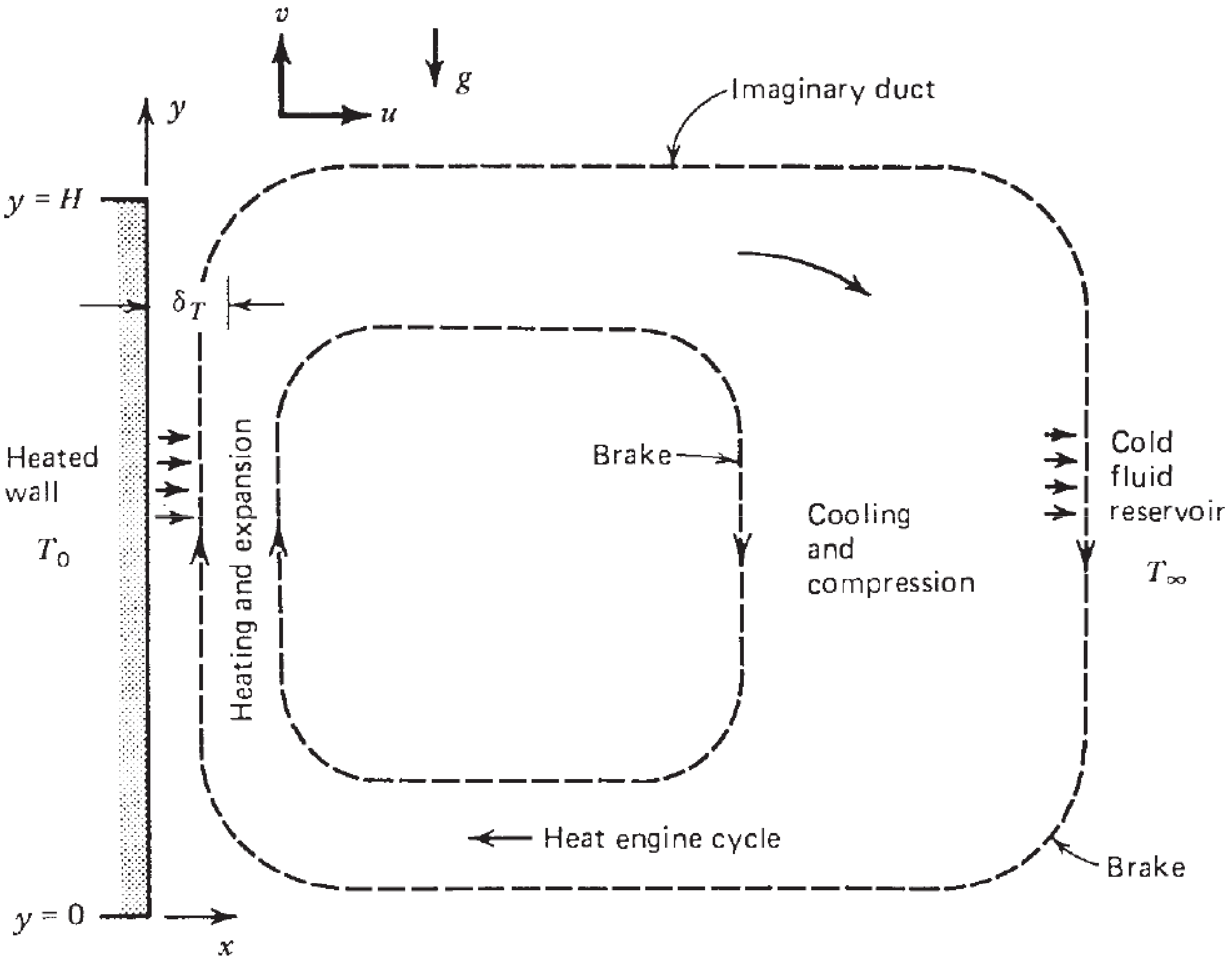
EXTERNAL NATURAL CONVECTION



heating → expansion → cooling → compression

LAMINAR BOUNDARY LAYER EQUATIONS

$Q = (HW)h_{0-H}(T_0 - T_\infty)$ HW is the wall area the scale of h_{0-H} is k/δ_T

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

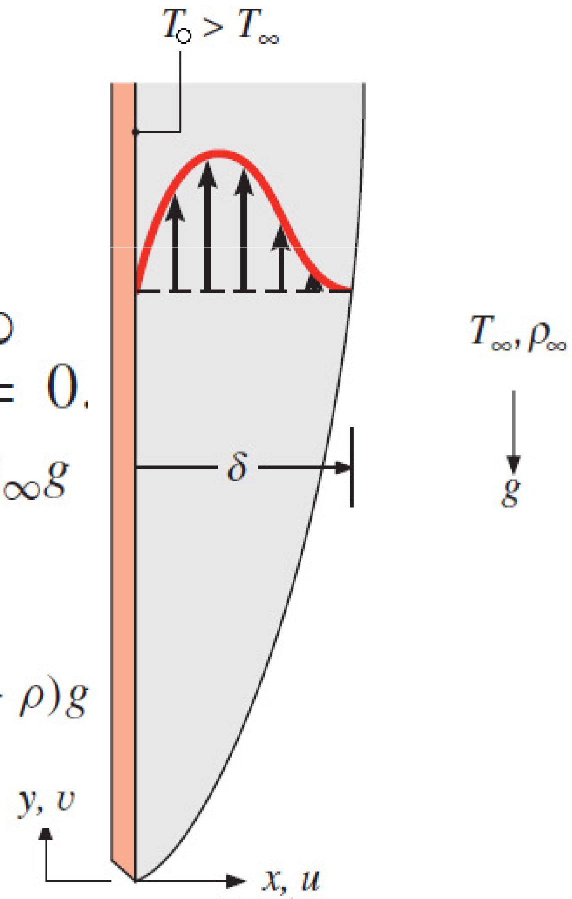
$(x \sim \delta_T,\ y \sim H, \text{ and } \delta_T \ll H)$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu \nabla^2 u$$

$u = 0 \Rightarrow \frac{\partial P}{\partial y} = \frac{dP}{dy} = \frac{dP_\infty}{dy}$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} + \mu \nabla^2 v - \rho g$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \nabla^2 T$$



as $x \rightarrow \infty$ $u = v = 0.$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{dP_\infty}{dy} + \mu\frac{\partial^2 v}{\partial x^2} - \rho g$$

$\Rightarrow dP_\infty/dy = -\rho_\infty g$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \mu\frac{\partial^2 v}{\partial x^2} + (\rho_\infty - \rho)g$$

$$P = \rho R T$$

$$\rho = \frac{P_\infty / R}{T} \quad \text{and} \quad \rho_\infty = \frac{P_\infty / R}{T_\infty} \Bigg\} \quad \rho - \rho_\infty = \rho \left(1 - \frac{T}{T_\infty}\right)$$

$$\frac{\rho_\infty - \rho}{\rho_\infty} \left(1 - \frac{\rho_\infty - \rho}{\rho_\infty}\right)^{-1} = \frac{T - T_\infty}{T_\infty}$$

$$(T - T_\infty) \ll T_\infty \Rightarrow \rho \simeq \rho_\infty \left[1 - \frac{1}{T_\infty}(T - T_\infty) + \cdots\right]$$

$$\rho \simeq \rho_\infty [1 - \beta (T - T_\infty) + \cdots]$$

volume expansion coefficient at constant pressure $\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$

$\beta(T - T_\infty)$ is considerably smaller than unity

Boussinesq approximation $\rho \simeq \rho_\infty [1 - \beta (T - T_\infty) + \cdots]$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=\mu\frac{\partial^2 v}{\partial x^2}+(\rho_\infty-\rho)g$$

$$\rho \simeq \rho_\infty[1-\beta(T-T_\infty)+\cdots] \Rightarrow \rho_\infty-\rho \simeq \rho_\infty\beta(T-T_\infty)$$

$\beta(T-T_\infty)$ is considerably smaller than unity

$$\Rightarrow 1-\beta(T-T_\infty)\simeq 1 \Rightarrow u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 v}{\partial x^2}+g\beta(T-T_\infty)$$

where g, β, T_∞, and $\nu=\mu/\rho_\infty$ are constants

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2} \rightarrow \alpha=k/\rho_\infty c_P \text{ is assumed constant}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\nu\frac{\partial^2 v}{\partial x^2}+g\beta(T-T_\infty)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}$$

$u=v=0$ and $T=T_0$ at $x=0$

$v=0$ and $T=T_\infty$ as $x\rightarrow\infty$

SCALE ANALYSIS

$\delta_T \times H$ region

$(x \sim \delta_T, y \sim H)$

$\Delta T = T_0 - T_\infty$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{u}{\delta_T} \sim \frac{v}{H}$$

$$\underbrace{u\frac{\Delta T}{\delta_T},\quad v\frac{\Delta T}{H}}_{\text{Convection}} \sim \underbrace{\alpha\frac{\Delta T}{\delta_T^2}}_{\text{Conduction}} \Rightarrow u\frac{\Delta T}{\delta_T} \sim v\frac{\Delta T}{H}$$

$$v\frac{\Delta T}{H} \sim \alpha\frac{\Delta T}{\delta_T^2} \Rightarrow \boxed{v \sim \frac{\alpha H}{\delta_T^2}}$$

δ_T is still unknown

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial x^2} + g\beta(T - T_\infty)$$

$$\underbrace{u\frac{v}{\delta_T},\quad v\frac{v}{H}}_{\text{Inertia}} \quad \underbrace{\frac{\nu v}{\delta_T^2}}_{\text{Friction}} \quad \underbrace{g\beta\,\Delta T}_{\text{Buoyancy}}$$

$$\left[\underbrace{u\frac{v}{\delta_T}, \quad v\frac{v}{H}}_{\text{Inertia}} \quad \underbrace{\frac{\nu v}{\delta_T^2}}_{\text{Friction}} \quad \underbrace{g\beta\,\Delta T}_{\text{Buoyancy}} \right] \Big/ \; g\beta\,\Delta T$$

Rayleigh number

$$\mathrm{Ra}_H = \frac{g\beta\,\Delta T\,H^3}{\alpha\nu}$$

$$\underbrace{\left(\frac{H}{\delta_T}\right)^4 \mathrm{Ra}_H^{-1}\,\mathrm{Pr}^{-1}}_{\text{Inertia}} \qquad \underbrace{\left(\frac{H}{\delta_T}\right)^4 \mathrm{Ra}_H^{-1}}_{\text{Friction}} \qquad \underbrace{1}_{\text{Buoyancy}}$$

High-Pr Fluids $\mathrm{Pr} \gg 1$ ➡ the friction–buoyancy balance

$$\underbrace{\left(\frac{H}{\delta_T}\right)^4 \mathrm{Ra}_H^{-1}}_{\text{Friction}} \sim \underbrace{1}_{\text{Buoyancy}} \quad \Rightarrow \quad \boxed{\delta_T \sim H\,\mathrm{Ra}_H^{-1/4}}$$

$$v \sim \frac{\alpha H}{\delta_T^2}$$

$$\boxed{v \sim \frac{\alpha}{H}\mathrm{Ra}_H^{1/2}}$$

the scale of h_{0-H} is k/δ_T ➡ $\boxed{\mathrm{Nu} = \frac{hH}{k} \sim \mathrm{Ra}_H^{1/4}}$

