به نام خدا

مدل سازی ماشین سنکرون

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فهرست

مقدمه

تحلیل پایداری سیستم ها

SISO

روش روث

نایکوئیست

مکان هندسی ریشه ها

MIMO

فضای حالت

مقدمه: تحلیل پایداری سیستم ها با استفاده از روش فضای حالت

هدف: استخراج مقادیر ویژه ازمعادلات دیفرانسیل غیر خطی متغیر با زمان در P.S

کاربرد

۱.تحلیل مقادیر ویژه ← شرط پایداری: مقادیر ویژه سمت چپ محور $j\omega$

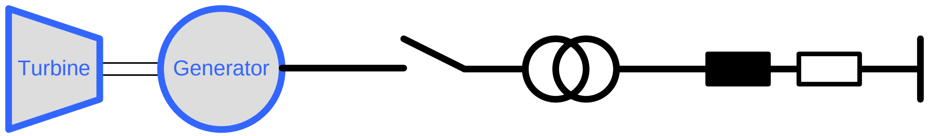
۲. حساسیت مقادیر ویژه به پارامترها

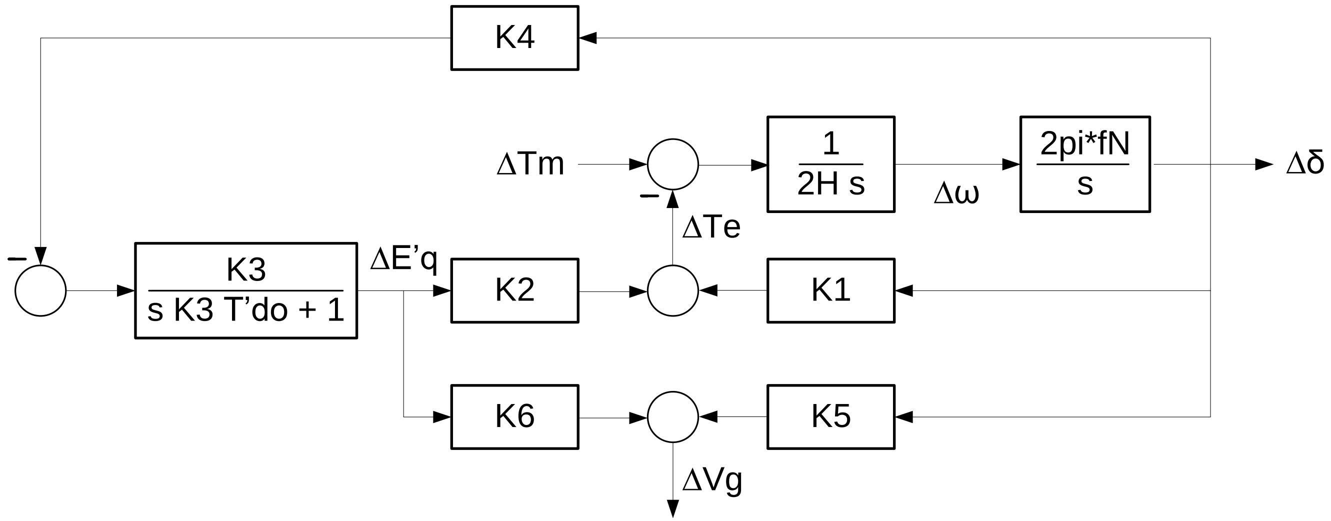
قدم اول:

مدلسازی المان های سیستم قدرت

۱. ماشین سنکرون
۲. سیستم تحریک
۳. توربین و گاورنر
۴. بار
۵. و ...

مقدمه: مدل تک ماشین متصل به شین بینهایت





A single generator connected to a large grid can be represented by the Phillips-Heffron model (assuming constant field voltage and mechanical torque)

مقدمه

مدلسازی ماشین سنکرون

۱. استخراج روابط شاردور و ولتاژ

۲. مستقل از زمان کردن ← تبدیل پارک

۳. خطی سازی ← خطی سازی حول نقطه کار

۴. پریونیت کردن ← انتخاب مبنای مناسب

۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

۱.۱ تعریف ماشین سنکرون ایده آل

- **Note:**

*** S. M. is a *rotating* magnetic element *with complex dynamic behavior*. It is the *heart* of P. S. It**

*** It provides *active and reactive power* to loads and has strong *power, frequency and voltage regulation/control capability* .**

*** To study S. M., mathematic models are developed for S. M.**

*** Special assumptions are made to simplify the modeling.**

۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

۱.۱ تعریف ماشین سنکرون ایده آل (ادامه)

- ۱. فرضیات برای ماشین سنکرون ایده آل

- **Machine magnetic permeability (μ) is a constant with magnetic saturation neglected. Eddy current, hysteresis, and skin effects are neglected, so *the machine is linear.***
- ***Symmetric rotor structure* in direct (d) and quadratic (q) axes.**
- ***Symmetric stator winding structure:* the three stator windings are 120 (electric) degrees apart in space with *same structure*.**
- **The stator and rotor have *smooth surface* with tooth and slot effects neglected. All windings generate *sinusoidal distributed magnetic field*.**

۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

۲.۱ معادلات ولتاژ در چارچوب abc

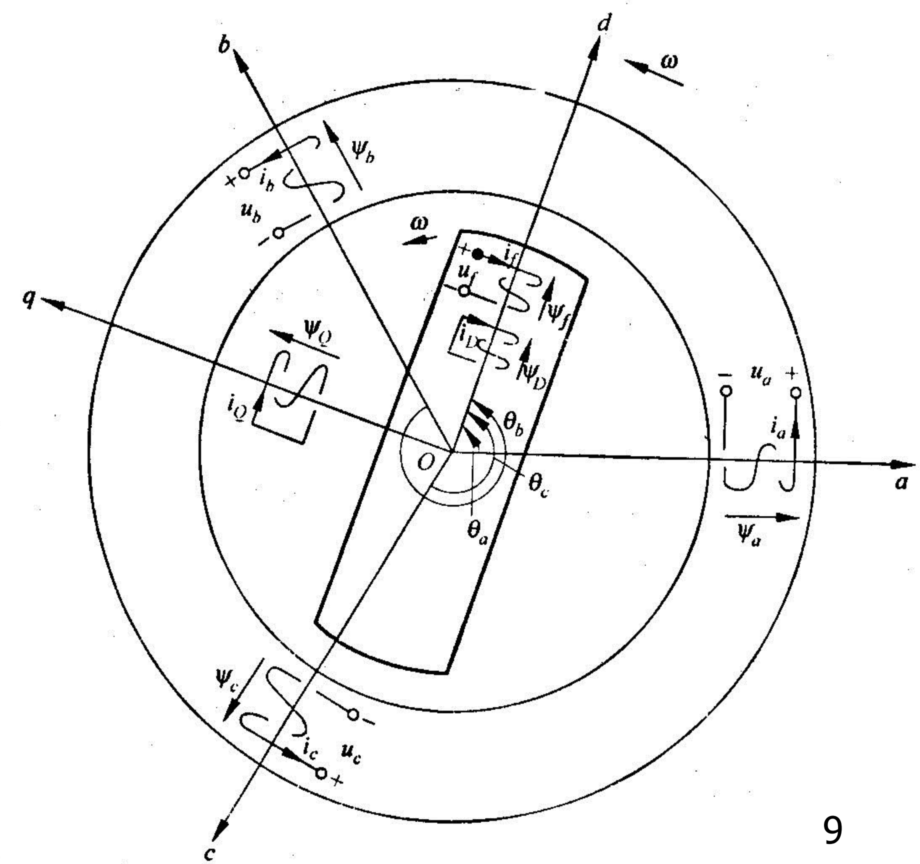
- **Positive direction setting:**
  - **dq and abc axes, speed direction**
  - **Angle definition:**

