}\rho U^2}$

This definition is essentially the same as skin friction coefficient

One important difference between $f$ and $C_{f,x}$ is

friction factor $f$ is $x$ independent because it is a *fully developed regime* concept

literature also uses $4f$ as the friction factor

برای تشخیص رابطه استفاده شده کافیست ضریب اصطکاک را
برای لوله حساب نمایید. برای رابطه فوق این ضریب برابر است با: $f = 16/Re_D$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the pressure drop $\Delta P$ across the duct is $\Delta P = f\frac{pL}{A}\left(\frac{1}{2}\rho U^2\right)$

Finally, note that $A/p$ is the linear dimension of the cross section:

$$r_h = \frac{A}{p} \quad \text{hydraulic radius} \quad \text{or} \quad D_h = 4r_h = \frac{4A}{p} \quad \text{hydraulic diameter}$$

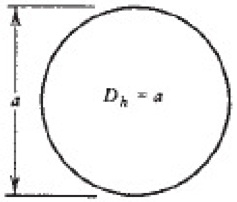
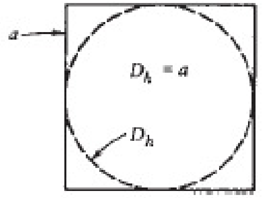
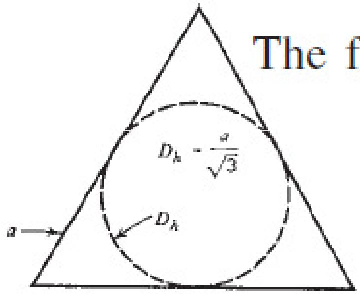
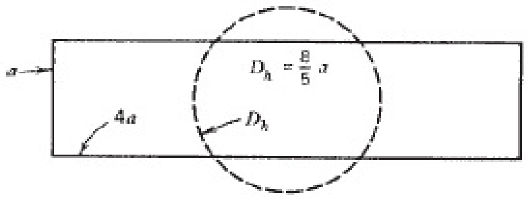
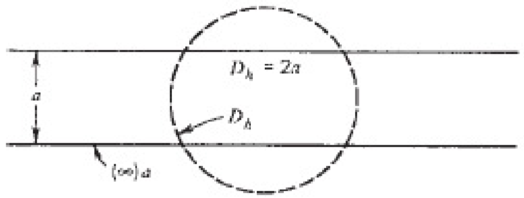
$$\Delta P = f\frac{pL}{A}\left(\frac{1}{2}\rho U^2\right) \Rightarrow \Delta P = f\frac{4L}{D_h}\left(\frac{1}{2}\rho U^2\right)$$

$$f = \frac{24}{\mathrm{Re}_{D_h}}, \quad D_h = 2D \quad \text{parallel plates } (D = \text{gap thickness})$$

$$f = \frac{16}{\mathrm{Re}_{D_h}}, \quad D_h = D \quad \text{round tube } (D = \text{tube diameter})$$

These formulas hold as long as the flow is in the laminar regime ($\mathrm{Re}_{D_h} < 2000$)

**Table 3.1 Scale drawings of five ducts that have the same hydraulic diameter**

| Cross Section | |
|---|---|
| Circular |   |
| Square |   |
| Equilateral triangle |   |
| Rectangular (4:1) |   |
| Infinite parallel plates |   |

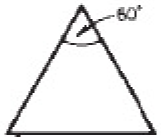
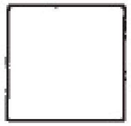
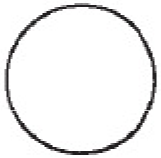
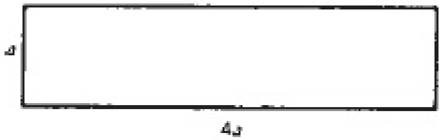
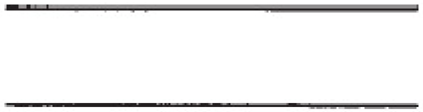
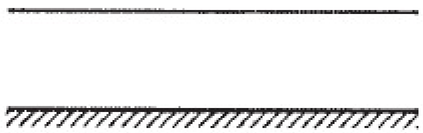
*Poiseuille number* $\mathrm{Po} = f\,\mathrm{Re}_{D_h}$

The product $f\,\mathrm{Re}_{D_h}$ is a number that depends only on the shape of the cross section

$$\frac{\Delta P/L}{\mu U/D_h^2} \sim f\,\mathrm{Re}_{D_h}$$

The fact that $f\,\mathrm{Re}_{D_h}$ is a constant expresses the balance between the only two forces that are present.

**Table 3.2 Effect of cross-sectional shape on $f$ and Nu in fully developed duct flow**

| Cross-sectional geometry | $f\,\mathrm{Re}_{D_h}$ | $B = \dfrac{\pi D_h^2/4}{A_{\text{duct}}}$ | $\mathrm{Nu} = hD_h/k$ | |
|---|---|---|---|---|
| | | | Uniform $q''$ | Uniform $T_0$ |
|   | 13.3 | 0.605 | 3 | 2.35 |
|  | 14.2 | 0.785 | 3.63 | 2.89 |
|  | 16 | 1 | 4.364 | 3.66 |
|   | 18.3 | 1.26 | 5.35 | 4.65 |
|  | 24 | 1.57 | 8.235 | 7.54 |
|  One side insulated | 24 | 1.57 | 5.385 | 4.86 |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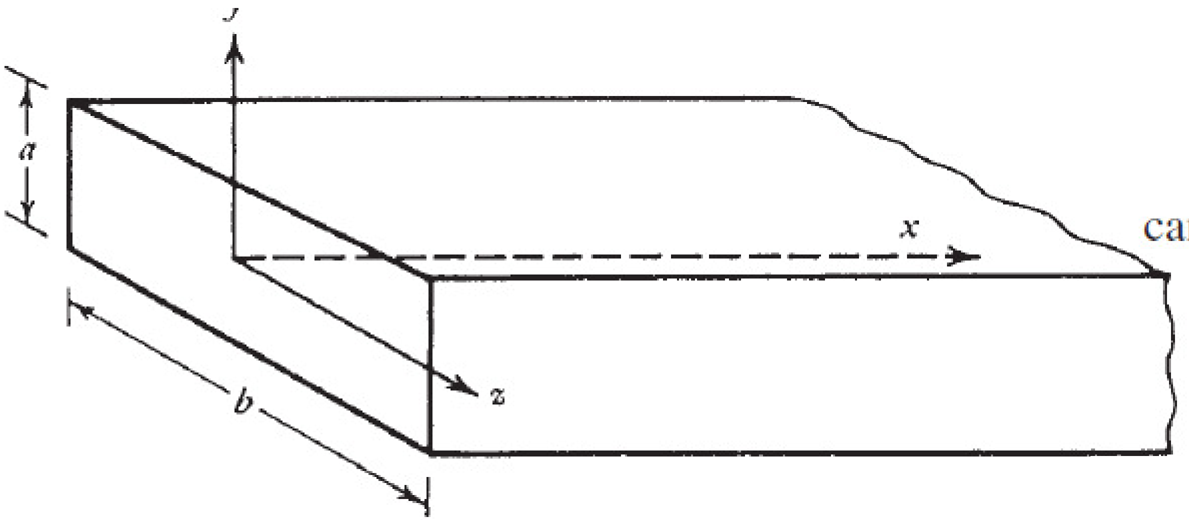


**Figure 3.5** Straight duct with rectangular cross section.

$$\frac{dP}{dx} = \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \text{constant}$$

can be solved for $u(y, z)$ by Fourier series

$$u(y,z) = u_0\left[1-\left(\frac{y}{a/2}\right)^2\right]\left[1-\left(\frac{z}{b/2}\right)^2\right]$$

where $u_0$ is the centerline (peak) velocity.