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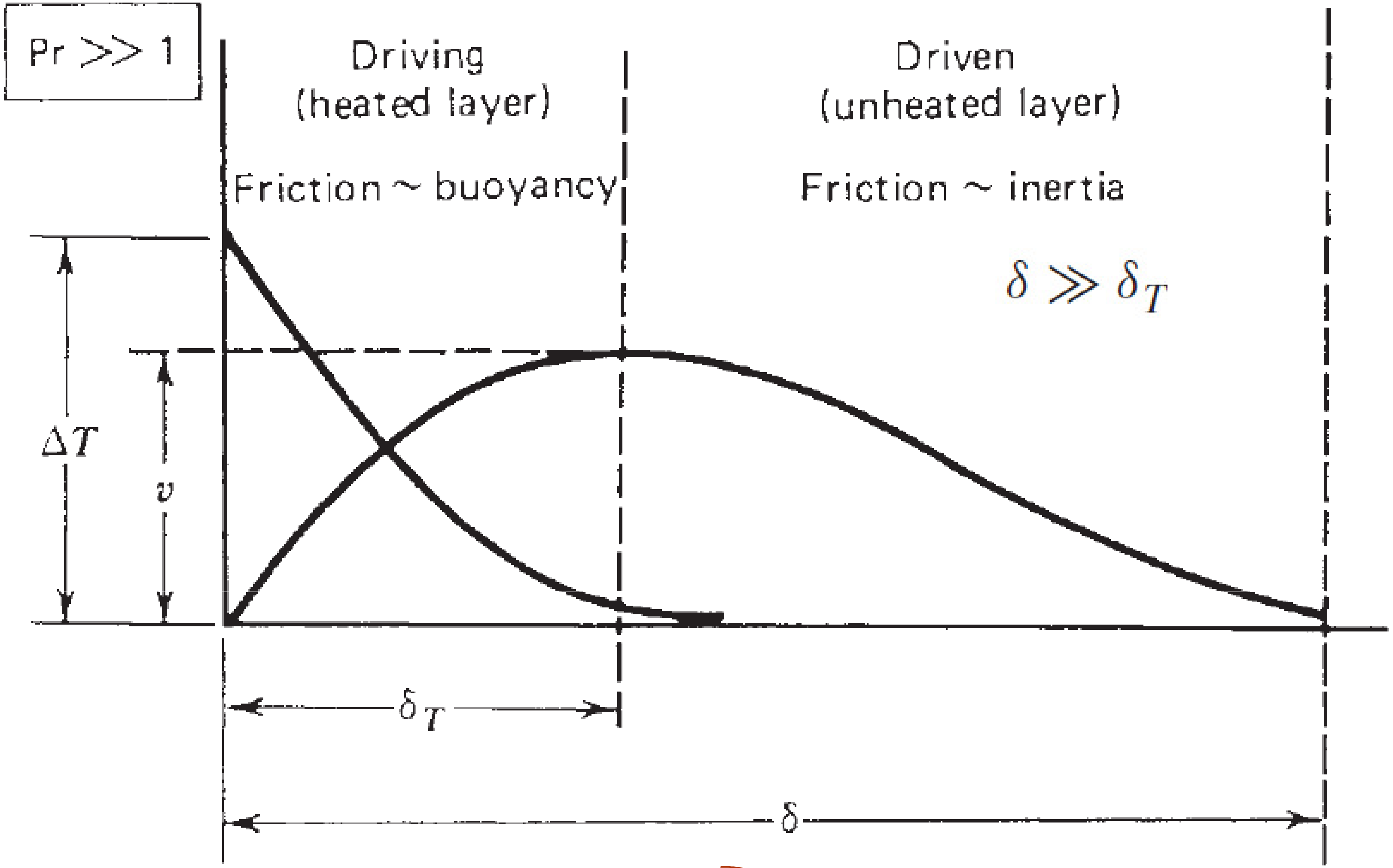

Friction ~ inertia → $v\frac{v}{H} \sim \nu\frac{v}{\delta^2}$

$$v \sim \frac{\alpha}{H}\mathrm{Ra}_H^{1/2}$$

$$\delta \sim H\,\mathrm{Ra}_H^{-1/4}\,\mathrm{Pr}^{1/2}$$

$$\frac{\delta}{\delta_T} \sim \mathrm{Pr}^{1/2} > 1$$

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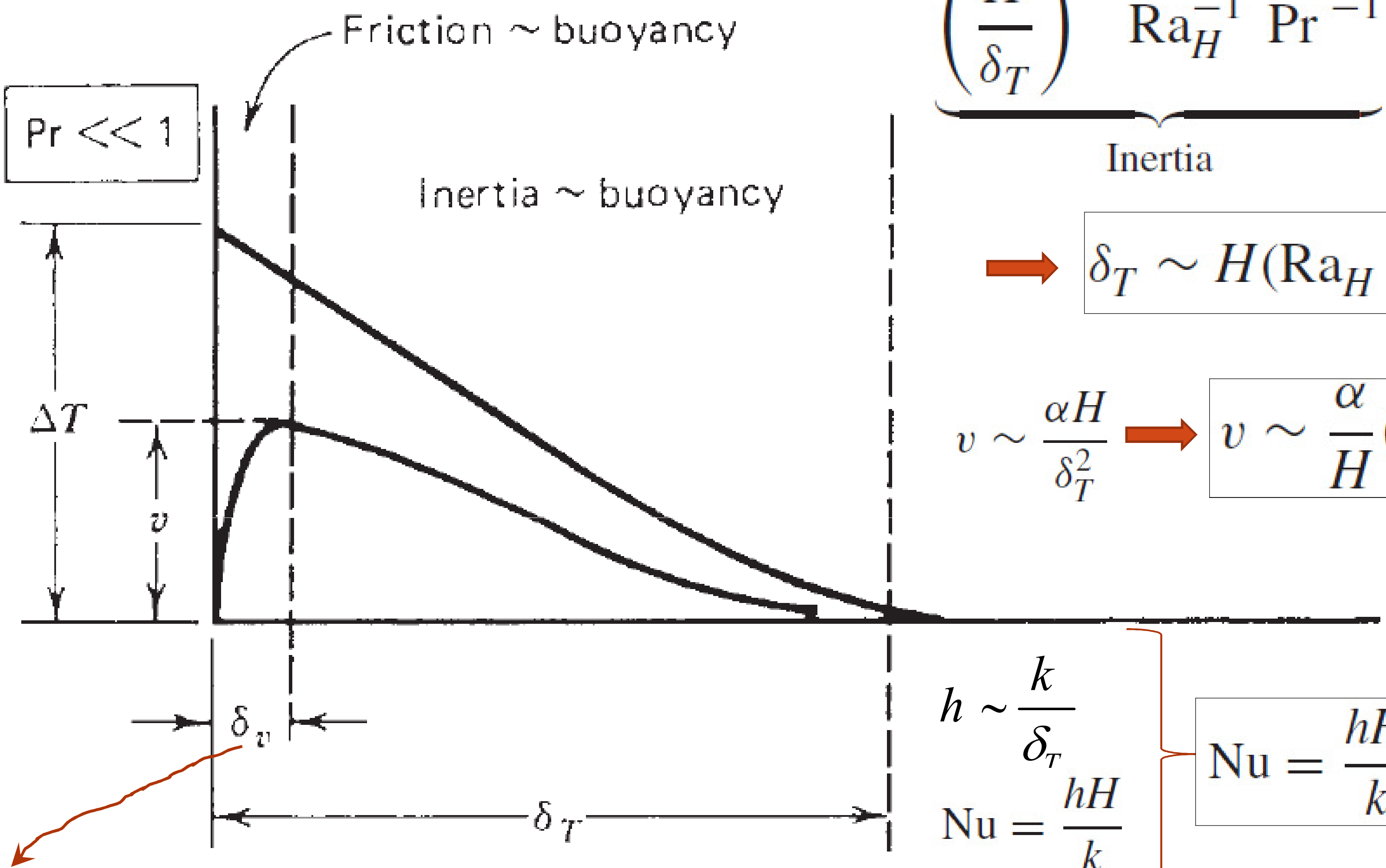
Low-Pr Fluids $\Pr \ll 1$



$$\underbrace{\left(\frac{H}{\delta_T}\right)^4 Ra_H^{-1}\ Pr^{-1}}_{\text{Inertia}} \sim \underbrace{1}_{\text{Buoyancy}}$$

$$\Rightarrow \quad \boxed{\delta_T \sim H(Ra_H\ Pr)^{-1/4}}$$

$$v \sim \frac{\alpha H}{\delta_T^2} \quad \Rightarrow \quad \boxed{v \sim \frac{\alpha}{H}(Ra_H\ Pr)^{1/2}}$$

$$h \sim \frac{k}{\delta_T}$$

$$Nu = \frac{hH}{k}$$

$$\boxed{Nu = \frac{hH}{k} \sim (Ra_H\ Pr)^{1/4}}$$

δ_v be the thickness of a very thin layer right near the wall.

$$\textit{Boussinesq number} \quad Bo_H = Ra_H\ Pr = \frac{g\beta\ \Delta T\ H^3}{\alpha^2}$$

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δ_v → buoyancy $\sim$ friction balance → $\nu \dfrac{v}{\delta_v^2} \sim g\beta\, \Delta T$

v scale is dictated by the δ_T layer scale $v \sim \dfrac{\alpha}{H}(\mathrm{Ra}_H\ \mathrm{Pr})^{1/2}$

$$\delta_v \sim H\, \mathrm{Gr}_H^{-1/4}$$

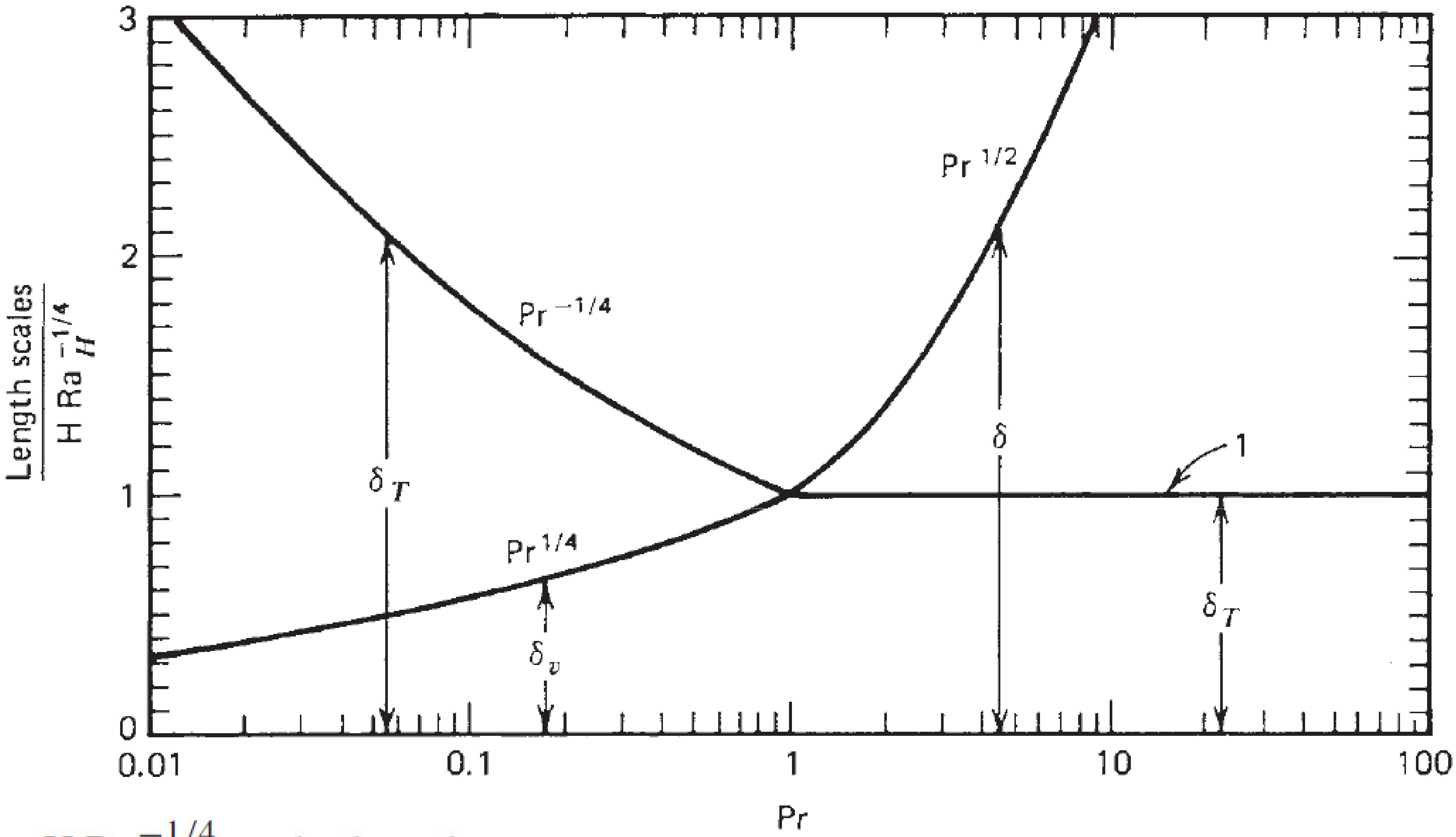
Grashof number is defined as $\mathrm{Gr}_H = \dfrac{g\beta\, \Delta T\, H^3}{\nu^2} = \dfrac{\mathrm{Ra}_H}{\mathrm{Pr}}$