$$\theta_a = \theta : (d \text{ leading ahead } a)$$

$$\theta_b = \theta_a - 120°, \theta_a + 240°$$

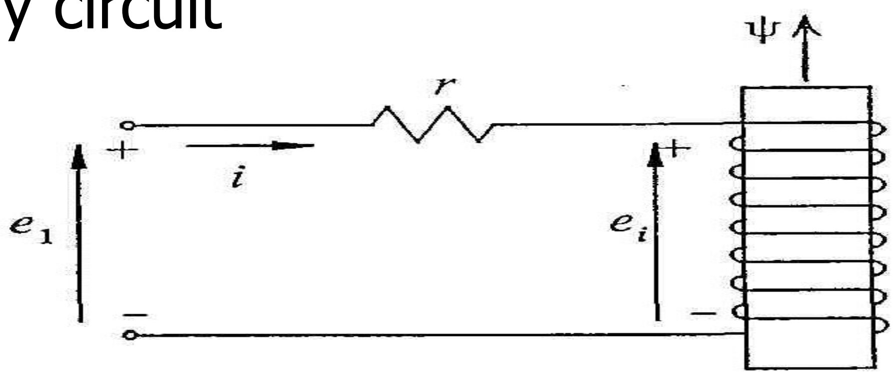
$$\theta_c = \theta_a - 240°, \theta_a + 120°$$

  - ***Ψ* directions for *abcfDQ* windings**
  - ***i* directions for *abcfDQ***
  - ***u* directions for *abcfDQ* ($u_D = u_Q = 0$)**



۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

Consider the elementary circuit



$$\mathbf{e_i = \frac{d\Psi}{dt}}$$

$$\mathbf{e_1 = \frac{d\Psi}{dt} + ri}$$

$$\mathbf{\Psi = Li}$$

۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

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۲.۱ معادلات ولتاژ در چارچوب abc (ادامه)

– معادلات ولتاژ سیم پیچی های سه فاز abc

$$\boxed{\begin{cases} u_a = p\Psi_a - r_a i_a \\ u_b = p\Psi_b - r_b i_b \\ u_c = p\Psi_c - r_c i_c \end{cases}}$$

where p= d / dt, t in sec.

r_{abc}: stator winding <u>resistance</u>, in Ω.

i_{abc} : stator winding <u>current</u>, in A.

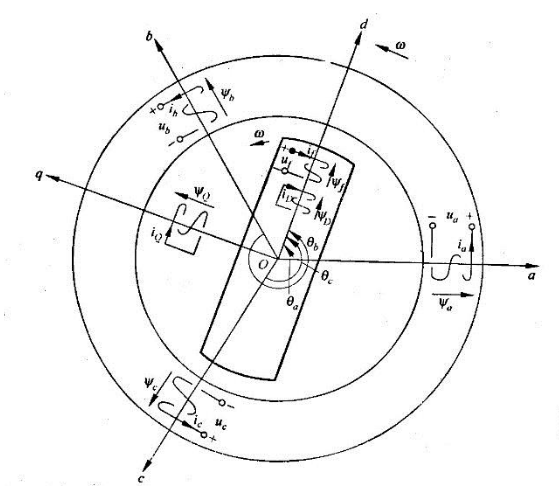
u_{abc}: stator winding <u>phase voltage</u>, in V.

ψ_{abc}: stator winding <u>flux linkage</u>, in Wb.

Note: * $p\psi_{abc}$: generate emf in abc windings

*** $\underline{u}_{abc}$~$\underline{i}_{abc}$: in <u>generator</u> conventional direction.**

*** $\underline{i}_{abc}$~ $\underline{\psi}_{abc}$: <u>positive</u> $\underline{i}_{abc}$ generates <u>negative</u> $\underline{\psi}_{abc}$ respectively**



۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

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۲.۱ معادلات ولتاژ در چارچوب abc (ادامه)

– معادلات ولتاژ سیم پیچی های fDQ

$$\begin{cases} u_f = p\Psi_f + r_f i_f \\ u_D = p\Psi_D + r_D i_D \equiv 0 \\ u_Q = p\Psi_Q + r_Q i_Q \equiv 0 \end{cases}$$

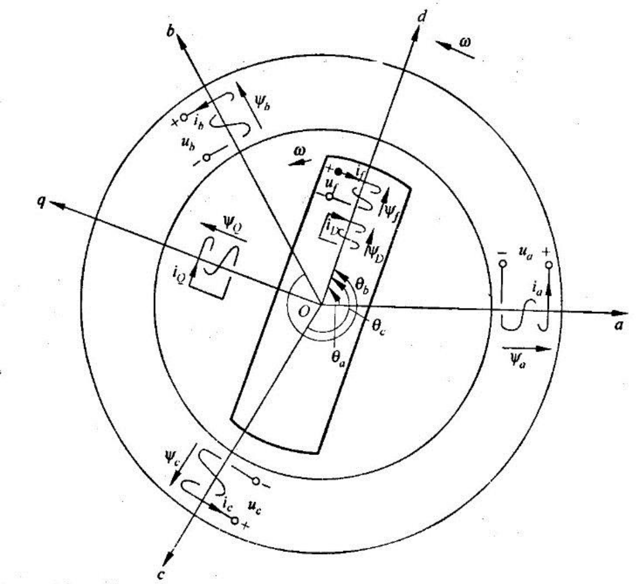
r_{fDQ}: rotor winding resistance, in Ω.

f: field winding,

D: damping winding in d-axis,

Q: damping winding in q-axis.

i_{fDQ}, u_{fDQ}, ψ_{fDQ}: rotor winding currents, voltages and flux linkages in A, V, Wb.