*integrated* over the entire cross section

$$ab\frac{dP}{dx} = \mu\int_{-a/2}^{a/2}\int_{-b/2}^{b/2}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)dz\,dy \rightarrow ab\frac{dP}{dx} = -\frac{16}{3}\mu u_0\left(\frac{b}{a}+\frac{a}{b}\right)$$

average velocity $U$.

$$abU = \int_{-a/2}^{a/2}\int_{-b/2}^{b/2} u\,dz\,dy \rightarrow$$

$$ab\frac{dP}{dx} = -\frac{16}{3}\mu u_0\left(\frac{b}{a}+\frac{a}{b}\right)$$

$$u_0 = \frac{9}{4}U$$

جایگزاری →

$$\Delta P = f\frac{4L}{D_h}\left(\frac{1}{2}\rho U^2\right) \rightarrow f = \frac{a^2+b^2}{(a+b)^2}\frac{24}{\mathrm{Re}_{D_h}}$$

$$D_h = \frac{4ab}{2(a+b)}$$


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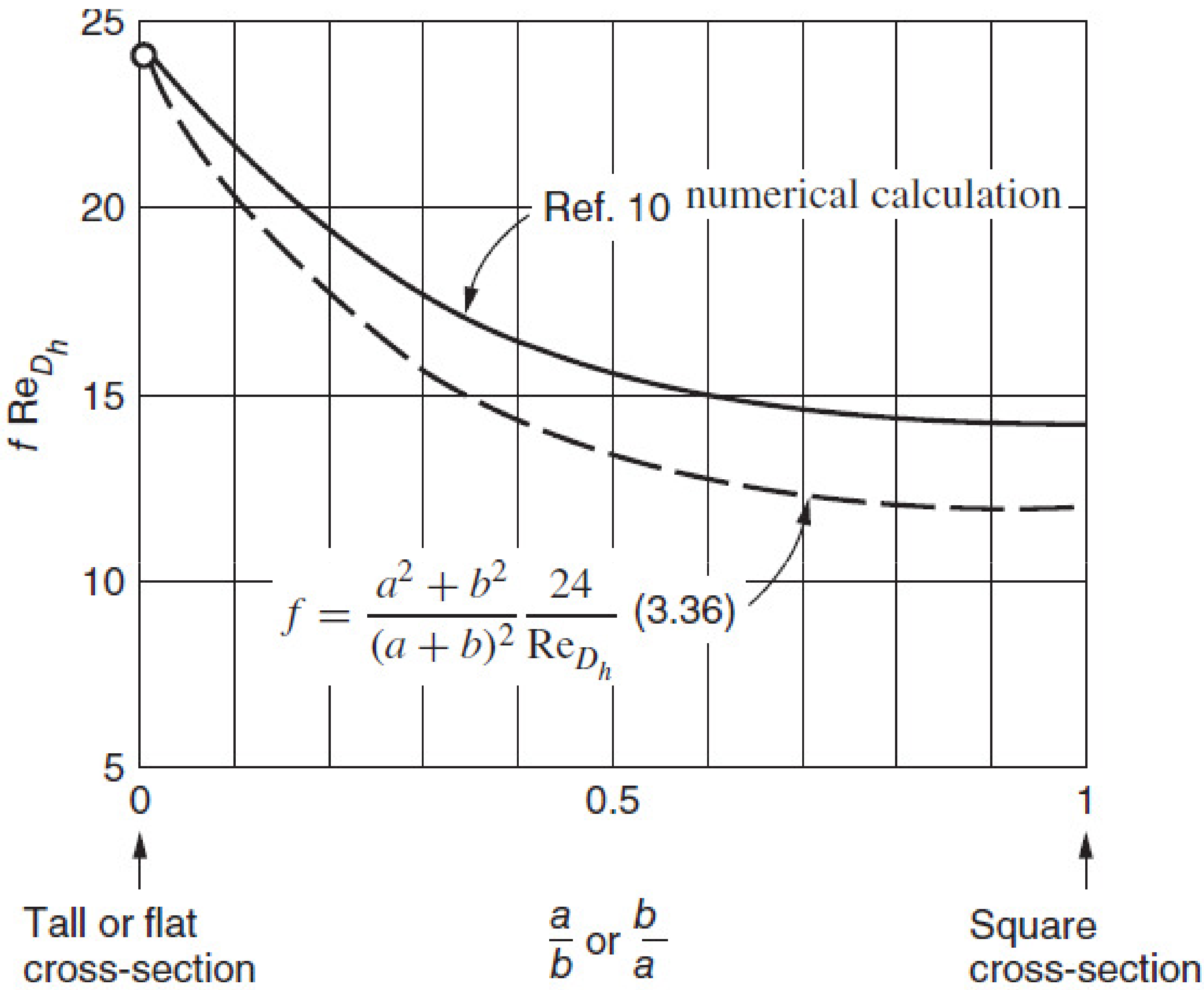
علت اختلاف مشاهده شده بدلیل تابع سرعت ارائه شده در کانال با مقطع مستطیلی است.




Friction factor for fully developed flow in a duct with rectangular cross section.