$\dfrac{\delta_v}{\delta_T} \sim \mathrm{Pr}^{1/2} < 1$

δ_v should not be confused with δ

Table 4.1 Summary of flow and heat transfer scales in a natural convection boundary layer along a vertical wall

Prandtl Number Range	Thermal Boundary Layer Thickness	Wall Jet Velocity Profile: Distance from Wall to Velocity Peak	Wall Jet Velocity Profile: Thickness of Wall Jet	Wall Jet Velocity Profile: Velocity Scale	Nusselt Number $\mathrm{Nu} = \dfrac{hH}{k}$
$\mathrm{Pr} > 1$	$H\,\mathrm{Ra}_H^{-1/4}$	$H\,\mathrm{Ra}_H^{-1/4}$	$\mathrm{Pr}^{1/2}(H\,\mathrm{Ra}_H^{-1/4})$	$\dfrac{\alpha}{H}\mathrm{Ra}_H^{1/2}$	$\mathrm{Ra}_H^{1/4}$
$\mathrm{Pr} < 1$	$\mathrm{Pr}^{-1/4}(H\,\mathrm{Ra}_H^{-1/4})$	$\mathrm{Pr}^{1/4}(H\,\mathrm{Ra}_H^{-1/4})$	$\mathrm{Pr}^{-1/4}(H\,\mathrm{Ra}_H^{-1/4})$	$\dfrac{\alpha}{H}(\mathrm{Pr}\ \mathrm{Ra}_H)^{1/2}$	$(\mathrm{Pr}\ \mathrm{Ra}_H)^{1/4}$



$H \, \mathrm{Ra}_H^{-1/4}$ as the length

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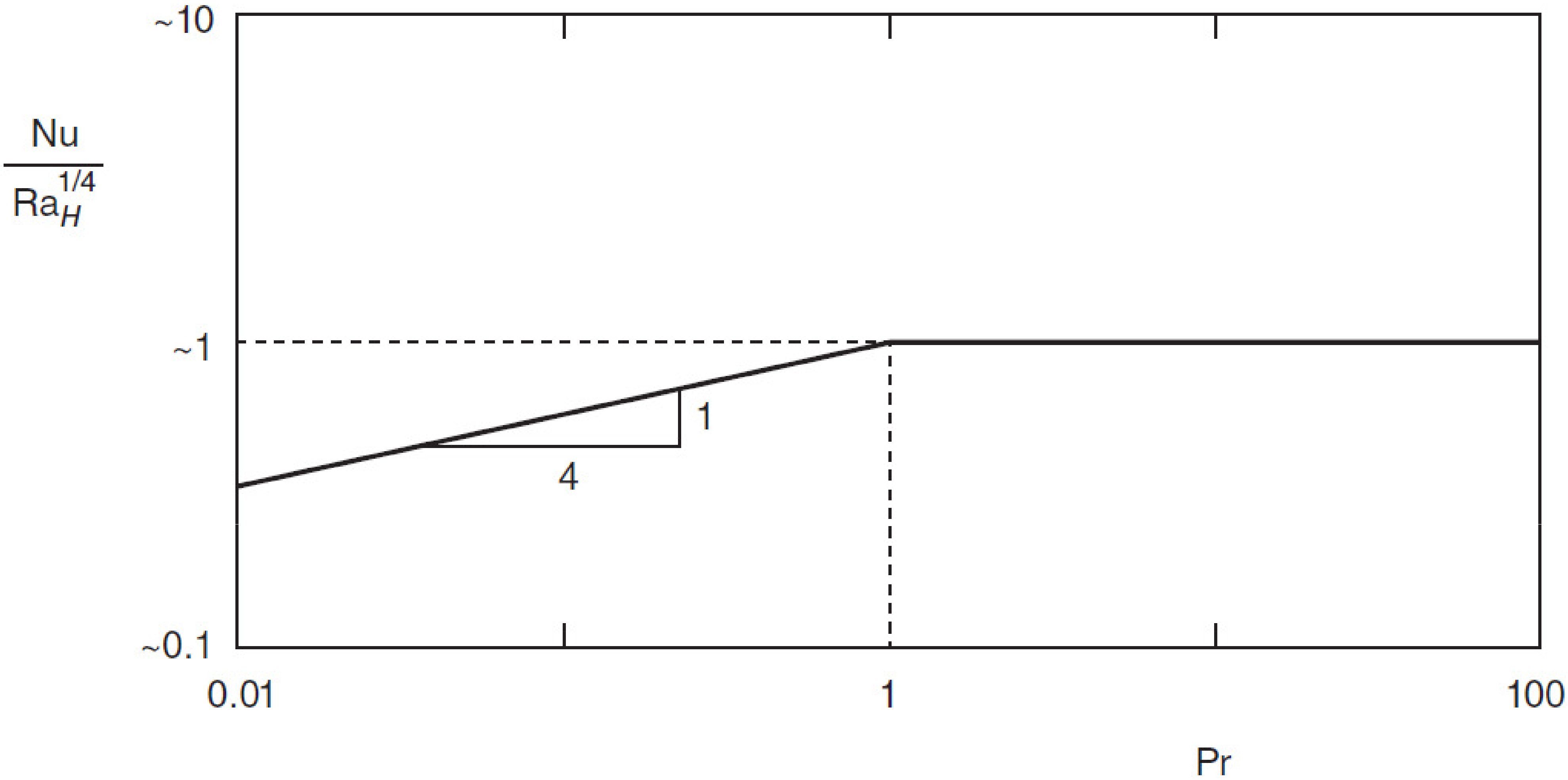

dimensionless numbers such as Ra_H, Bo_H, and Gr_H have no meaning

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$$\mathrm{Ra}_H^{1/4} \sim \frac{\text{wall height}}{\text{thermal boundary layer thickness}} \qquad (\mathrm{Pr} > 1)$$

$$\mathrm{Bo}_H^{1/4} \sim \frac{\text{wall height}}{\text{thermal boundary layer thickness}} \qquad (\mathrm{Pr} < 1)$$

$$\mathrm{Gr}_H^{1/4} \sim \frac{\text{wall height}}{\text{wall shear layer thickness}} \qquad (\mathrm{Pr} < 1)$$

The meaning of $\mathrm{Ra}_H^{1/4}$, $\mathrm{Bo}_H^{1/4}$, and $\mathrm{Gr}_H^{1/4}$ is purely *geometric*

INTEGRAL SOLUTION

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Integrating from the wall ($x = 0$) to a far enough plane $x = X$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial x^2} + g\beta(T - T_\infty) \rightarrow \frac{d}{dy}\int_0^X v^2\,dx = -\nu\left(\frac{\partial v}{\partial x}\right)_{x=0} + g\beta\int_0^X (T - T_\infty)\,dx$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2} \rightarrow \frac{d}{dy}\int_0^X v(T_\infty - T)\,dx = \alpha\left(\frac{\partial T}{\partial x}\right)_{x=0}$$

High-Pr Fluids $\Pr > 1$

$$T - T_\infty = \Delta T e^{-x/\delta_T}$$

$$v = V e^{-x/\delta}(1 - e^{-x/\delta_T})$$

where V, δ_T, and δ are unknown functions

$$\Delta T = T_0 - T_\infty = \text{constant}$$

جایگذاری →

$$\frac{d}{dy}\int_0^X v^2\,dx = -\nu\left(\frac{\partial v}{\partial x}\right)_{x=0} + g\beta\int_0^X (T - T_\infty)\,dx$$

$$\frac{d}{dy}\int_0^X v(T_\infty - T)\,dx = \alpha\left(\frac{\partial T}{\partial x}\right)_{x=0}$$

→ $X \to \infty$ →

$$\frac{d}{dy}\left[\frac{V^2\delta q^2}{2(2+q)(1+q)}\right] = -\frac{\nu V q}{\delta} + g\beta\,\Delta T\frac{\delta}{q}$$

$$q(\Pr) = \frac{\delta}{\delta_T} \qquad \frac{d}{dy}\left[\frac{V\delta}{(1+q)(1+2q)}\right] = \frac{\alpha}{\delta}$$

three unknowns: $V(y), \delta(y)$, and $q(\Pr)$ → The third equation, necessary

Squire's [15] avoided this problem by assuming that $\delta_T = \delta$

However $\delta \neq \delta_T$

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in the no-slip layer $0 < x < 0^+$ the inertia terms ➡ $0 = \nu \frac{\partial^2 v}{\partial x^2} + g\beta (T_0 - T_\infty)$

The third equation

$\delta \sim y^{1/4}$ and $V \sim y^{1/2}$ $\qquad \mathrm{Pr} = \frac{5}{6} q^2 \frac{q + \frac{1}{2}}{q+2}$

local Nusselt number

$$\mathrm{Nu} = \frac{q''}{T_0 - T_\infty} \frac{y}{k} = \left[\frac{3}{8} \frac{q^3}{(q+1)\left(q + \frac{1}{2}\right)(q+2)} \right]^{1/4} \mathrm{Ra}_y^{1/4}$$

$\mathrm{Pr} \to \infty$ ➡ $\frac{\delta}{\delta_T} = \left(\frac{6}{5} \mathrm{Pr} \right)^{1/2}$ and $\mathrm{Nu} = 0.783 \mathrm{Ra}_y^{1/4} \sim \mathrm{Ra}_H^{1/4}$

Low-Pr Fluids

$\mathrm{Pr} < 1 \qquad v = V_1 e^{-x/\delta_T} \left(1 - e^{-x/\delta_v}\right)$