Note: * u_D=u_Q=0

*** u_{fDQ}~i_{fDQ}: in load convention**

*** i_{fDQ} ~ψ_{fDQ}: positive i_{fDQ} generates positive ψ_{fDQ} respectively**

*** q-axis *leads* d-axis by 90 (electr.) deg.**

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۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

۲.۱ معادلات ولتاژ در چارچوب abc (ادامه)

– ماتریس معادلات ولتاژ :

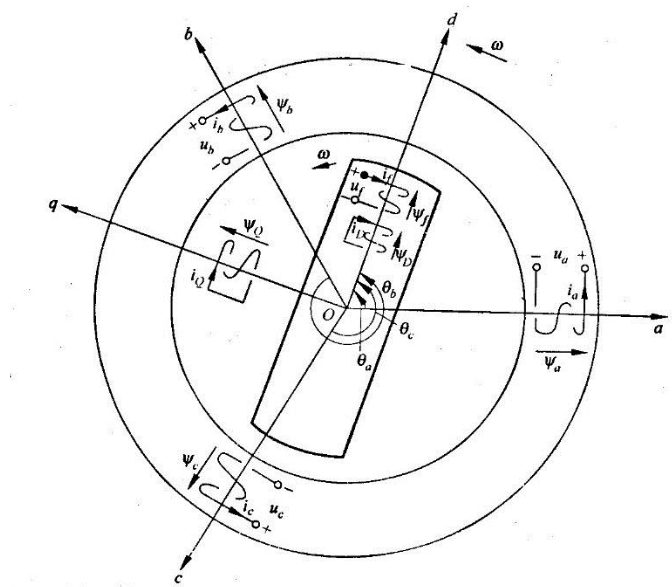
$$\boldsymbol{u} = p\boldsymbol{\Psi} + \boldsymbol{ri}$$

$$\boldsymbol{u} = (u_a, u_b, u_c, u_f, u_D, u_Q)^{\mathrm{T}}$$

$$\boldsymbol{\Psi} = (\Psi_a, \Psi_b, \Psi_c, \Psi_f, \Psi_D, \Psi_Q)^{\mathrm{T}}$$

$$\boldsymbol{r} = \mathrm{diag}(r_a, r_b, r_c, r_f, r_D, r_Q)^{\mathrm{T}}$$

$$\mathbf{i} = (-i_a, -i_b, -i_c, i_f, i_D, i_Q)^{\mathrm{T}}$$

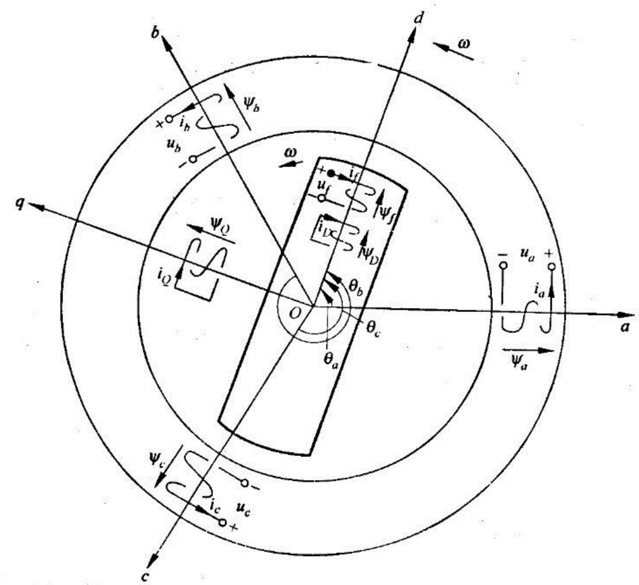


where '–' before i_{abc} is caused by generator convention of stator windings.

۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

۳.۱ معادلات شار دور/درگیر شونده (Flux linkage) در چارچوب abc

$$\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \\ \cdots \\ \Psi_f \\ \Psi_D \\ \Psi_Q \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & \vdots & L_{af} & L_{aD} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & \vdots & L_{bf} & L_{bD} & L_{bQ} \\ L_{ca} & L_{cb} & L_{cc} & \vdots & L_{cf} & L_{cD} & L_{cQ} \\ \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots \\ L_{fa} & L_{fb} & L_{fc} & \vdots & L_{ff} & L_{fD} & L_{fQ} \\ L_{Da} & L_{Db} & L_{Dc} & \vdots & L_{Df} & L_{DD} & L_{DQ} \\ L_{Qa} & L_{Qb} & L_{Qc} & \vdots & L_{Qf} & L_{QD} & L_{QQ} \end{bmatrix} \begin{bmatrix} -i_a \\ -i_b \\ -i_c \\ \cdots \\ i_f \\ i_D \\ i_Q \end{bmatrix}$$



$$\text{or} \begin{bmatrix} \psi_{abc} \\ \psi_{fDQ} \end{bmatrix} = \begin{bmatrix} L_{11(3\times3)} & L_{12(3\times3)} \\ L_{21(3\times3)} & L_{22(3\times3)} \end{bmatrix} \begin{bmatrix} -i_{abc} \\ i_{fDQ} \end{bmatrix}; \quad \psi_{(6\times1)} = L_{(6\times6)} i_{(6\times1)}$$

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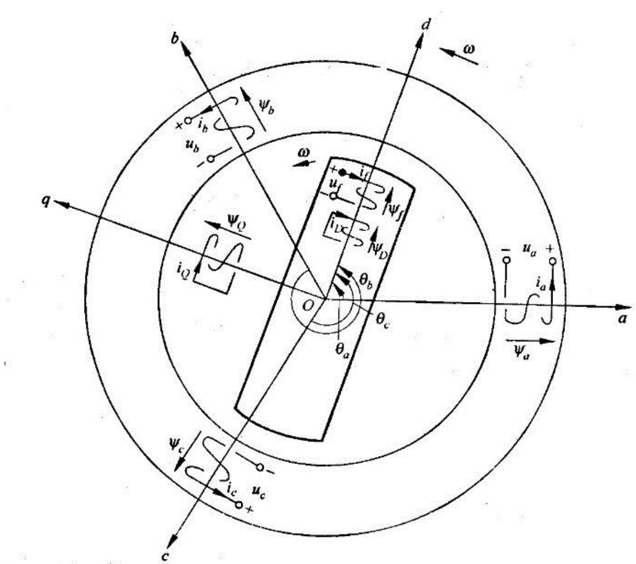
۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

۳.۱ معادلات شار دور (درگیر شونده) در چارچوب abc (ادامه)



- **In Flux linkage eqn.:**

$L_{\underline{ij}}$(i, j = a, b, c, f, D, Q): self and mutual inductances,

$L_{\underline{11}}$: stator winding self and mutual inductance , $L_{\underline{22}}$: rotor winding self and mutual inductances,

$L_{\underline{12}}$, $L_{\underline{21}}$: mutual inductances among stator and rotor windings ,

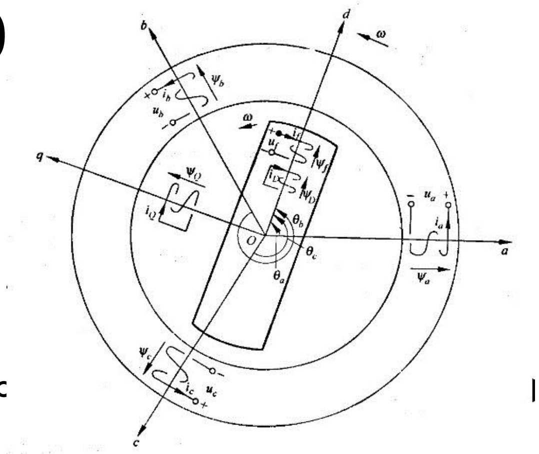
ψ, i : same definition as voltage eqn..