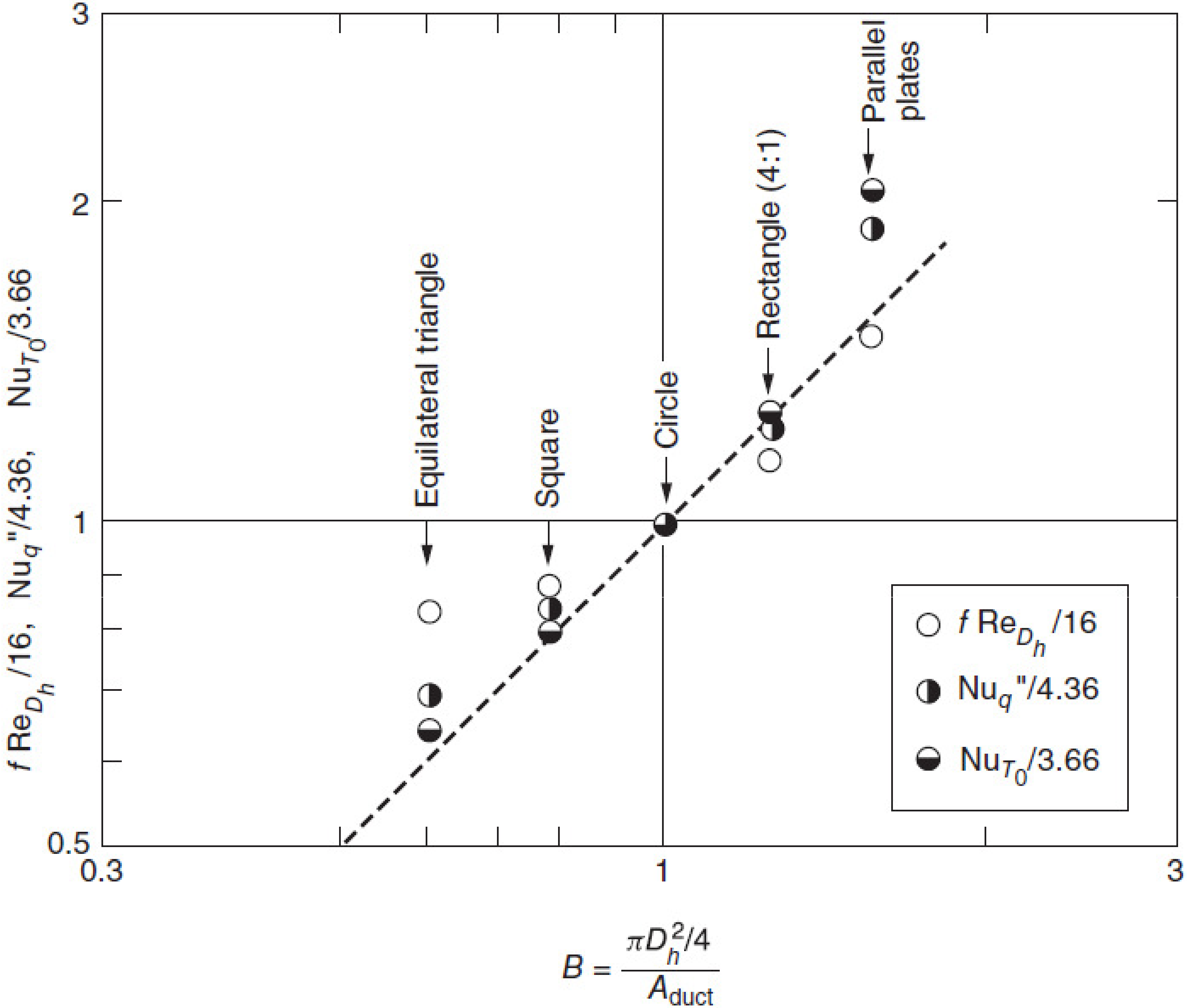
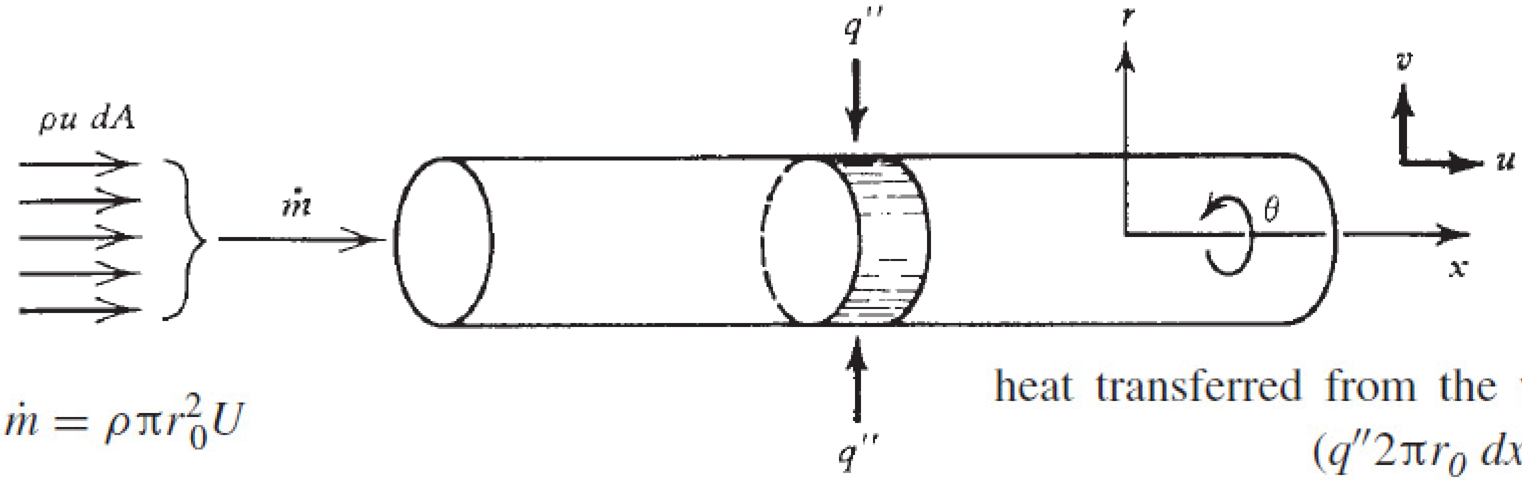