V_1, δ_T, and δ_v are unknown functions

$\delta_T \sim y^{1/4}$, $\delta_v \sim y^{1/4}$, and $V_1 \sim y^{1/2}$

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$$\mathrm{Pr} = \frac{5}{3}\left(\frac{q_1}{1+q_1}\right)^2, \qquad q_1 = \frac{\delta_v}{\delta_T}$$

$$\mathrm{Nu} = \frac{q''}{T_0 - T_\infty}\frac{y}{k} = \left(\frac{3}{8}\right)^{1/4}\left(\frac{q_1}{2q_1+1}\right)^{1/2}\mathrm{Ra}_y^{1/4}$$

limit $\mathrm{Pr} \to 0$

$$\frac{\delta_v}{\delta_T} = \left(\frac{3}{5}\mathrm{Pr}\right)^{1/2} \qquad \text{and} \qquad \mathrm{Nu} = 0.689\,(\mathrm{Pr}\;\mathrm{Ra}_y)^{1/4}$$

SIMILARITY SOLUTION

$$\eta = \frac{x}{y}\mathrm{Ra}_y^{1/4} \qquad u = \partial\psi/\partial y,\ v = -\partial\psi/\partial x \qquad \frac{\partial\psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2}$$

$$-\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x^2} + \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\partial y} = -\nu\frac{\partial^3\psi}{\partial x^3} + g\beta(T - T_\infty)$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \theta(\eta,\ \mathrm{Pr})$$

$\Pr > 1$

$$\left.\begin{aligned} v &= \frac{\alpha}{y}\,\mathrm{Ra}_y^{1/2}\,G(\eta,\ \Pr) \\ v &= -\partial\psi/\partial x \end{aligned}\right\}$$

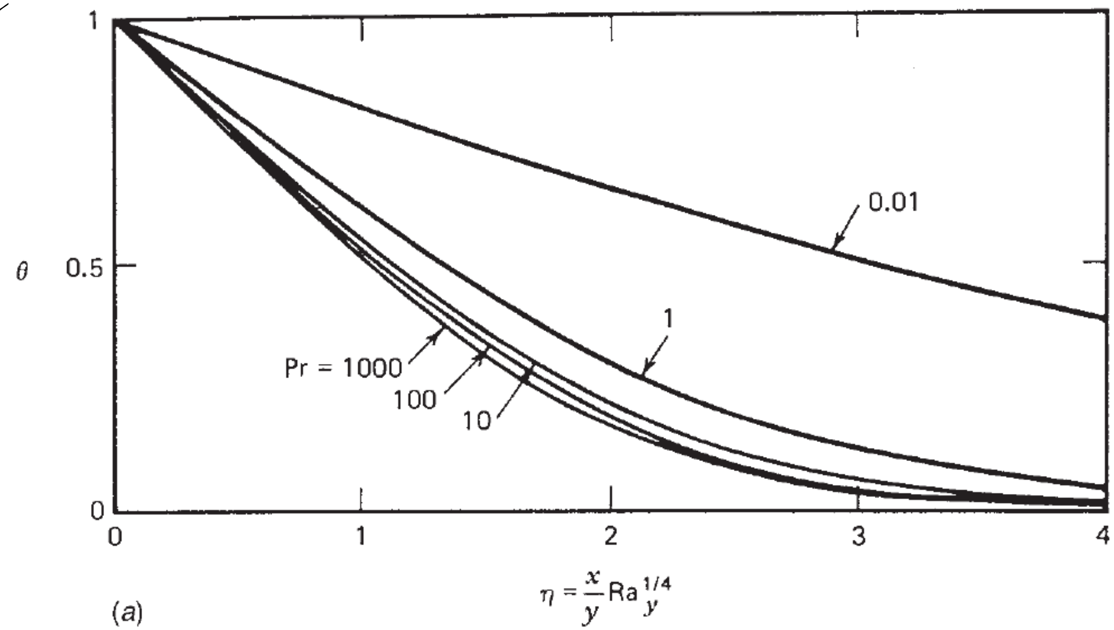
$$\psi = \alpha\,\mathrm{Ra}_y^{1/4}\,F(\eta,\ \Pr)$$

$$G = -\partial F/\partial \eta$$

$$\tfrac{3}{4}F\theta' = \theta''$$

$$\frac{1}{\Pr}\left(\frac{1}{2}F'^2 - \frac{3}{4}FF''\right) = -F''' + \theta$$

(i) At $x = 0$,	$u = 0$	$F = 0$
$(\eta = 0)$	$v = 0$	$F' = 0$
	$T = T_0$	$\theta = 1$
(ii) As $x \to \infty$,	$v = 0$	$F' = 0$
$(\eta \to \infty)$	$T = T_\infty$	$\theta = 0$

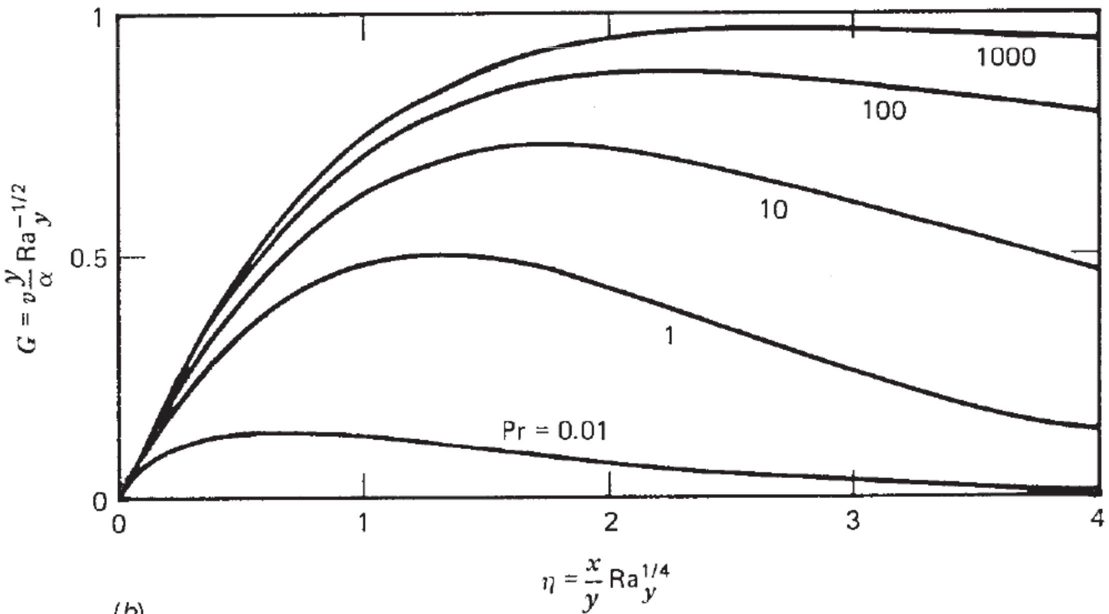


(a)

temperature profiles

$$\mathrm{Nu} = \frac{hy}{k} = -(\theta')_{\eta=0}\ \mathrm{Ra}_y^{1/4}$$

Pr > 1 fluid



(b)

vertical velocity profiles

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Table 4.2 Similarity solution heat transfer results for natural convection boundary layer along a vertical isothermal wall[a]

Pr	0.01	0.72	1	2	10	100	1000
$\mathrm{Nu}\,\mathrm{Ra}_y^{-1/4}$	0.162	0.387	0.401	0.426	0.465	0.490	0.499

[a]Numerical values calculated from Ostrach's solution [16].

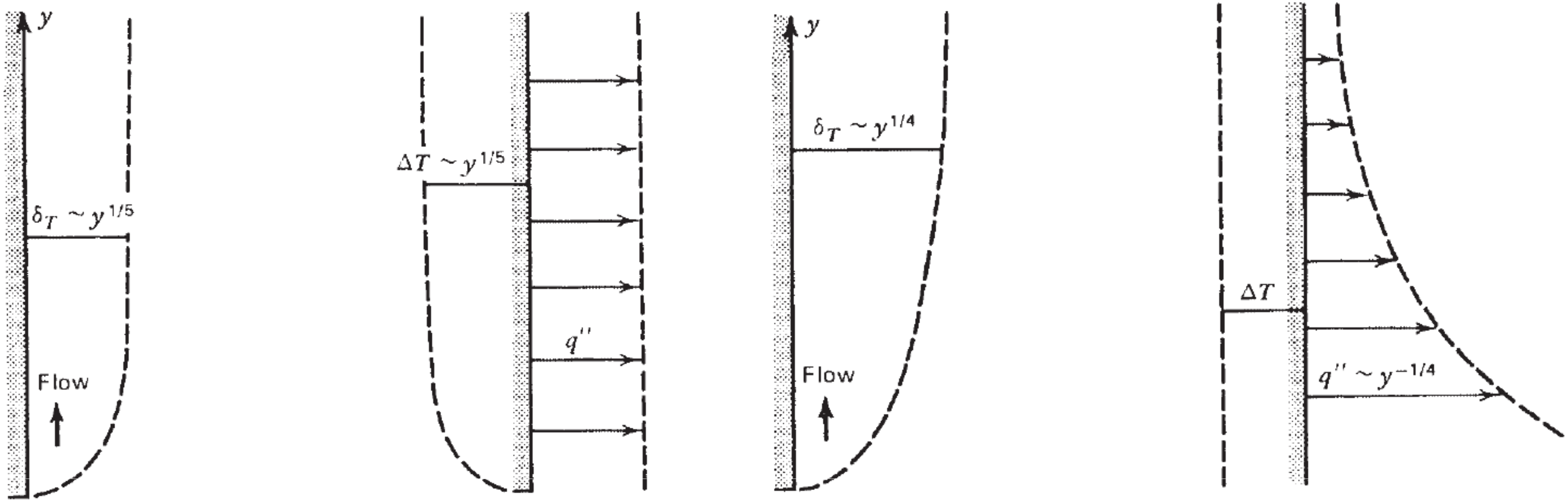
$$\mathrm{Nu} = 0.503\,\mathrm{Ra}_y^{1/4} \qquad \text{as } \mathrm{Pr} \to \infty$$

$$\mathrm{Nu} = 0.6(\mathrm{Ra}_y\ \mathrm{Pr})^{1/4} \qquad \text{as } \mathrm{Pr} \to 0$$

$$\mathrm{Nu}_{0-H} = 0.671\ \mathrm{Ra}_H^{1/4} \qquad \text{as } \mathrm{Pr} \to \infty$$

$$\mathrm{Nu}_{0-H} = 0.8(\mathrm{Ra}_H\ \mathrm{Pr})^{1/4} \qquad \text{as } \mathrm{Pr} \to 0$$

UNIFORM WALL HEAT FLUX



(b) Uniform wall heat flux (a) Isothermal wall

$$q'' \sim k\frac{\Delta T}{\delta_T}$$

an isothermal wall

both ΔT and the product $q''\delta_T$ are independent of y.

in the case of constant q'', ΔT and δ_T functions of y.

$$\delta_T \sim H\,\mathrm{Ra}_H^{-1/4} \Rightarrow \delta_T \sim H\left(\frac{g\beta\,\Delta T\,H^3}{\alpha\nu}\right)^{-1/4}$$