Note: * Positive $\underline{i}_{abc}$ generates negative $\underline{\psi}_{abc}$ respectively.

*** The negative signs of $\underline{i}_{abc}$ make L_{aa}, L_{bb}, L_{cc}>0**

۳.۱ معادلات شار دور (درگیر شونده) در چارچوب abc (ادامه)

- **Stator winding self/mutual inductance (L_{11})**
  - **Stator winding self inductance (L_{aa}, L_{bb}, L_{cc})**

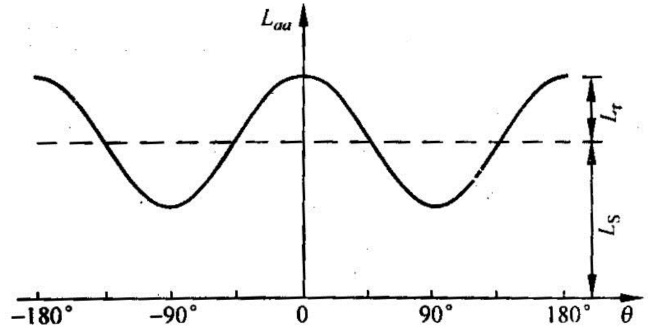


$$L_{aa} = \frac{\Psi_a}{-i_a} > 0 \quad (i_b, i_c, i_f, i_D, i_Q = 0)$$

L_{aa}: reach max @ d-a aligning (when θ_a=0, 180°)
reach min @ d-a perpendicular (when θ_a=90, 270°)
L_{aa}~ θ_a: 'sin'-curve, with period of 180°

$$L_{aa} = L_S + L_t \cos 2\theta_a = L_S + L_t \cos 2\theta$$
$$L_{bb} = L_S + L_t \cos 2\theta_b = L_S + L_t \cos 2(\theta - 120°)$$
$$L_{cc} = L_S + L_t \cos 2\theta_c = L_S + L_t \cos 2(\theta + 120°)$$

(Ls>Lt>0, for round rotor: Lt=0)



۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

۳.۱ معادلات شار دور (درگیر شونده) در چارچوب abc (ادامه)

- **Stator winding self/mutual inductance (L_{11})**
  - **Stator winding mutual inductance**

$$L_{ab} = \frac{\psi_a}{-i_b} < 0 \; (i_{a,c,\,fDQ} = 0); \; L_{ba} = \frac{\psi_b}{-i_a} = L_{ab} < 0 \; (i_{b,c,\,fDQ} = 0)$$

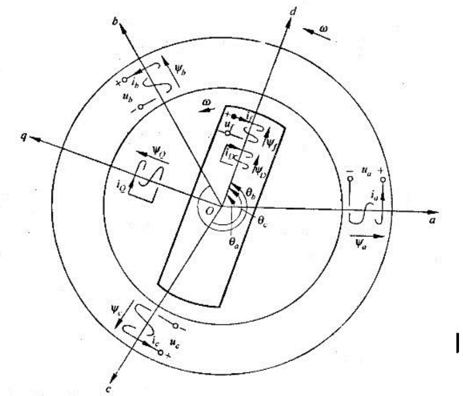
L_{ab}: reach max |.| when θ_a= -30, 150°

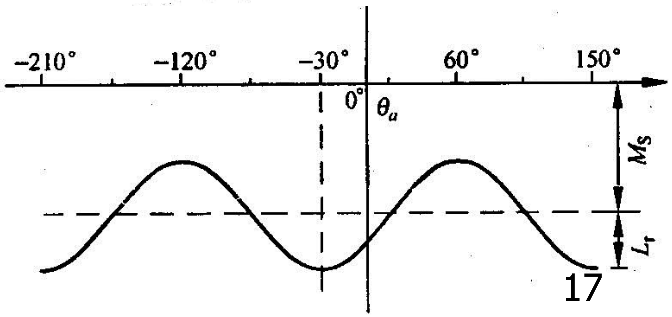
reach min |.| when θ_a= 60, 240°

L_{aa}~ θ_a: ‘sin’-curve, with period of 180°

$$L_{ab} = L_{ba} = -M_s - L_t \cos 2(\theta_a + 30°)$$
$$= -(M_s + L_t \cos 2(\theta + 30°))$$
$$L_{bc} = L_{cb} = -\,(M_s + L_t \cos 2(\theta - 90°))$$
$$L_{ca} = L_{ac} = -(M_s + L_t \cos 2(\theta - 150°))$$

(Ms>Lt>0, for round rotor: Lt=0)





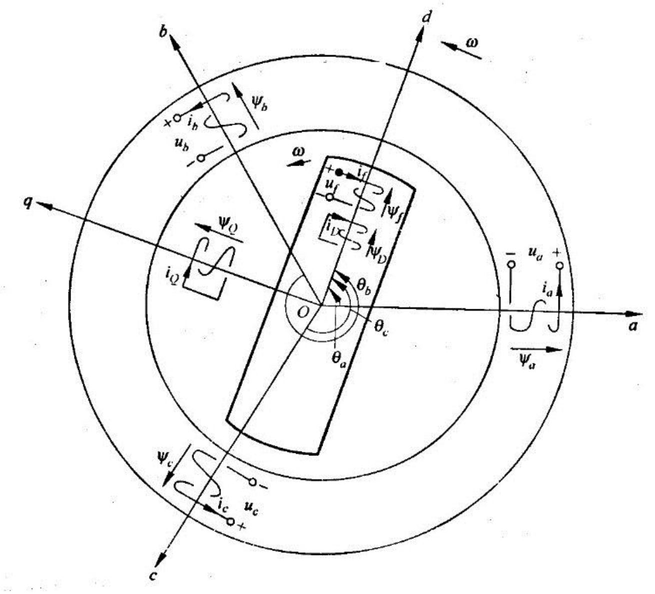
۳.۱ معادلات شار دور (درگیر شونده) در چارچوب abc (ادامه)