**Figure 3.7** Cross-sectional shape number $B$ and fully developed friction and heat transfer in straight ducts.

# HEAT TRANSFER TO FULLY DEVELOPED DUCT FLOW

## Mean Temperature




$\dot{m} = \rho \pi r_0^2 U$

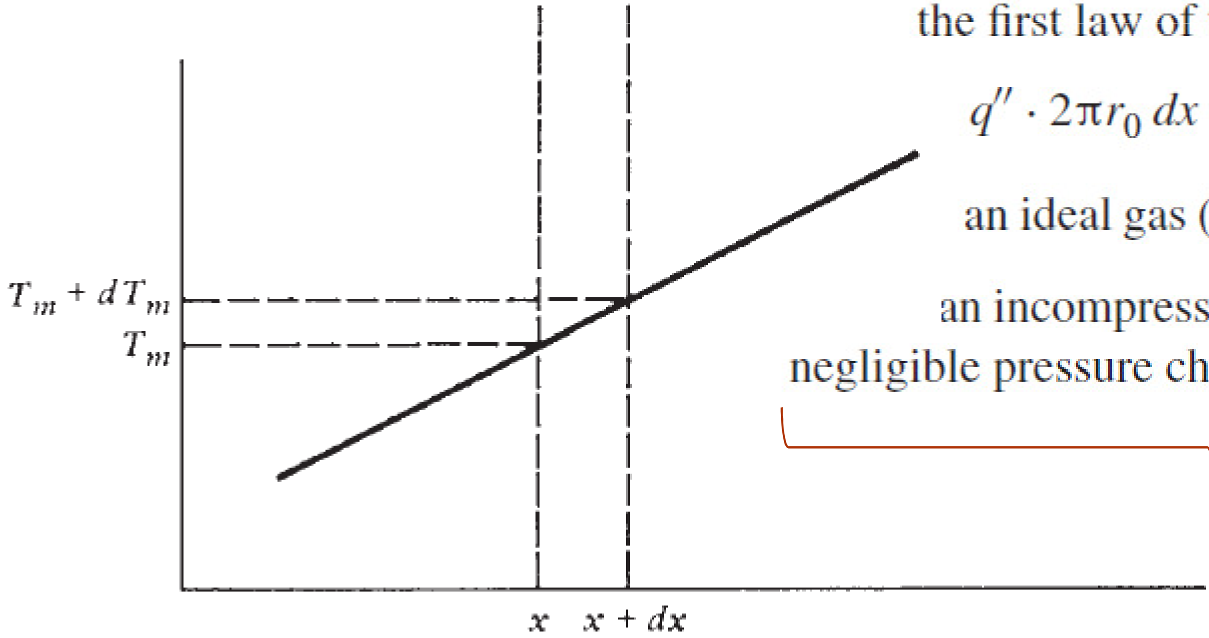
heat transferred from the wall to the stream
$(q'' 2\pi r_0\, dx)$

the first law of thermodynamics

$$q'' \cdot 2\pi r_0\, dx = \dot{m}(h_{x+dx} - h_x)$$

an ideal gas $(dh = c_P dT_m)$

an incompressible liquid with negligible pressure changes $(dh \cong c dT_m)$




$$\frac{dT_m}{dx} = \frac{2}{r_0} \frac{q''}{\rho c_P U}$$

$$q'' \cdot 2\pi r_0\, dx = d \iint_A \rho u c_P T\, dA$$

$$\frac{dT_m}{dx} = \frac{2}{r_0} \frac{q''}{\rho c_P U}$$

$$T_m \rho c_P U A = \iint_A \rho c_p u T\, dA$$

For constant-property tube flow $T_m = \frac{1}{\pi r_0^2 U} \int_0^{2\pi} \int_0^{r_0} u T r\, dr\, d\theta$ $\qquad h = \frac{q''}{T_0 - T_m} = \frac{k(\partial T/\partial r)_{r=r_0}}{T_0 - T_m}$

## Fully Developed Temperature Profile

flow through a round tube, the energy equation

$$\frac{1}{\alpha}\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r}\right) = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}$$

In the *hydrodynamic* fully developed region, we have $v = 0$ and $u = u(r)$;

$$\frac{u(r)}{\alpha}\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}$$

$\partial T/\partial x \sim q''/(D\rho c_P U)$

Convection: $\frac{U}{\alpha}\left(\frac{q''}{D\rho c_P U}\right)$,

Conduction — radial: $\frac{\Delta T}{D^2}$, longitudinal: $\frac{1}{x}\left(\frac{q''}{D\rho c_P U}\right)$

$$D^2/\Delta T \times \left[ \frac{U}{\alpha}\left(\frac{q''}{D\rho c_p U}\right), \quad \frac{\Delta T}{D^2}, \quad \frac{1}{x}\left(\frac{q''}{D\rho c_p U}\right) \right]$$

$h = q''/\Delta T$.

Convection: $\frac{hD}{k}$,

Conduction — radial: 1, longitudinal: $\left(\frac{hD}{k}\right)^2 \left(\frac{\alpha}{UD}\right)^2$

$$\mathrm{Pe}_D = \frac{UD}{\alpha} \gg 1$$

the longitudinal conduction effect is negligible

$$\mathrm{Nu} = \frac{hD}{k} \sim 1$$

Nusselt number is a constant of order 1

$(\mathrm{Pe}_D \gg 1)$.

$$\frac{u(r)}{\alpha}\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}$$

the fully developed temperature

$$\frac{T_0 - T}{T_0 - T_m} = \phi\left(\frac{r}{r_0}\right)$$

where, in general, $T$, $T_0$, and $T_m$ can be functions of $x$.

or

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right]_{\mathrm{fd},t} = 0$$

$$\mathrm{Nu} = \frac{hD}{k} = \frac{D}{k}\frac{q''}{T_0 - T_m}$$

the scaling law $\mathrm{Nu} \sim 1$

$$\mathrm{Nu} = D\frac{(\partial T/\partial r)_{r=r_0}}{T_0 - T_m} \sim 1$$

$$\frac{\partial T/\partial (r/r_0)}{T_0(x) - T_m(x)} = f_1\left(\frac{r}{r_0}\right) = O(1)$$

the $x$ variation of $(\partial T/\partial r)_{r=r_0}$ must be the same as that of $T_0(x) - T_m(x)$

$$\frac{\partial T/\partial (r/r_0)}{T_0(x) - T_m(x)} = f_1\left(\frac{r}{r_0}\right) \rightarrow T\left(x, \frac{r}{r_0}\right) = (T_0 - T_m) f_2\left(\frac{r}{r_0}\right) + f_3(x)$$

**Uniform Wall Heat Flux**

$$\frac{T_0 - T}{T_0 - T_m} = \phi\left(\frac{r}{r_0}\right)$$

$$h = q''/\Delta T$$

$$\Delta T = T_0 - T_m$$

$$T(x,r) = T_0(x) - \frac{q''}{h}\phi\left(\frac{r}{r_0}\right) \rightarrow \boxed{\frac{\partial T}{\partial x} = \frac{dT_0}{dx}}$$

$$q''D/k[T_0(x) - T_m(x)] \sim 1 \rightarrow \boxed{\frac{dT_0}{dx} = \frac{dT_m}{dx}}$$