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$$q'' \sim k\frac{\Delta T}{\delta_T}$$

$$\delta_T \sim H\left(\frac{g\beta\,\Delta T\,H^3}{\alpha\nu}\right)^{-1/4}$$

$$\delta_T \sim H\,\mathrm{Ra}_{*H}^{-1/5}$$

$$\mathrm{Ra}_{*H} = \frac{g\beta H^4 q''}{\alpha\nu k}$$

$$\Delta T \sim \frac{q''}{k} H\,\mathrm{Ra}_{*H}^{-1/5}$$

$(\mathrm{Pr} \gg 1)$

both δ_T and ΔT are proportional to $H^{1/5}$

$$\mathrm{Nu} = \frac{q''}{T_0(y) - T_\infty}\frac{y}{k} \xrightarrow{\mathrm{Pr} \gtrsim 1} \mathrm{Nu} \sim \frac{H}{\delta_T} \sim \mathrm{Ra}_{*H}^{1/5}$$

low–Prandtl number

$$\delta_T \sim H(\mathrm{Ra}_{*H}\,\mathrm{Pr})^{-1/5}$$

$$\Delta T \sim \frac{q''}{k} H(\mathrm{Ra}_{*H}\ \mathrm{Pr})^{-1/5}$$

$$\mathrm{Nu} \sim (\mathrm{Ra}_{*H}\ \mathrm{Pr})^{1/5}$$

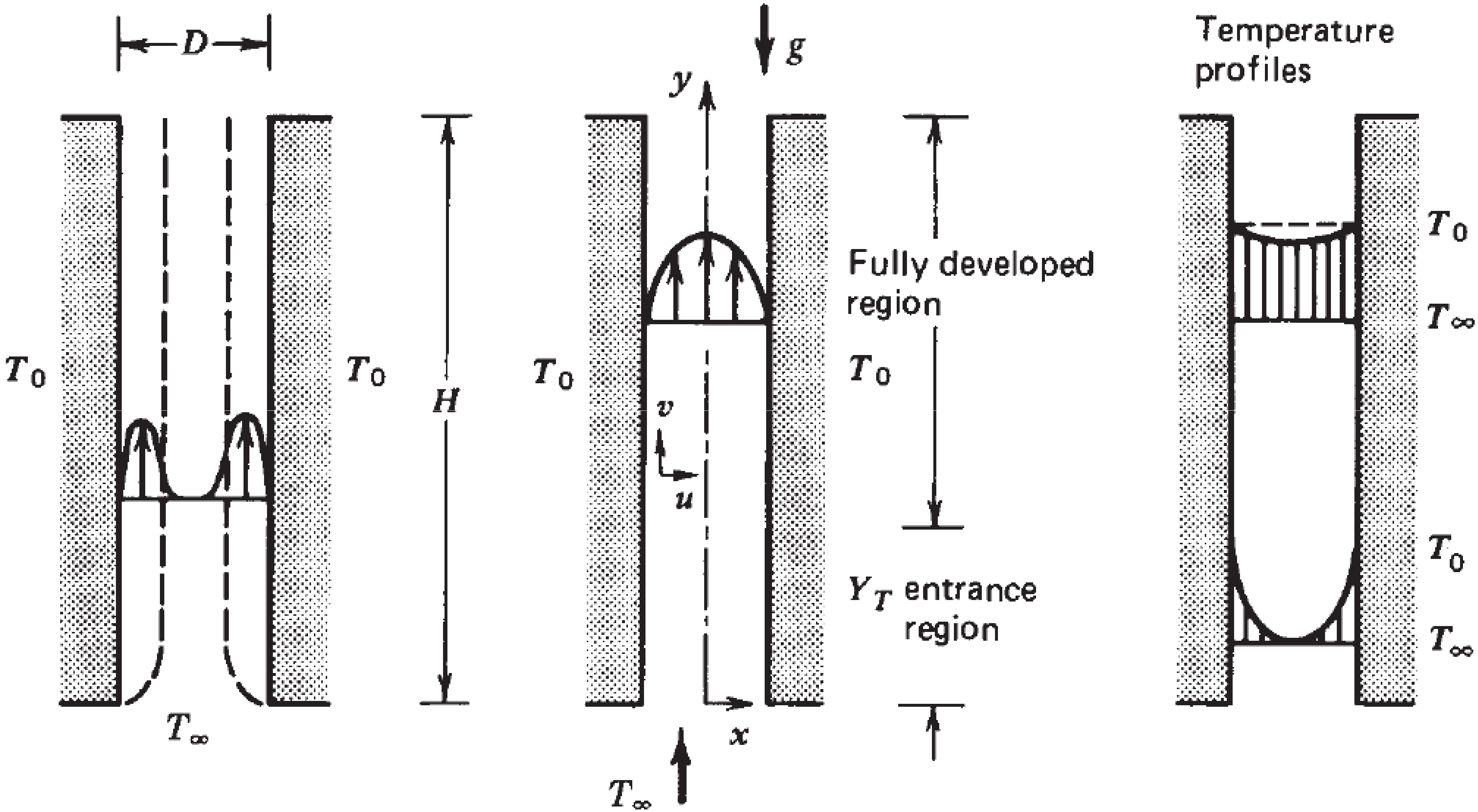
Sparrow [19]

$$\mathrm{Nu} = \frac{2}{360^{1/5}}\left(\frac{\mathrm{Pr}}{\frac{4}{5} + \mathrm{Pr}}\right)^{1/5} \mathrm{Ra}_{*y}^{1/5}$$

Sparrow and Gregg [20]

$$\mathrm{Nu} = \begin{cases} 0.616\mathrm{Ra}_{*y}^{1/5} & (\mathrm{Pr} \to \infty) \\ 0.644\mathrm{Ra}_{*y}^{1/5}\ \mathrm{Pr}^{1/5} & (\mathrm{Pr} \to 0) \end{cases}$$

VERTICAL CHANNEL FLOW



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$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+\mu\,\nabla^2 v-\rho g$$

fully developed flow $u=0$ and $\frac{\partial v}{\partial y}=0$

because both ends of the channel are open to the ambient of density ρ_∞

$$\frac{\partial P}{\partial y}=\frac{dP}{dy}=-\rho_\infty g$$

➡ $$\frac{d^2 v}{dx^2}=-\frac{g\beta}{v}(T-T_\infty)$$

the temperature difference can be approximated by $T_0 - T_\infty$

$$T_0-T\ll T_0-T_\infty \longrightarrow T-T_\infty\sim T_0-T_\infty$$

➡ $$v=\frac{g\beta D^2(T_0-T_\infty)}{8\nu}\left[1-\left(\frac{x}{D/2}\right)^2\right]$$

$$\dot{m}=\frac{\rho g\beta D^3(T_0-T_\infty)}{(12)\nu}$$

Total heat transfer rate between stream and channel walls

$$q' = \dot{m}(\text{outlet enthalpy} - \text{inlet enthalpy})$$
$$= \dot{m}c_P(T_0 - T_\infty)$$

Average heat flux:

$$q''_{0-H} = q'/(2H)$$

Overall Nusselt number. $$\frac{q''_{0-H}H}{(T_0 - T_\infty)k} = \frac{\mathrm{Ra}_D}{24}$$ $$\mathrm{Ra}_D = \frac{g\beta D^3(T_0 - T_\infty)}{\alpha\nu}$$

The Rayleigh number range of its validity

the thermal entrance length Y_T be much smaller than the channel height H,

$Y_T < H$

$$Y_T \mathrm{Ra}_{Y_T}^{-1/4} \sim \frac{D}{2} \qquad (\mathrm{Pr} > 1)$$

$$Y_T \mathrm{Bo}_{Y_T}^{-1/4} \sim \frac{D}{2} \qquad (\mathrm{Pr} < 1)$$

Evaluating Y_T from above, criterion →

$$\mathrm{Ra}_D^{1/4} < 2\left(\frac{H}{D}\right)^{1/4} \qquad (\mathrm{Pr} > 1)$$

$$\mathrm{Bo}_D^{1/4} < 2\left(\frac{H}{D}\right)^{1/4} \qquad (\mathrm{Pr} < 1)$$

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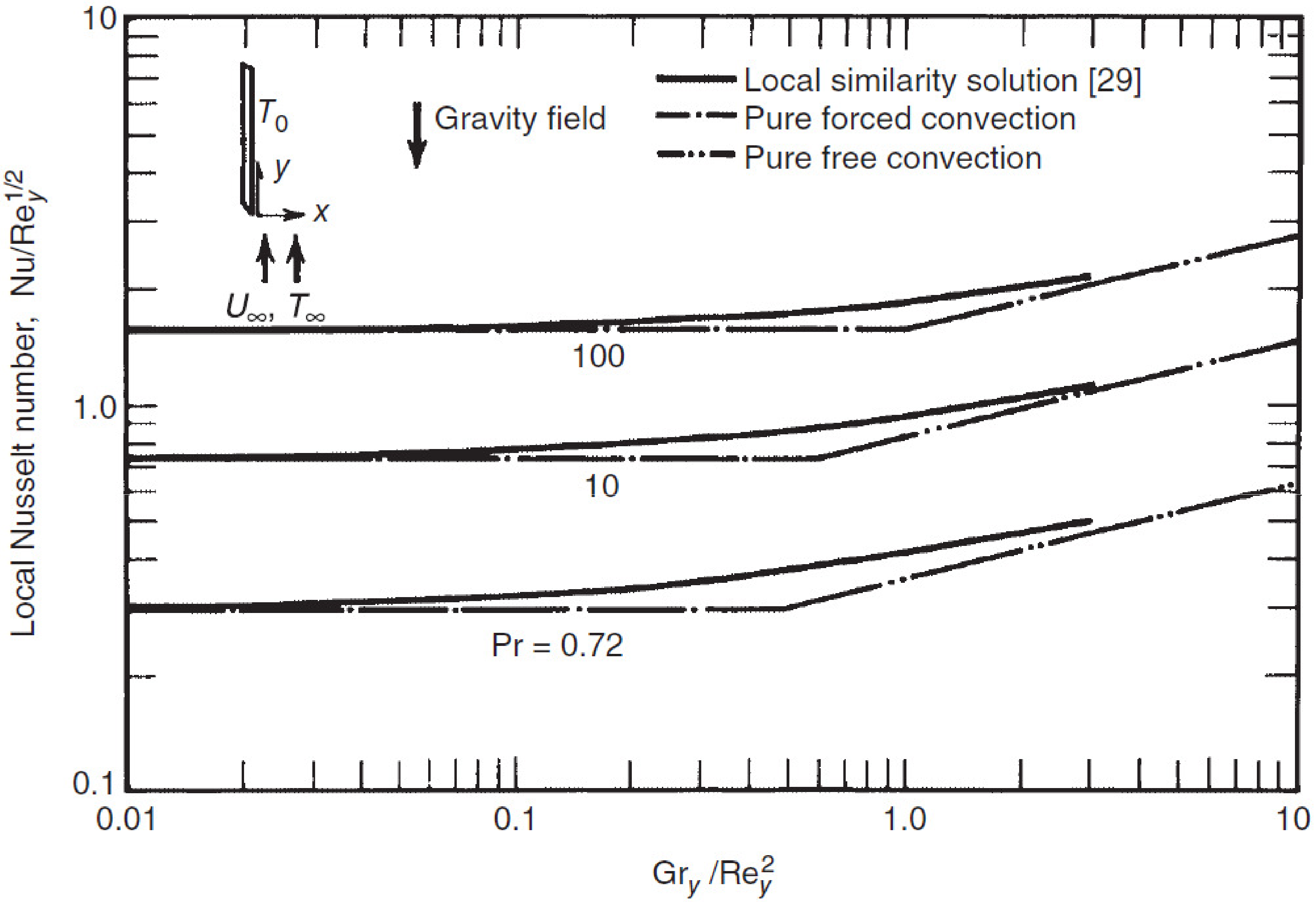
COMBINED NATURAL AND FORCED CONVECTION (MIXED CONVECTION)



Heat transfer by natural and forced convection along a vertical wall. (After Ref. 29.)

the mechanism is natural convection, $(\delta_T)_{NC} \sim y\,Ra_y^{-1/4} \qquad (Pr > 1)$

the mechanism is forced convection, $(\delta_T)_{FC} \sim y\,Re_y^{-1/2} Pr^{-1/3} \qquad (Pr > 1)$