- **Rotor winding self/mutual inductance (L_{22})**
  - **Rotor winding self inductance (constant: why?)**

$$L_{ff} = L_f = const. > 0$$
$$L_{DD} = L_D = const. > 0$$
$$L_{QQ} = L_Q = const. > 0$$

  - **Rotor winding mutual inductance**

$$L_{fQ} = L_{Qf} = 0\,, \quad L_{DQ} = L_{QD} = 0$$
$$L_{fD} = L_{Df} = M_R = const. > 0$$



۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

۳.۱ معادلات شار دور (درگیر شونده) در چارچوب abc (ادامه)

- **Stator and rotor winding mutual inductance** (**L_{12}; L_{21}**)
  - ***abc~f:*** **(M_f=const.>0, period: 360°, max. when *d-abc* align)**

$$L_{af} = L_{fa} = M_f \cos\theta_a = M_f \cos\theta$$

$$L_{bf} = L_{fb} = M_f \cos(\theta - 120°)$$

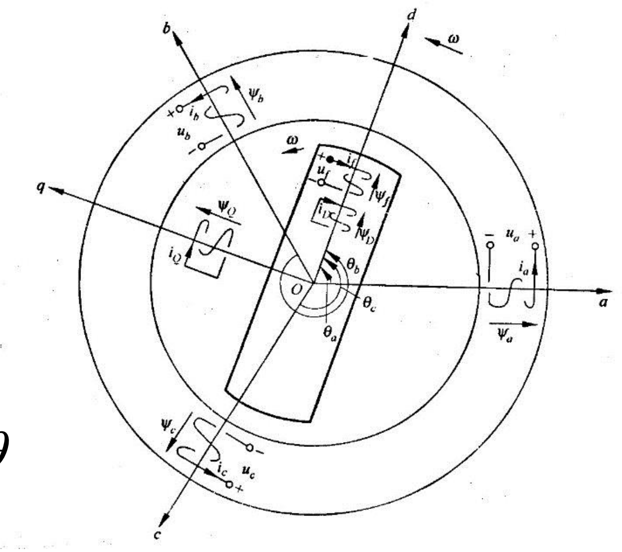
$$L_{cf} = L_{fc} = M_f \cos(\theta + 120°)$$

  - **abc~D: similar to abc~f, $M_f \rightarrow M_D > 0$**
  - **abc~Q:(M_Q=const.>0, period: 360°,**
  - **max. when *q-abc* align)**

$$L_{aQ} = L_{Qa} = M_Q \cos(\theta_a + 90°) = -M_Q \sin\theta$$

$$L_{bQ} = L_{Qb} = -M_Q \sin(\theta - 120°)$$

$$L_{cQ} = L_{Qc} = -M_Q \sin(\theta + 120°)$$



۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

۳.۱ معادلات شار دور (درگیر شونده) در چارچوب abc (ادامه)

- **Time varying *L*-matrix : related to rotor position**
- **L_{11} (abc~abc): 180° period; L_{12}, L_{21}(abc~fDQ): 360° period.**
- **Non-sparse *L*-matrix: most mutual inductances ≠ 0**
- ***L*-matrix: non-user friendly, lead to abc → dq0 coordinates!**

$$\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \\ \cdots \\ \Psi_f \\ \Psi_D \\ \Psi_Q \end{bmatrix} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} & \vdots & L_{af} & L_{aD} & L_{aQ} \\ L_{ba} & L_{bb} & L_{bc} & \vdots & L_{bf} & L_{bD} & L_{bQ} \\ L_{ca} & L_{cb} & L_{cc} & \vdots & L_{cf} & L_{cD} & L_{cQ} \\ \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots \\ L_{fa} & L_{fb} & L_{fc} & \vdots & L_f & M_R & 0 \\ L_{Da} & L_{Db} & L_{Dc} & \vdots & M_R & L_D & 0 \\ L_{Qa} & L_{Qb} & L_{Qc} & \vdots & 0 & 0 & L_Q \end{bmatrix} \begin{bmatrix} -i_a \\ -i_b \\ -i_c \\ \cdots \\ i_f \\ i_D \\ i_Q \end{bmatrix}$$

۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

۴.۱ خلاصه مدل ماشین سنکرون در چارچوب abc و واحدهای SI

- **6 volt. Differential Equations(DEs). (abcfDQ):** $u = p\psi + ri$
- **6 flux linkage Algebraic Equations(AEs). (abcfDQ):** $\psi = Li$
- **2 rotor motion eqns. (ω, θ):** $\frac{1}{p_P} J \frac{d\omega}{dt} = T_m - T_e; \quad \frac{d\theta}{dt} = \omega$

$$T_e = p_P \frac{1}{\sqrt{3}}[\psi_a(i_b - i_c) + \psi_b(i_c - i_a) + \psi_c(i_a - i_b)]$$

- **Totally 14 eqns. with 8 DEs and 6 AEs.**
  - **8^{th} order nonlinear model.**
  - **8 state variables are: ψ (6×1) and ω, θ (related to 8 DEs)**
- **Totally 19 variables: *u*: 4 (v_D=v_Q=0), *i*: 6, ψ: 6, plus (T_m, ω, θ).**
- **If 5 variables are known, remaining 14 variables can be solved.**
- **Usually u_f and T_m are known (as input signals), 3 network interface eqns. (3 v_{abc}-i_{abc} relations from network) are known.**

۱. ماشین سنکرون ایده آل و مدل آن در چارچوب abc

۴.۱ خلاصه مدل ماشین سنکرون در چارچوب abc (ادامه)

- **Request of transformation of S. M. model:**
  - **abc to dq0 coordinates: Park's transformation, Park's eqns.**
  - **per unit system and S. M. pu model**
  - **Reduced-order practical models:**

    -- Neglect stator abc winding transients (8th order$\rightarrow$ 5th order).

    -- Introduce practical variables (E'_{dq}, E''_{dq}, E_f etc.)

۲. تبدیل park (چارچوب abc به dq0)

- **Definition of Park's transformation:**

$$\boldsymbol{f}_{dq0} = \begin{bmatrix} f_d \\ f_q \\ f_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta_a & \cos\theta_b & \cos\theta_c \\ -\sin\theta_a & -\sin\theta_b & -\sin\theta_c \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} \overset{\Delta}{=} \boldsymbol{D} \cdot \boldsymbol{f}_{abc}$$

$$\boldsymbol{f}_{abc} = \boldsymbol{D}^{-1} \boldsymbol{f}_{dq0}$$

Here, $\boldsymbol{D}^{-1} = \begin{bmatrix} \cos\theta_a & -\sin\theta_a & 1 \\ \cos\theta_b & -\sin\theta_b & 1 \\ \cos\theta_c & -\sin\theta_c & 1 \end{bmatrix}$ **,** $\begin{cases} \theta_a = \theta \\ \theta_b = \theta - 120^\circ \\ \theta_c = \theta + 120^\circ \end{cases}$

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۳. مدل ماشین سنکرون در چارچوب dq0

۱.۳ معادلات ولتاژ در چارچوب dq0

Voltage equation in dq0 coord.