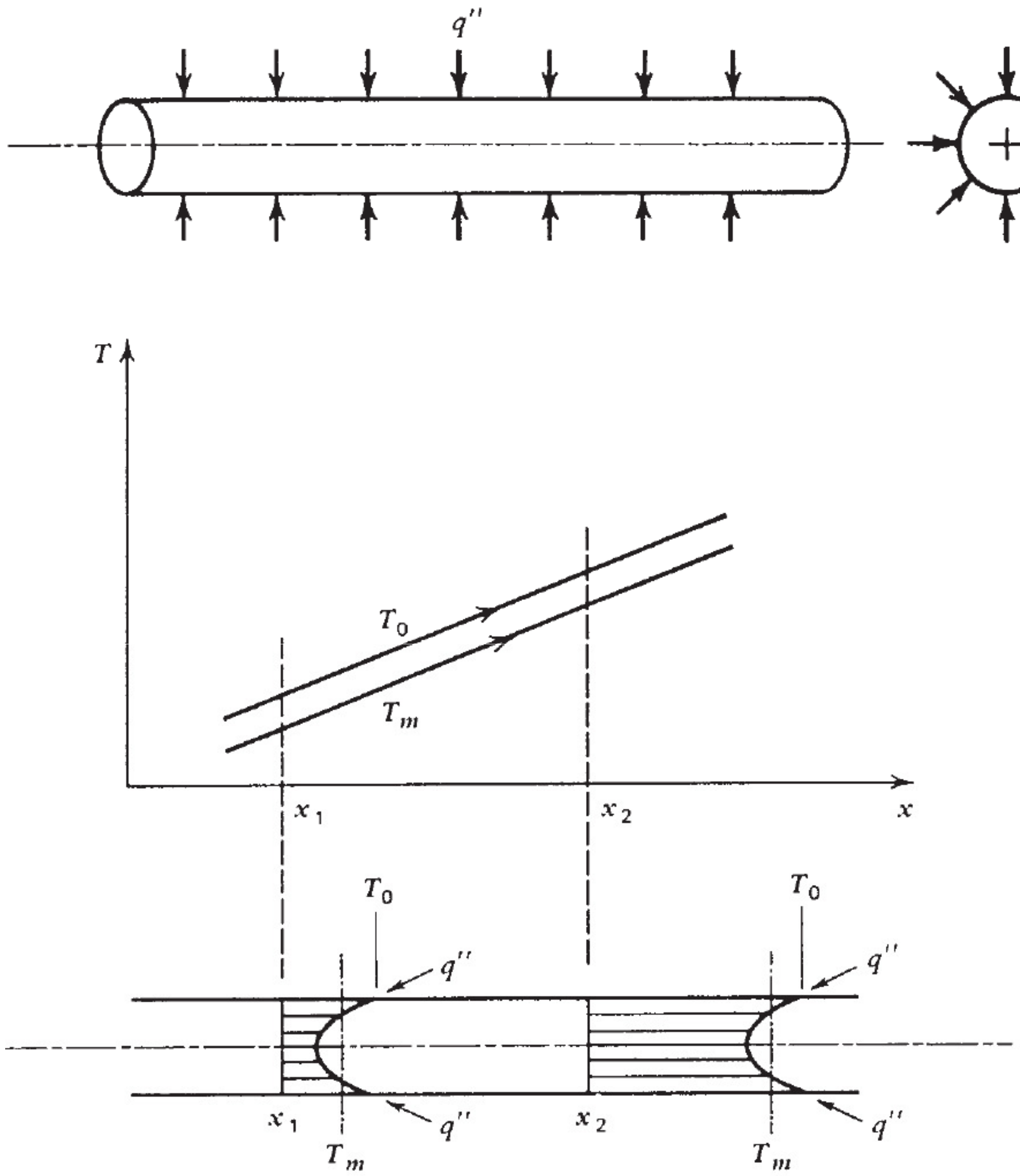



Figure 3.9 Fully developed temperature profile in a round tube with uniform heat flux.

or

$$\frac{\partial}{\partial x}\left[\frac{T_s(x)-T(r,x)}{T_s(x)-T_m(x)}\right]_{\text{fd},t}=0 \Rightarrow \left.\frac{\partial T}{\partial x}\right|_{\text{fd},t}=\left.\frac{dT_s}{dx}\right|_{\text{fd},t}-\left.\frac{(T_s-T)}{(T_s-T_m)}\frac{dT_s}{dx}\right|_{\text{fd},t}+\left.\frac{(T_s-T)}{(T_m-T_m)}\frac{dT_m}{dx}\right|_{\text{fd},t}$$

$$\left.\frac{dT_s}{dx}\right|_{\text{fd},t}=\left.\frac{dT_m}{dx}\right|_{\text{fd},t} \qquad q_s''=\text{constant}$$

$$\left.\frac{\partial T}{\partial x}\right|_{\text{fd},t}=\left.\frac{dT_s}{dx}\right|_{\text{fd},t} \Rightarrow \left.\frac{\partial T}{\partial x}\right|_{\text{fd},t}=\left.\frac{dT_m}{dx}\right|_{\text{fd},t}$$

$$\frac{dT_m}{dx}=\frac{2}{r_0}\frac{q''}{\rho c_P U}$$

$$\left.\frac{\partial T}{\partial x}\right|_{\text{fd},t}=\left.\frac{dT_m}{dx}\right|_{\text{fd},t}$$

$$\frac{\partial T}{\partial x}=\frac{2}{r_0}\frac{q''}{\rho c_P U}=\text{constant}$$

$\phi(r_*)$, where $r_*=r/r_0$

$$\frac{u(r)}{\alpha}\frac{\partial T}{\partial x}=\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}$$

$$u=2U\left[1-\left(\frac{r}{r_0}\right)^2\right]$$

$$-2\frac{hD}{k}(1-r_*^2)=\frac{d^2\phi}{dr_*^2}+\frac{1}{r_*}\frac{d\phi}{dr_*}$$

$$\Rightarrow \phi=C_2-2\text{Nu}\left(\frac{r_*^2}{4}-\frac{r_*^4}{16}\right)$$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$$T(x,r) = T_0(x) - \frac{q''}{h}\,\phi\left(\frac{r}{r_0}\right)$$

$$\phi = C_2 - 2\text{Nu}\left(\frac{r_*^2}{4} - \frac{r_*^4}{16}\right)$$

$T = T_0$ at $r_* = 1$ to determine $C_2$

$$T = T_0 - (T_0 - T_m)\,\text{Nu}\left(\frac{3}{8} - \frac{r_*^2}{2} + \frac{r_*^4}{8}\right)$$

$$T_m = \frac{1}{\pi r_0^2 U}\int_0^{2\pi}\int_0^{r_0} uTr\,dr\,d\theta$$

$$T_0 - T_m = \frac{1}{\pi r_0^2 U}\int_0^{2\pi}\int_0^{r_0}(T_0 - T)ur\,dr\,d\theta = 4\int_0^1 (T_0 - T)(1 - r_*^2)r_*\,dr_*$$

$$1 = 4\text{Nu}\int_0^1\left(\frac{3}{8} - \frac{r_*^2}{2} + \frac{r_*^4}{8}\right)(1 - r_*^2)r_*\,dr_* = \frac{11}{48}\,\text{Nu}$$