$(\delta_T)_{NC} < (\delta_T)_{FC}$ natural convection

$(\delta_T)_{NC} > (\delta_T)_{FC}$ forced convection

In other words, for $Pr > 1$ fluids,

$$\frac{Ra_y^{1/4}}{Re_y^{1/2}\,Pr^{1/3}} \begin{cases} > O(1) & \text{natural convection} \\ < O(1) & \text{forced convection} \end{cases}$$

در مراجع قبلی گفته شد

Gr_y / Re_y^2,

$$\frac{Gr_y}{Re_y^2} = \left(\frac{Ra_y^{1/4}}{Re_y^{1/2}\,Pr^{1/3}} \right)^4 Pr^{1/3} \neq \frac{Ra_y^{1/4}}{Re_y^{1/2}\,Pr^{1/3}}$$

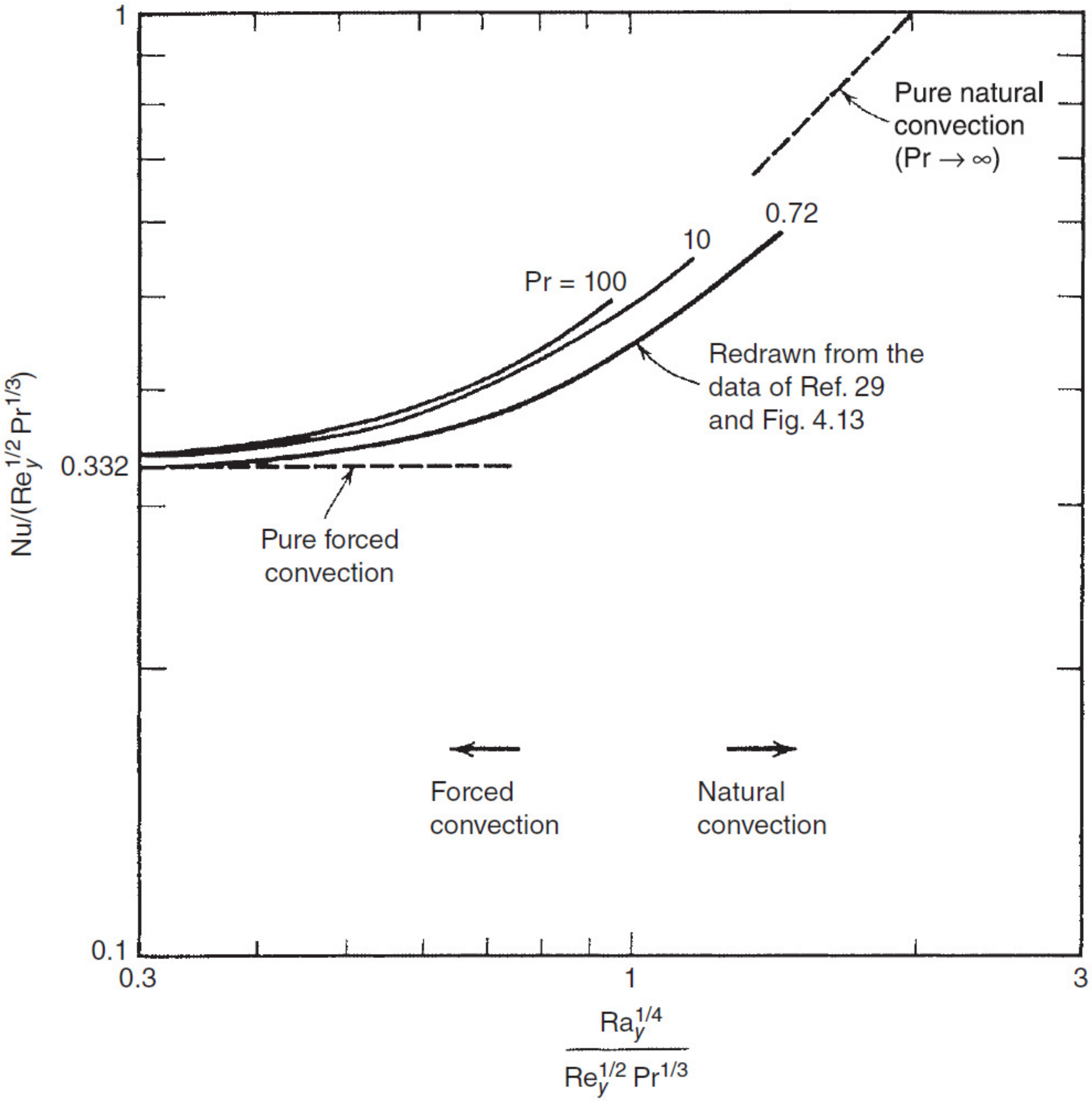


Figure 4.14 Correct transition between natural and forced convection on a vertical wall when $Pr \geq 1$.

for $Pr < 1$ fluids,

$$\frac{Bo_y^{1/4}}{Pe_y^{1/2}} \begin{cases} > O(1) & \text{natural convection} \\ < O(1) & \text{forced convection} \end{cases}$$

Note that

the dimensionless group $Bo_y^{1/4} \; Pe_y^{-1/2}$ is equal to Gr_y/Re_y^2

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HEAT TRANSFER RESULTS INCLUDING THE EFFECT OF TURBULENCE

Vertical Walls

$\mathrm{Ra}_y \sim 10^9$, Bejan and Lage [32] → *Grashof number of order* 10^9

$$\mathrm{Gr}_y \sim 10^9 \qquad (10^{-3} \leq \mathrm{Pr} \leq 10^3)$$

$$\left.\begin{aligned} \mathrm{Ra}_y &= \mathrm{Gr}_y\,\mathrm{Pr}, \\ \mathrm{Gr}_y &\sim 10^9 \end{aligned}\right\} \mathrm{Ra}_y \sim 10^9\,\mathrm{Pr} \qquad (10^{-3} \leq \mathrm{Pr} \leq 10^3)$$

Churchill and Chu [33]

$$\overline{\mathrm{Nu}}_y = \left\{0.825 + \frac{0.387\mathrm{Ra}_y^{1/6}}{[1 + (0.492/\,\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$$

for $10^{-1} < \mathrm{Ra}_y < 10^{12}$ and for all Prandtl numbers

physical properties
film temperature $(T_w + T_\infty)/2$

In the *laminar range*, $\mathrm{Gr}_y < 10^9$

$$\overline{\mathrm{Nu}}_y = 0.68 + \frac{0.67\mathrm{Ra}_y^{1/4}}{[1 + (0.492/\,\mathrm{Pr})^{9/16}]^{4/9}}$$

$$\overline{\mathrm{Nu}}_y = 0.68 + 0.515\mathrm{Ra}_y^{1/4} \qquad (\mathrm{Pr} = 0.72)$$

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$q''_w = \text{constant}$

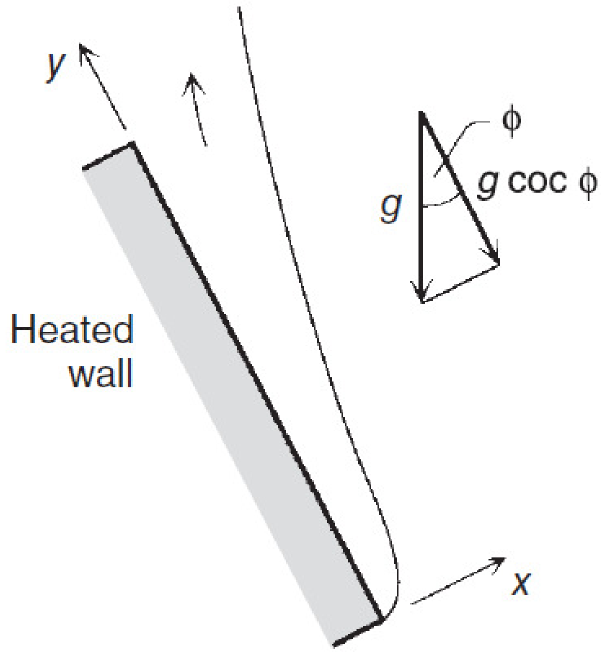
Vliet and Liu [34]

$$\left.\begin{aligned} Nu_y &= 0.6 Ra_{*y}^{1/5} \\ \overline{Nu}_y &= 0.75 Ra_{*y}^{1/5} \end{aligned}\right\} \quad \text{laminar, } 10^5 < Ra_{*y} < 10^{13}$$

$$\left.\begin{aligned} Nu_y &= 0.568 Ra_{*y}^{0.22} \\ \overline{Nu}_y &= 0.645 Ra_{*y}^{0.22} \end{aligned}\right\} \quad \text{turbulent, } 10^{13} < Ra_{*y} < 10^{16}$$

Inclined Walls

The angle between the plane and the vertical direction ϕ is restricted to the range $-60^\circ < \phi < 60^\circ$ ➡ g ➡ $g \cos\phi$



for laminar flow

$$Ra_y = \frac{g \cos\phi \, \beta (T_w - T_\infty) y^3}{\alpha \nu}$$

uniform heat flux

$$Ra_{*y} = \frac{g \cos\phi \, \beta q''_w y^4}{\alpha \nu k}$$

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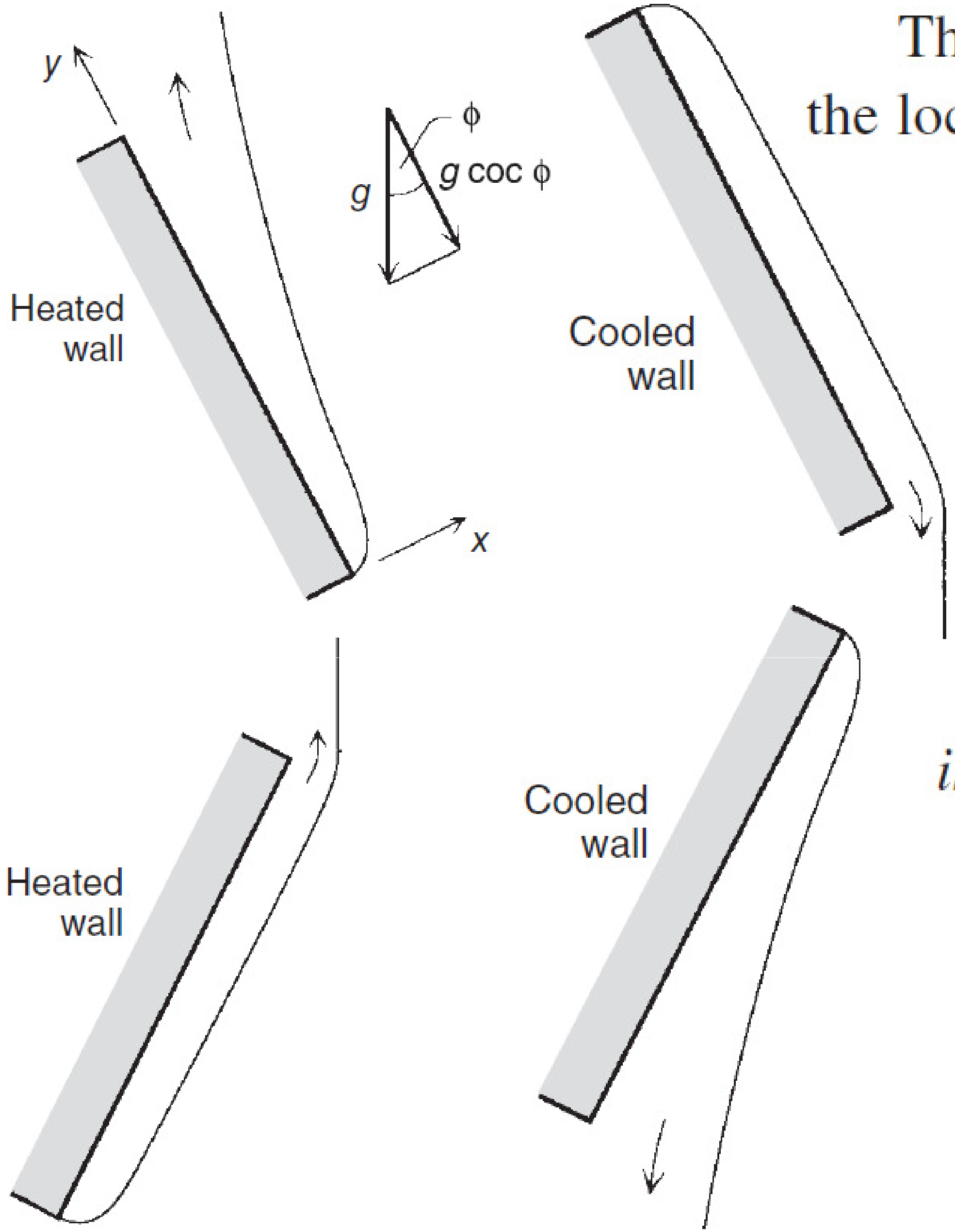