- **Rewrite volt. eqn as:**

$$\begin{bmatrix} \boldsymbol{u}_{abc} \\ \boldsymbol{u}_{fDQ} \end{bmatrix} = \begin{bmatrix} p\boldsymbol{\Psi}_{abc} \\ p\boldsymbol{\Psi}_{fDQ} \end{bmatrix} + \begin{bmatrix} \boldsymbol{r}_{abc} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{r}_{fDQ} \end{bmatrix} \begin{bmatrix} -\boldsymbol{i}_{abc} \\ \boldsymbol{i}_{fDQ} \end{bmatrix}$$

Where: $f_{abc} = (f_a, f_b, f_c)^{\mathrm{T}}$ **can be** u, Ψ **or** i

f_{fDQ} **is similar to** f_{abc}

$$\boldsymbol{r}_{abc} = diag(r_a, r_b, r_c), \boldsymbol{r}_{fDQ} = diag(r_f, r_D, r_Q)$$

- **Define a linear transformation matrix:**

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{D}_{(3\times3)} & \boldsymbol{0}_{(3\times3)} \\ \boldsymbol{0}_{(3\times3)} & \boldsymbol{I}_{(3\times3)} \end{bmatrix}_{(6\times6)}$$

Where D is Park's transformation matrix

I is a unity matrix

- **Multiply T to both-sides of volt. eqn., we have:**

$$\text{l.h.s.} = \begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{u}_{abc} \\ \boldsymbol{u}_{fDQ} \end{bmatrix} = \begin{bmatrix} \boldsymbol{u}_{dq0} \\ \boldsymbol{u}_{fDQ} \end{bmatrix}$$

$$\text{r.h.s.} = \begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \cdot \begin{bmatrix} p\boldsymbol{\Psi}_{abc} \\ p\boldsymbol{\Psi}_{fDQ} \end{bmatrix} + \begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \cdots$$

$$\cdot \begin{bmatrix} \boldsymbol{r}_{abc} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{r}_{fDQ} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{D}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I}^{-1} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \cdot \begin{bmatrix} -\boldsymbol{i}_{abc} \\ \boldsymbol{i}_{fDQ} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \boldsymbol{u}_{dq0} \\ \boldsymbol{u}_{fDQ} \end{bmatrix} = \begin{bmatrix} \boldsymbol{D}p\boldsymbol{\Psi}_{abc} \\ p\boldsymbol{\Psi}_{fDQ} \end{bmatrix} + \begin{bmatrix} \boldsymbol{r}_{dq0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{r}_{fDQ} \end{bmatrix} \begin{bmatrix} -\boldsymbol{i}_{dq0} \\ \boldsymbol{i}_{fDQ} \end{bmatrix}$$

where:

$$\mathbf{r}_{dq0} = \mathbf{r}_{abc} = \mathrm{diag}(r_a, r_b, r_c)$$

$$Dp\boldsymbol{\Psi}_{abc} \Rightarrow$$ ***<u>discussed below</u>*.**

۱.۳ معادلات ولتاژ در چارچوب dq0 (ادامه)

Voltage equation in dq0 coord. (cont.)

- **Derive the expression for the term "$Dp\Psi_{abc}$"**

$$\boldsymbol{D}p\boldsymbol{\Psi}_{abc} = p(\boldsymbol{D}\cdot\boldsymbol{\Psi}_{\mathrm{abc}}) - (p\boldsymbol{D})\cdot\boldsymbol{\Psi}_{abc} = p\boldsymbol{\Psi}_{dq0} - (p\boldsymbol{D})\boldsymbol{D}^{-1}\boldsymbol{\Psi}_{dq0}$$

Where:

$$(p\boldsymbol{D})\boldsymbol{D}^{-1} = \frac{2}{3}\begin{bmatrix} -\sin\theta_a & -\sin\theta_b & -\sin\theta_c \\ -\cos\theta_a & -\cos\theta_b & -\cos\theta_c \\ 0 & 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} \cos\theta_a & -\sin\theta_a & 1 \\ \cos\theta_b & -\sin\theta_b & 1 \\ \cos\theta_c & -\sin\theta_c & 1 \end{bmatrix}\cdot\frac{d\theta}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\cdot\omega$$

$$\therefore \boldsymbol{D}p\boldsymbol{\Psi}_{abc} = p\boldsymbol{\Psi}_{\mathrm{dq0}} - \omega\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\boldsymbol{\Psi}_{dq0} = p\boldsymbol{\Psi}_{dq0} + \begin{bmatrix} -\omega\boldsymbol{\Psi}_q \\ \omega\boldsymbol{\Psi}_d \\ 0 \end{bmatrix}$$

- **The final volt. eqn. in dq0 coord.**

$$\begin{bmatrix} \boldsymbol{u}_{dqo} \\ \boldsymbol{u}_{fDQ} \end{bmatrix} = p\begin{bmatrix} \boldsymbol{\Psi}_{dq0} \\ \boldsymbol{\Psi}_{fDQ} \end{bmatrix} + \begin{bmatrix} \boldsymbol{S}_{dq0} \\ \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{r}_{dq0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{r}_{fDQ} \end{bmatrix}\begin{bmatrix} -\boldsymbol{i}_{dq0} \\ \boldsymbol{i}_{fDQ} \end{bmatrix} \qquad \textbf{where:}\quad \boldsymbol{S}_{dq0} = \begin{bmatrix} -\omega\boldsymbol{\Psi}_q \\ \omega\boldsymbol{\Psi}_d \\ 0 \end{bmatrix}$$

۳. مدل ماشین سنکرون در چارچوب dq0

۱.۳ معادلات ولتاژ در چارچوب dq0 (ادامه)

- **Discussion of volt. eqn. in dq0 coord.**
  - **The first term of the r.h.s. : “*Transformer potential*”**
  - **The second term of the r.h.s. : “*speed potential*”**
    - **It is zero when rotor speed** $\omega = 0$
    - **It is <u>caused by cutting the rotating mag. field lines</u>.**

      (usually when the observation coordinates (e.g. dq0) have relative motion to the physical windings (e.g. abc), this term will appear)
  - **The third term of the r.h.s. : “*ohm potential*”**
    - **It is caused by winding resistances.**
  - **At the steady-state,** $\boldsymbol{u}_{dq0} = \boldsymbol{S}_{dq0}$ **if** $\boldsymbol{r}_{dq0} \cong \boldsymbol{0}$

<u>The second term is corresponding to the electromechanic power transferred from rotor to stator.</u>

۳. مدل ماشین سنکرون در چارچوب dq0

۲.۳ معادلات شار دور در چارچوب dq0

Flux linkage eqn. in dq0 coord.