$$\boxed{\text{Nu} = \frac{48}{11} = 4.36}$$



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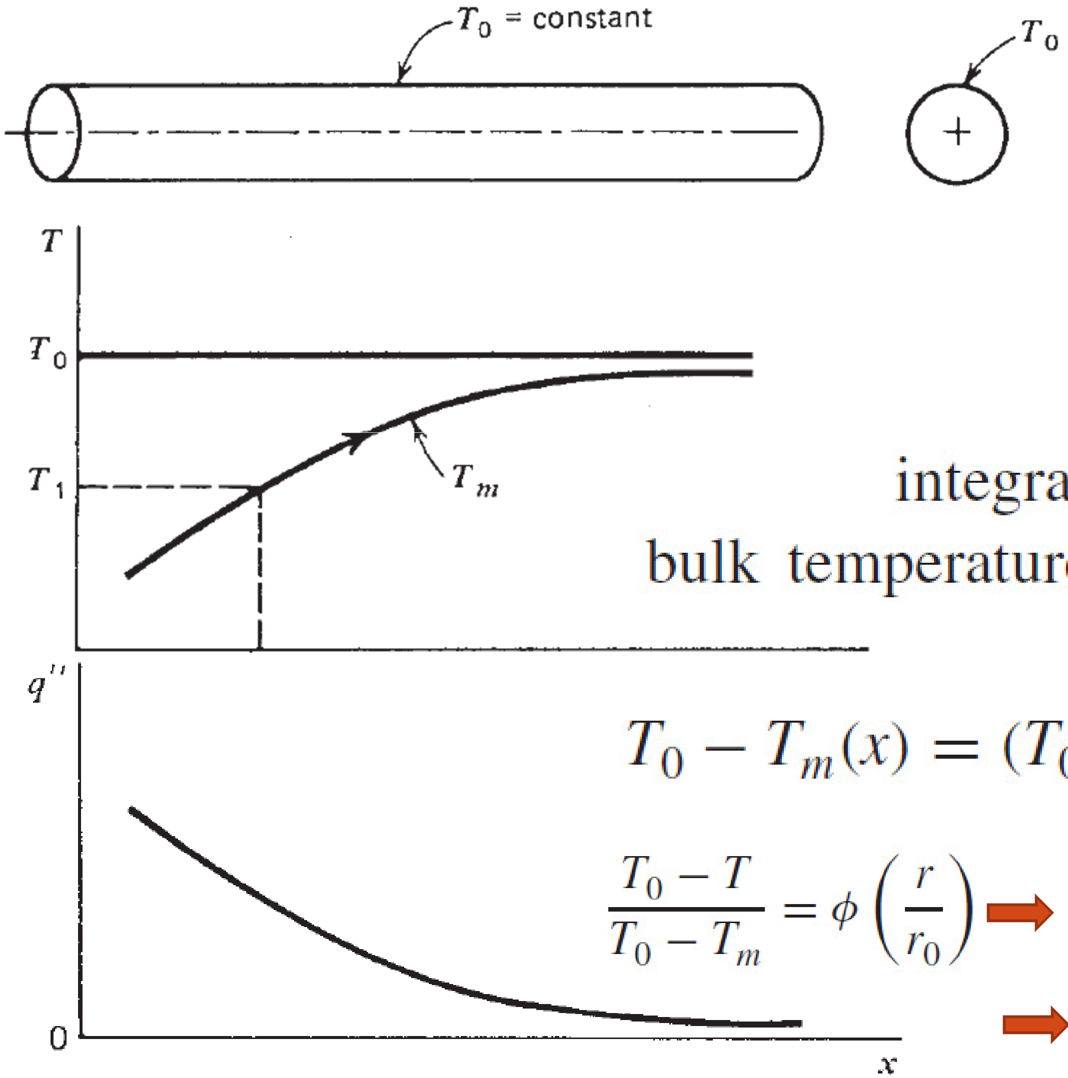
For noncircular cross sections, $\mathrm{Nu} = \dfrac{hD_h}{k}$

## Uniform Wall Temperature




$$q''(x) = h[T_0 - T_m(x)]$$

$$\frac{dT_m}{dx} = \frac{2}{r_0}\frac{q''}{\rho c_P U}$$

integrating from $T_m = T_1$ at $x = x_1$

bulk temperature is $T_1$ at some place $x = x_1$

$$T_0 - T_m(x) = (T_0 - T_1)\exp\left[-\frac{\alpha\ \mathrm{Nu}}{r_0^2 U}(x - x_1)\right]$$

$$\frac{T_0 - T}{T_0 - T_m} = \phi\left(\frac{r}{r_0}\right) \Rightarrow T = T_0 - \phi(T_0 - T_m)$$

$$\Rightarrow \frac{\partial T}{\partial x} = \frac{\partial}{\partial x}[T_0 - \phi(T_0 - T_m)] = \phi\frac{dT_m}{dx}$$

$$\frac{u(r)}{\alpha}\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}$$

$$u = 2U\left[1 - \left(\frac{r}{r_0}\right)^2\right]$$

$$\frac{\partial T}{\partial x} = \phi\frac{dT_m}{dx}$$

for the unknown $\phi(r_*)$

$$-2\mathrm{Nu}(1 - r_*^2)\phi = \frac{d^2\phi}{dr_*^2} + \frac{1}{r_*}\frac{d\phi}{dr_*}$$

$$\mathrm{Nu} = -2\left(\frac{d\phi}{dr_*}\right)_{r_*=1}$$

boundary conditions

$d\phi/dr_* = 0$ at $r_* = 0$ radial symmetry

$\phi = 0$ at $r_* = 1$ isothermal wall

to solve the problem numerically ➡ guess the value of Nu

the differential equation is first approximated by finite differences

integrated from $r_* = 1$ to $r_* = 0$ ➡ Nu ➡ $-2\mathrm{Nu}(1 - r_*^2)\phi = \frac{d^2\phi}{dr_*^2} + \frac{1}{r_*}\frac{d\phi}{dr_*}$

مقایسه عدد ناسلت به دست آمده با مقدار حدس زده شده، در صورت وجود اختلاف، جایگزینی عدد ناسلت جدید در معادله و تکرار تا رسیدن به همگرایی