The angle ϕ has a noticeable effect on the location of the laminar–turbulent transition

uniform-flux wall (Pr ≅ 6.5)

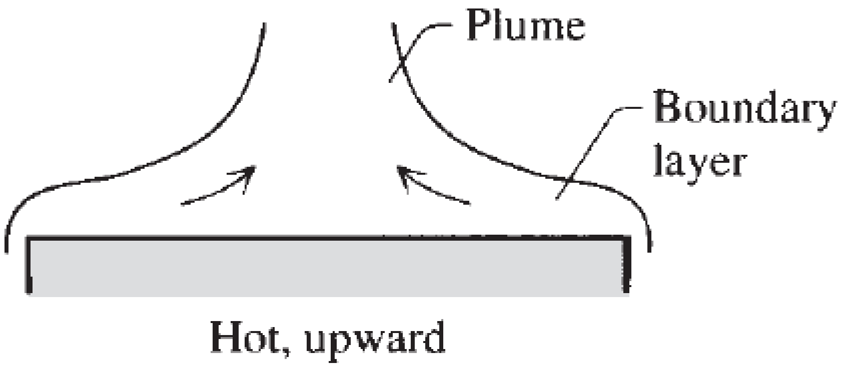
ϕ	Ra_{*y}
0°	5×10^{12}–10^{14}
30°	3×10^{10}–10^{12}
60°	6×10^{7}–6×10^{9}

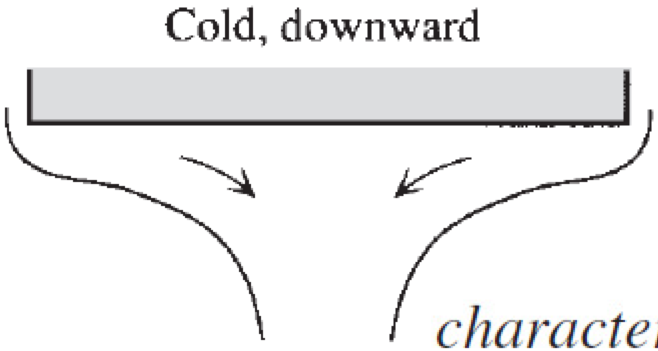
the beginning and the end of the transition region

isothermal wall in water (Pr ∼ 6)

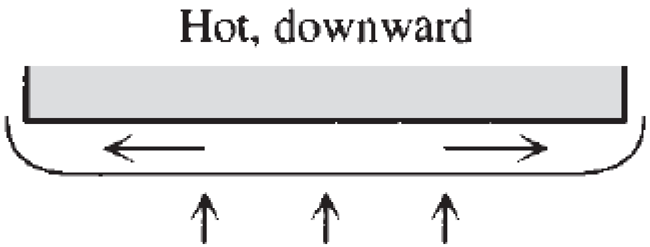
ϕ	Ra_y
0°	8.7×10^{8}
20°	2.5×10^{8}
45°	1.7×10^{7}
60°	7.7×10^{5}

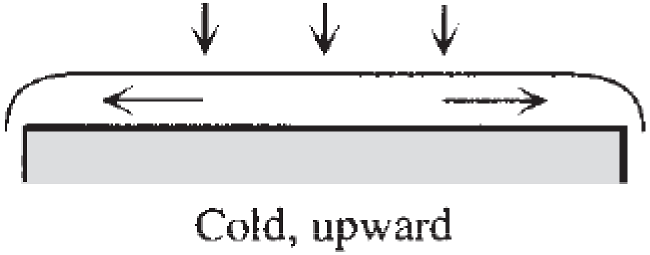
Horizontal Walls

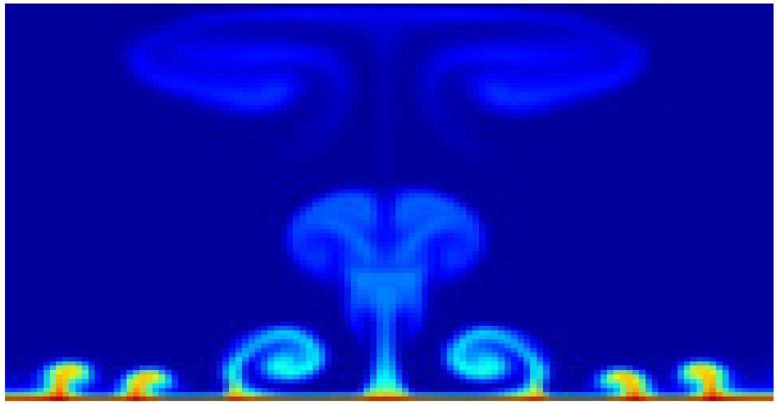


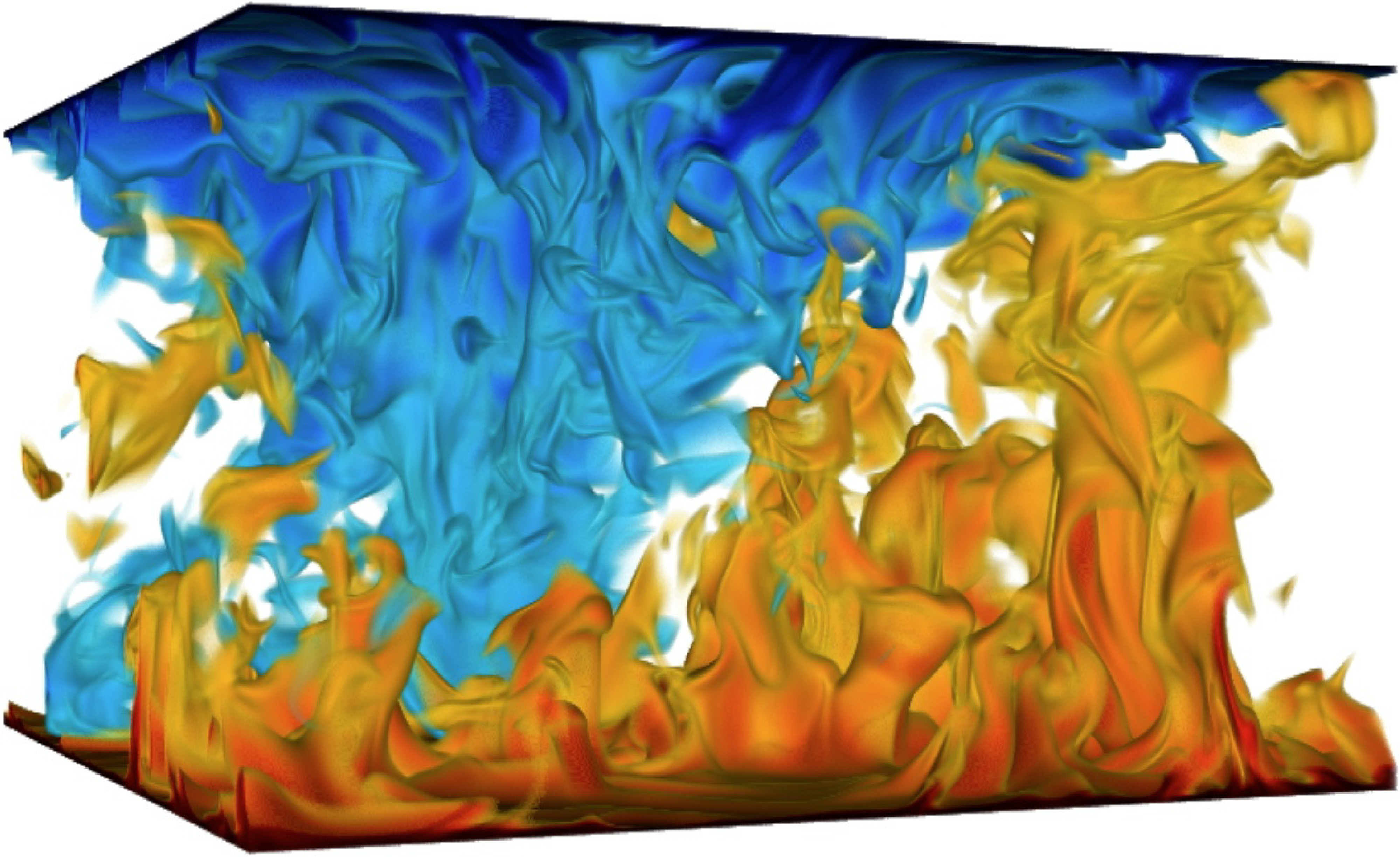


characteristic length $L = \frac{A}{p}$









In the case of hot surfaces facing upward or cold surfaces facing downward

$$\overline{\mathrm{Nu}}_L = \begin{cases} 0.54\mathrm{Ra}_L^{1/4} & (10^4 < \mathrm{Ra}_L < 10^7) \\ 0.15\mathrm{Ra}_L^{1/3} & (10^7 < \mathrm{Ra}_L < 10^9) \end{cases}$$

for isothermal surfaces

The corresponding correlation for hot surfaces facing downward or cold surfaces facing upward

$$\overline{\mathrm{Nu}}_L = 0.27\mathrm{Ra}_L^{1/4} \qquad (10^5 < \mathrm{Ra}_L < 10^{10})$$

for isothermal surfaces

for uniform-flux surfaces

averaged temperature difference between the surface and the surrounding fluid.

The flux Rayleigh number Ra_{*L}

noting that $\mathrm{Ra}_L = \mathrm{Ra}_{*L}/\overline{\mathrm{Nu}}_L$.