- **Rewrite flux linkage eqn. in abc coord.**

$$\begin{bmatrix} \boldsymbol{\Psi}_{abc} \\ \boldsymbol{\Psi}_{fDQ} \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}_{11} & \boldsymbol{L}_{12} \\ \boldsymbol{L}_{21} & \boldsymbol{L}_{22} \end{bmatrix} \begin{bmatrix} -\boldsymbol{i}_{abc} \\ \boldsymbol{i}_{fDQ} \end{bmatrix}$$

- **Multiply the both-sides by T**
- **Yield flux linkage eqn. in dq0 coord.**

$$\begin{bmatrix} \boldsymbol{\Psi}_{dq0} \\ \boldsymbol{\Psi}_{fDQ} \end{bmatrix} = \begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{L}_{11} & \boldsymbol{L}_{12} \\ \boldsymbol{L}_{21} & \boldsymbol{L}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{D}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{D} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} -\boldsymbol{i}_{abc} \\ \boldsymbol{i}_{fDQ} \end{bmatrix}$$

$$= \begin{bmatrix} \boldsymbol{D}\boldsymbol{L}_{11}\boldsymbol{D}^{-1} & \boldsymbol{D}\boldsymbol{L}_{12} \\ \boldsymbol{L}_{21}\boldsymbol{D}^{-1} & \boldsymbol{L}_{22} \end{bmatrix} \begin{bmatrix} -\boldsymbol{i}_{dq0} \\ \boldsymbol{i}_{fDQ} \end{bmatrix}$$

$$\triangleq \begin{bmatrix} \boldsymbol{L}_{SS} & \boldsymbol{L}_{SR} \\ \boldsymbol{L}_{RS} & \boldsymbol{L}_{RR} \end{bmatrix} \begin{bmatrix} -\boldsymbol{i}_{dq0} \\ \boldsymbol{i}_{fDQ} \end{bmatrix}$$

where

$$\boldsymbol{L}_{SS} = \boldsymbol{D}\boldsymbol{L}_{11}\boldsymbol{D}^{-1} = \mathrm{diag}(L_{\mathrm{d}}, L_q, L_0)$$

$$\begin{cases} L_d = L_S + M_S + \dfrac{3}{2}L_t = \mathrm{const} > 0 \\ L_{\mathrm{q}} = L_S + M_S - \dfrac{3}{2}L_t = \mathrm{const} > 0 \\ L_0 = L_S - 2M_S = \mathrm{const} > 0 \end{cases}$$

$$\boldsymbol{L}_{RR} = \boldsymbol{L}_{22} = \begin{bmatrix} L_f & M_R & 0 \\ M_R & L_D & 0 \\ 0 & 0 & L_Q \end{bmatrix}$$

$$\boldsymbol{L}_{SR} = \boldsymbol{D}\boldsymbol{L}_{12} = \begin{bmatrix} M_f & M_D & 0 \\ 0 & 0 & M_Q \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{L}_{RS} = \boldsymbol{L}_{21}\boldsymbol{D}^{-1} = \begin{bmatrix} \frac{3}{2}M_f & 0 & 0 \\ \frac{3}{2}M_D & 0 & 0 \\ 0 & \frac{3}{2}M_Q & 0 \end{bmatrix} \neq \boldsymbol{L}_{SR}^{\mathrm{T}}$$

۳. مدل ماشین سنکرون در چارچوب dq0

۲.۳ معادلات شار دور در چارچوب dq0 (ادامه)

Flux linkage eqn. in dq0 coord. (cont.)

- $L_{RS} \neq L_{SR}^{\mathrm{T}}$ **(caused by '2/3' of Park's transf.)**
  - The problem will be solved in per unit system.
- **Zero-axis flux linkage has:** $\Psi_0 = -L_0 i_0$
  - <u>decoupled with d and q-axes</u>.
- **The windings on the d-axis (windings d, D, f) are <u>decoupled</u> with those on the q-axis (windings q, Q).**
- **The matrix L in dq0 coord. is <u>constant and sparse</u>, very good for dynamic analysis.**
- **All the elements in L are positive.**
- **Clear phy. meanings.**

$$L = \begin{bmatrix} L_{SS} & L_{SR} \\ L_{RS} & L_{RR} \end{bmatrix}$$

$$= \left[\begin{array}{ccc|ccc} L_d & 0 & 0 & M_f & M_D & 0 \\ 0 & L_q & 0 & 0 & 0 & M_Q \\ 0 & 0 & L_0 & 0 & 0 & 0 \\ \hline \frac{3}{2}M_f & 0 & 0 & L_f & M_R & 0 \\ \frac{3}{2}M_D & 0 & 0 & M_R & L_D & 0 \\ 0 & \frac{3}{2}M_Q & 0 & 0 & 0 & L_Q \end{array}\right]$$

$$= \text{const.}$$

۳. مدل ماشین سنکرون در چارچوب dq0

۳.۳ خلاصه مدل ماشین سنکرون در چارچوب dq0 (معادلات پارک)

- **6 voltage differential equations:** $\boldsymbol{u} = p\boldsymbol{\Psi} + \boldsymbol{S} + \boldsymbol{ri}$
- **6 flux linkage algebraic equations:** $\boldsymbol{\Psi} = \boldsymbol{Li}$
- **2 rotor motion equations:**

  where T_e **can be eliminated by** $T_e = p_p \dfrac{3}{2}(\Psi_d i_q - \Psi_q i_d)$
- **<u>Discussion</u>:**
  - **This model after converted to per unit system is widely used in the power system.**
  - **The most significant features are**
    - **<u>constant inductances</u> under the dq0 coord.,** $(\boldsymbol{L} = \text{const})$
    - **decoupling of dq0 axes flux,** $(\boldsymbol{L}:\text{sparse})$
    - **clear physical meaning.**
  - **The model is a <u>non-linear</u> dynamic model.** $\left(\because \begin{cases} '\Psi i' \text{ terms in } T_e \text{ eqn.} \\ '\omega\Psi' \text{ terms in volt eqn.} \end{cases} \right)$

۴. سیستم پریونیت برای ماشین سنکرون

۱.۴ مقدمه

Advantages

- **Using per unit system, machines parameters of different capacities are close to each other and can also tell the physical feature.**

  e.g. The d-axis syn. reactance X_d will be 0.6~2.5.