$$\mathrm{Nu} = 3.66$$

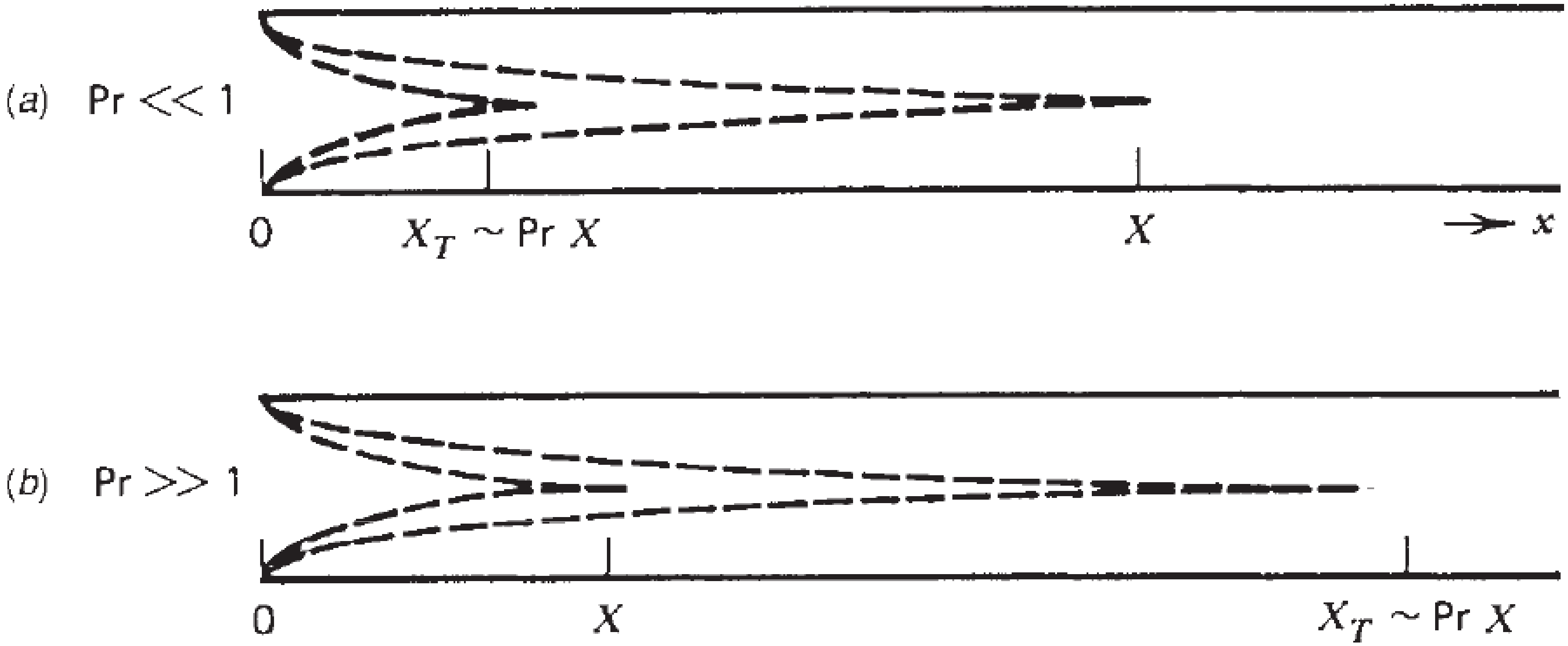
**Table 3.3 Friction factors and Nusselt numbers for heat transfer to laminar flow through ducts with regular polygonal cross sections**

| Cross Section | $f\,\mathrm{Re}_{D_h}$ Fully Developed Flow | $\mathrm{Nu} = hD_h/k$ Uniform Heat Flux Fully Developed Flow | Uniform Heat Flux Slug Flow | Isothermal Wall Fully Developed Flow | Isothermal Wall Slug Flow (Pr → 0) |
|---|---|---|---|---|---|
| Square | 14.167 | 3.614 | 7.083 | 2.980 | 4.926 |
| Hexagon | 15.065 | 4.021 | 7.533 | 3.353 | 5.380 |
| Octagon | 15.381 | 4.207 | 7.690 | 3.467 | 5.526 |
| Circle | 16 | 4.364 | 7.962 | 3.66 | 5.769 |

*Source*: Data from Ref. 12.

velocity profile remains uniform over the cross section, $u = U$,

# HEAT TRANSFER TO DEVELOPING FLOW




## Scale Analysis

$\delta_T(X_T) \sim D_h$ **Pr ≪ 1.** $\delta_T(x) \sim x \ \mathrm{Pr}^{-1/2} \ \mathrm{Re}_x^{-1/2}$

Because at the end of thermal development $x \sim X_T$ and $\delta_T \sim D_h$:

$$X_T \ \mathrm{Pr}^{-1/2} \ \mathrm{Re}_{X_T}^{-1/2} \sim D_h \rightarrow \left(\frac{X_T/D_h}{\mathrm{Re}_{D_h} \ \mathrm{Pr}}\right)^{1/2} \sim 1$$

other publications

$$\frac{X_T/D_h}{\mathrm{Re}_{D_h} \ \mathrm{Pr}} \sim 0.1$$

**Pr ≫ 1** $\delta_T(x) \sim x\mathrm{Pr}^{-1/2}\mathrm{Re}_x^{-1/2}$

$\frac{X_T}{X} \sim \mathrm{Pr}$ the thermally developing section ($x \ll X_T$)

$$\mathrm{Nu} = \frac{hD_h}{k} \sim \frac{q''}{\Delta T}\frac{D_h}{k} \sim \frac{D_h}{\delta_T} \sim \left(\frac{x/D_h}{\mathrm{Re}_{D_h}\mathrm{Pr}}\right)^{-1/2}$$

**Thermally Developing Hagen–Poiseuille Flow**

*Graetz problem*

Uniform wall temperature, $T_0 = \text{constant}$

Symmetry about the centerline, $\partial T/\partial r = 0$ at $r = 0$

$$\frac{1}{2}(1 - r_*^2)\frac{\partial \theta_*}{\partial x_*} = \frac{\partial^2 \theta_*}{\partial r_*^2} + \frac{1}{r_*}\frac{\partial \theta_*}{\partial r_*}$$

$$\frac{\partial \theta_*}{\partial r_*} = 0 \quad \text{at} \quad r_* = 0 \qquad \theta_* = 0 \quad \text{at} \quad r_* = 1$$

$$\theta_* = 1 \quad \text{at} \quad x_* = 0$$

$$\theta_* = \frac{T - T_0}{T_{\mathrm{IN}} - T_0}, \qquad r_* = \frac{r}{r_0}, \qquad x_* = \frac{x/D}{\mathrm{Re}_D\ \mathrm{Pr}}$$