Horizontal Cylinder

isothermal cylinder

$$\overline{Nu}_D = \left\{ 0.6 + \frac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}} \right\}^2$$

$$10^{-5} < Ra_D < 10^{12}$$

$$Ra_D = \frac{g\beta \, \Delta T \, D^3}{\alpha \nu}$$

Sphere

$$\overline{Nu}_D = 2 + \frac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$$

$$Pr \gtrsim 0.7 \text{ and } Ra_D < 10^{11}$$

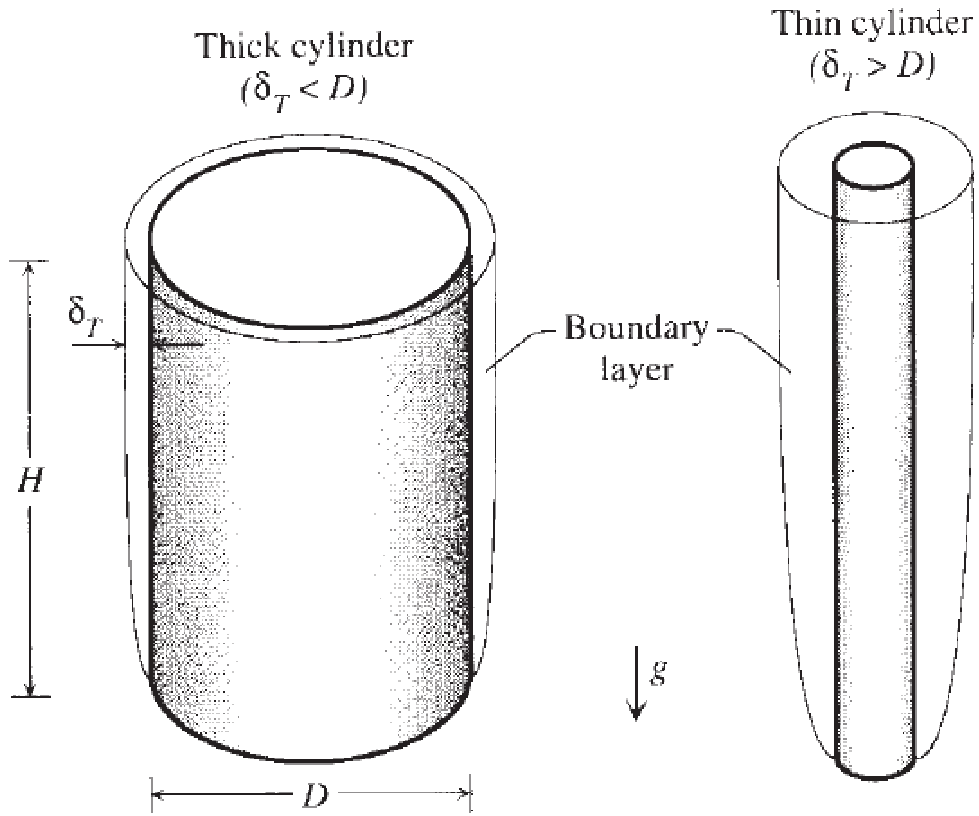
Vertical Cylinder

δ_T is much smaller than the cylinder diameter

the Nusselt number can be calculated with the vertical-wall formulas

if $Pr \gtrsim 1$, the δ_T criterion requires that

$$\frac{D}{H} > Ra_H^{-1/4}$$



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$$\overline{\mathrm{Nu}}_H = \frac{4}{3}\left[\frac{7\mathrm{Ra}_H\ \mathrm{Pr}}{5\,(20+21\,\mathrm{Pr})}\right]^{1/4} + \frac{4(272+315\,\mathrm{Pr})H}{35(64+63\,\mathrm{Pr})D}$$ in the laminar regime

$\mathrm{Ra}_H = (g\beta\ \Delta T\ H^3)/\alpha\nu$,

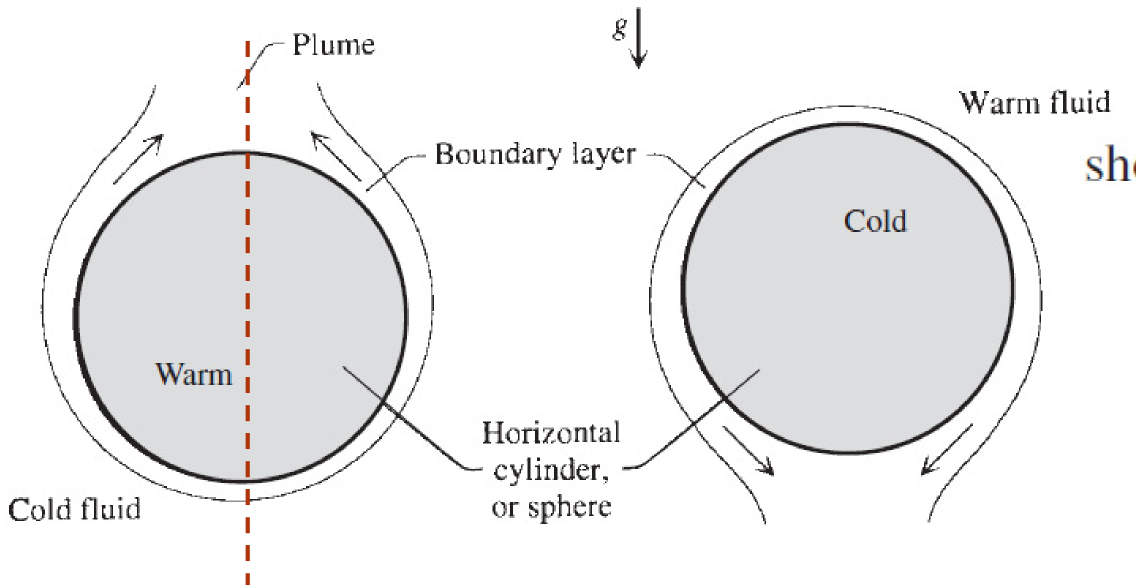
Other Immersed Bodies

The simplest formula $\overline{\mathrm{Nu}}_l = hl/k,\ \mathrm{Ra}_l = (g\beta\ \Delta T\ \ l^3)/\alpha\nu$

Lienhard [44] $\overline{\mathrm{Nu}}_l \cong 0.52\mathrm{Ra}_l^{1/4}$

طول l در این رابطه فاصله ای است که لایه مرزی در تماس با سطح جسم می باشد.

برای مثال در سیلندر افقی $l = \pi D/2$.



should be accurate within 10 percent $\mathrm{Pr} \gtrsim 0.7$

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Yovanovich [45] $0 < Ra_{\mathfrak{L}} < 10^8$ $\mathfrak{L} = A^{1/2}$ → $Ra_{\mathfrak{L}} = (g\beta \, \Delta T \, \mathfrak{L}^3)/\alpha\nu$

$$\overline{Nu}_{\mathfrak{L}} = \overline{Nu}_{\mathfrak{L}}^0 + \frac{0.67 G_{\mathfrak{L}} \, Ra_{\mathfrak{L}}^{1/4}}{[1 + (0.492/Pr)^{9/16}]^{4/9}}$$

pure conduction $Ra_{\mathfrak{L}} \to 0$.

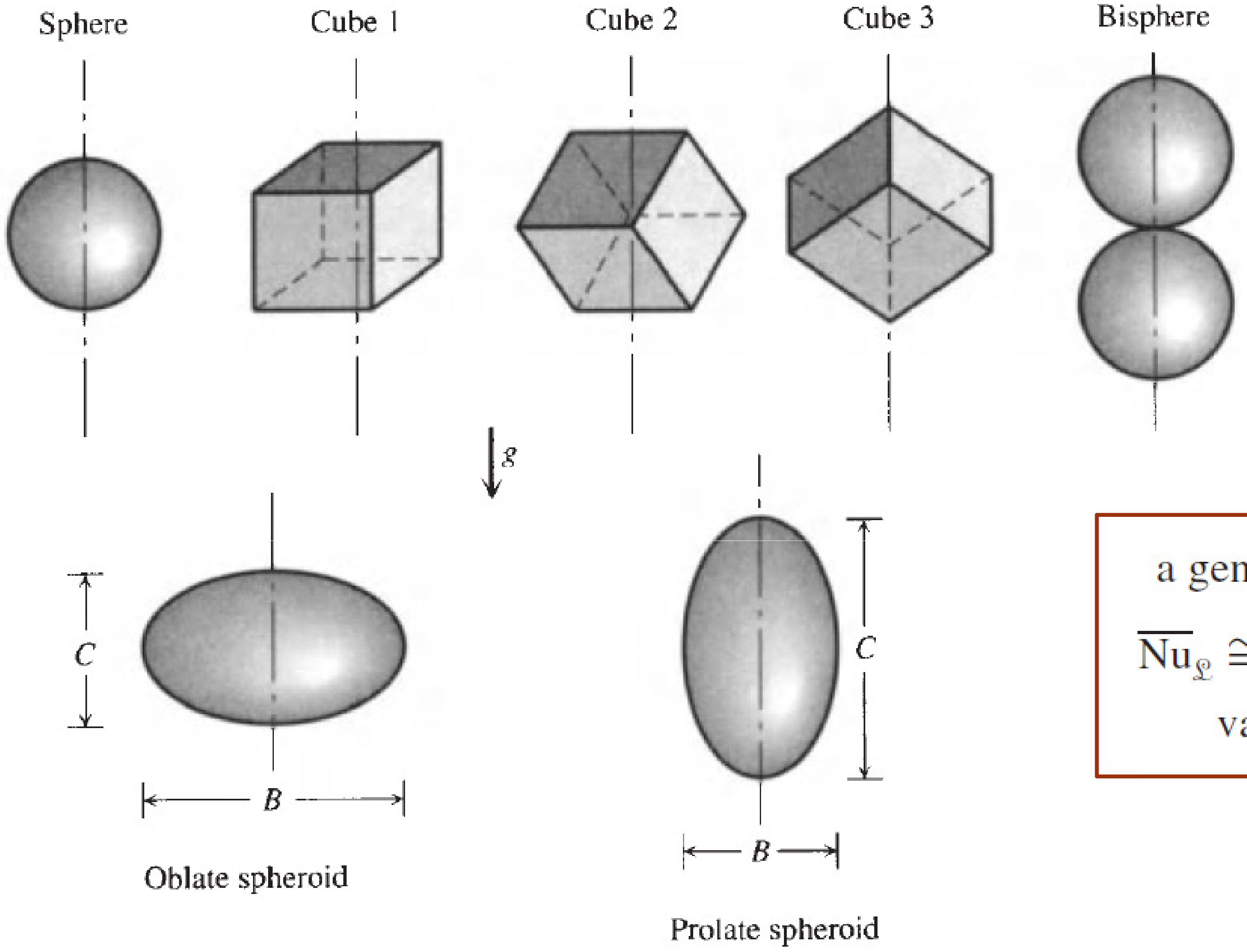
the conduction-limit Nusselt number $\overline{Nu}_{\mathfrak{L}}^0$

the average values of $\overline{Nu}_{\mathfrak{L}}^0 \cong 3.47$

Table 4.3 Constants for Yovanovich's correlation [45] for laminar natural convection heat transfer from immersed bodies (Fig. 4.20)

Body Shape	$\overline{Nu}_{\mathfrak{L}}^0$	$G_{\mathfrak{L}}$
Sphere	3.545	1.023
Bisphere	3.475	0.928
Cube 1	3.388	0.951
Cube 2	3.388	0.990
Cube 3	3.388	1.014
Vertical cylinder [a]	3.444	0.967
Horizontal cylinder [a]	3.444	1.019
Cylinder [a] at 45°	3.444	1.004
Prolate spheroid ($C/B = 1.93$)	3.566	1.012
Oblate spheroid ($C/B = 0.5$)	3.529	0.973
Oblate spheroid ($C/B = 0.1$)	3.342	0.768

[a] Short cylinder, $H = D$.



a general expression

$$\overline{\mathrm{Nu}}_{\mathfrak{L}} \cong 3.47 + 0.51\mathrm{Ra}_{\mathfrak{L}}^{1/4}$$

valid for $\mathrm{Pr} \gtrsim 0.7$

Figure 4.20 Shapes and orientations of bodies immersed in a fluid (Table 4.3).