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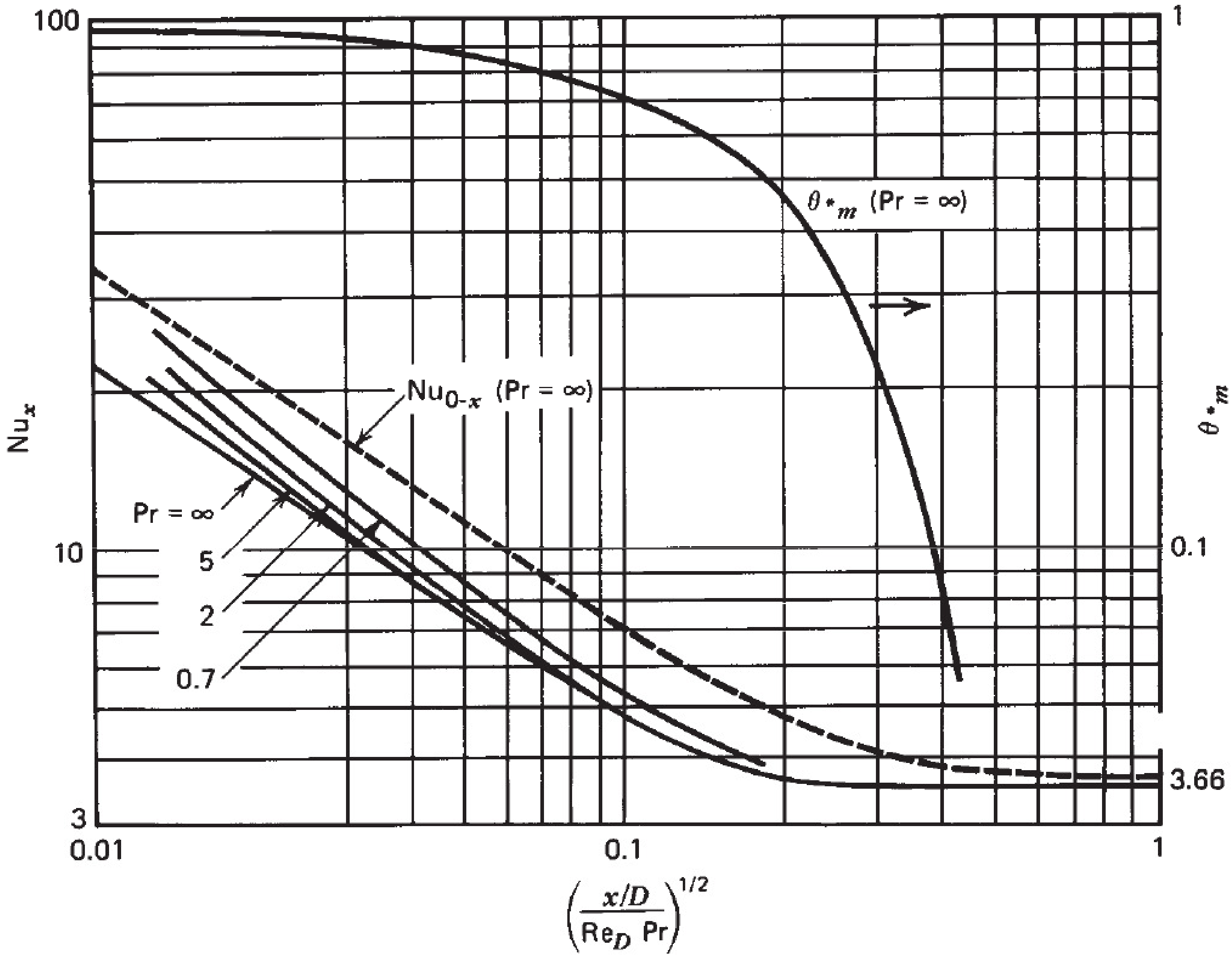
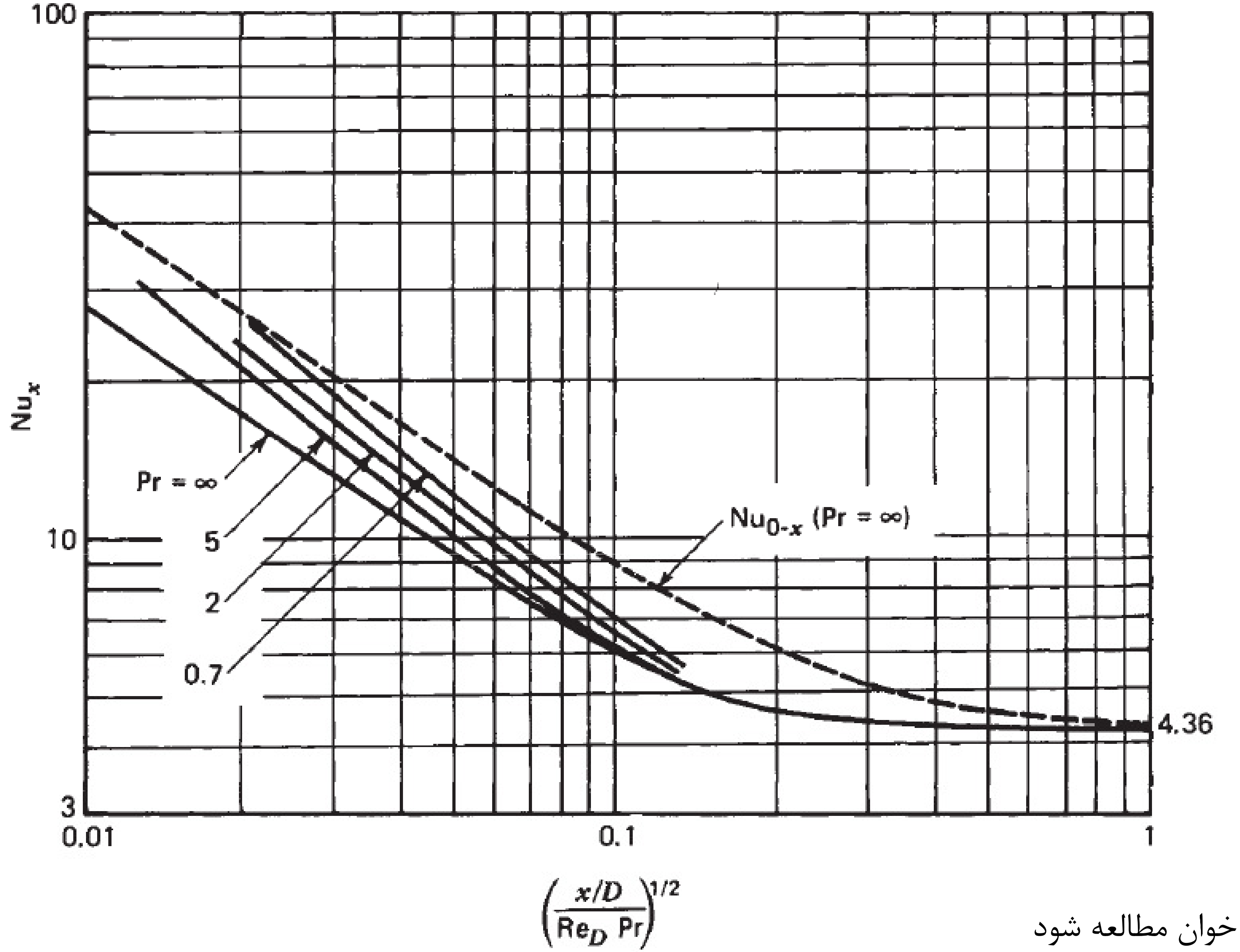



**Figure 3.12** Heat transfer in the entrance region of a round tube with isothermal wall. (Based on data from Refs. 10 and 14.)




بقیه قسمتها خود خوان مطالعه شود

**Figure 3.13** Heat transfer in the entrance region of a round tube with uniform heat flux. (Based on data from Refs. 10 and 14.)