  - **The smaller value means a larger air gap.**
  - **We can check the data via p.u. values.**
- **The manufacturers provide the data in per unit values.**
- **Easy to handle three-phase power, line/phase variables etc.**
- **All the power system programs use per unit values. (load flow, stability analysis, etc.)**

۴. سیستم پریونیت برای ماشین سنکرون

۱.۴ مقدمه (ادامه)

Rules for setting up p. u. system

- **Rule 1**: **Base values obey elect. and mag. laws.**

  e.g. The bases of voltage (V_B), current (I_B), and resist. (R_B) should obey Ohm's law: $V_B = I_B \cdot R_B$.

  - **Thus the expression $V = I \cdot R$ is true for both MKS unit and per unit.**

- **Rule 2**: **The base values of the mutual inductances between the stator and rotor windings should make $\mathbf{L}_{RS} = \mathbf{L}_{SR}^{\mathrm{T}}$ in p. u.**

- **Rule 3**: **"X_d, X_{ad}, X_q, X_{aq}" etc. can be kept in the p. u. equations of syn. mac.**

۱.۵ معادلات ولتاژ

The dq0 coord. volt. eqn. is:

$$\begin{bmatrix} u_d \\ u_q \\ u_0 \\ u_f \\ u_D=0 \\ u_Q=0 \end{bmatrix} = \frac{d}{dt}\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_0 \\ \psi_f \\ \psi_D \\ \psi_Q \end{bmatrix} + \begin{bmatrix} -\omega\psi_q \\ \omega\psi_d \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -r_a i_d \\ -r_a i_q \\ -r_a i_0 \\ r_f i_f \\ r_D i_D \\ r_Q i_Q \end{bmatrix}$$

We know the following relations

$$\begin{cases} u_{aB} = \omega_B \psi_{aB} = R_{aB} i_{aB} \\ u_{fB} = \omega_B \psi_{fB} = R_{fB} i_{fB} \\ u_{DB} = \omega_B \psi_{DB} = R_{DB} i_{DB} \\ u_{QB} = \omega_B \psi_{QB} = R_{QB} i_{QB} \end{cases}$$

The stator volt. eqn's in p.u. system will be:

$$\begin{bmatrix} u_{d*} \\ u_{q*} \\ u_{0*} \end{bmatrix} = \frac{d}{\omega_B dt}\begin{bmatrix} \psi_{d*} \\ \psi_{q*} \\ \psi_{0*} \end{bmatrix} + \begin{bmatrix} -\omega_* \psi_{q*} \\ +\omega_* \psi_{d*} \\ 0 \end{bmatrix} + \begin{bmatrix} -r_{a*} i_{d*} \\ -r_{a*} i_{q*} \\ -r_{a*} i_{0*} \end{bmatrix}$$

where: $\dfrac{d}{\omega_B dt} = \dfrac{d}{d(t/t_B)} = \dfrac{d}{dt_*}$

Similarly, the rotor volt. eqn's in p.u. values should be:

$$\begin{bmatrix} u_{f*} \\ u_{D*} \equiv 0 \\ u_{Q*} \equiv 0 \end{bmatrix} = \frac{d}{dt_*}\begin{bmatrix} \psi_{f*} \\ \psi_{D*} \\ \psi_{Q*} \end{bmatrix} + \begin{bmatrix} r_{f*} i_{f*} \\ r_{D*} i_{D*} \\ r_{Q*} i_{Q*} \end{bmatrix}$$

۵. معادلات ماشین سنکرون در چارچوب dq0 و سیستم پریونیت

۱.۵ معادلات ولتاژ (ادامه)

We can unify the syn. mach. p. u. volt. eqn. as (subscript * is neglected):

$$\begin{bmatrix} \boldsymbol{u}_{dq0} \\ \boldsymbol{u}_{fDQ} \end{bmatrix} = p\begin{bmatrix} \Psi_{dq0} \\ \Psi_{fDQ} \end{bmatrix} + \begin{bmatrix} \boldsymbol{S}_{dq0} \\ 0 \end{bmatrix} + \begin{bmatrix} \boldsymbol{r}_{dq0} & 0 \\ 0 & \boldsymbol{r}_{fDQ} \end{bmatrix}\begin{bmatrix} -\boldsymbol{i}_{dq0} \\ \boldsymbol{i}_{fDQ} \end{bmatrix}$$

where

$$\boldsymbol{S}_{dq0} = \begin{bmatrix} -\omega\Psi_q \\ \omega\Psi_d \\ 0 \end{bmatrix} \qquad p = \frac{d}{dt_*}$$

t_* is in p.u.

$$t_* = \frac{t\ (\text{sec})}{t_B} = \omega_B \cdot t\ (\text{sec})$$

۲.۵ معادلات شار دور

The dq0 coord. flux linkage eqn. is:

$$\begin{bmatrix} \psi_d \\ \psi_q \\ \psi_o \\ \psi_f \\ \psi_D \\ \psi_Q \end{bmatrix} = \begin{bmatrix} L_d & 0 & 0 & L_{df} & L_{dD} & 0 \\ 0 & L_q & 0 & 0 & 0 & L_{qQ} \\ 0 & 0 & L_o & 0 & 0 & 0 \\ L_{fd} & 0 & 0 & L_f & L_{fD} & 0 \\ L_{Dd} & 0 & 0 & L_{Df} & L_D & 0 \\ 0 & L_{Qq} & 0 & 0 & 0 & L_Q \end{bmatrix}\begin{bmatrix} -i_d \\ -i_q \\ -i_o \\ i_f \\ i_D \\ i_Q \end{bmatrix}$$

where:

$$L_{fd} = \frac{3}{2}L_{df} = \frac{3}{2}M_f, \qquad L_{Dd} = \frac{3}{2}L_{dD} = \frac{3}{2}M_D$$

$$L_{Qq} = \frac{3}{2}L_{qQ} = \frac{3}{2}M_Q, \qquad L_{fD} = L_{Df} = M_R$$

۲.۵ معادلات شار دور (ادامه)

From base value definition we know

$$
\begin{cases}
\Psi_{aB} = L_{aB} i_{aB} = L_{afB} i_{fB} = L_{aDB} i_{DB} = L_{aQB} i_{QB} \\
\Psi_{fB} = L_{fB} i_{fB} = L_{faB} i_{aB} = L_{fDB} i_{DB} \\
\Psi_{DB} = L_{DB} i_{DB} = L_{DaB} i_{aB} = L_{DfB} i_{fB} \\
\Psi_{QB} = L_{QB} i_{QB} = L_{QaB} i_{aB}
\end{cases}
$$

Dividing the dq0 coord. flux linkage eqn. in MKS units by the appropriate terms of the above eqn., we have flux linkage eqn. in p. u. systems:

$$
\begin{bmatrix} \Psi_{d*} \\ \Psi_{q*} \\ \Psi_{0*} \\ \hline \Psi_{f*} \\ \Psi_{D*} \\ \Psi_{Q*} \end{bmatrix}
=
\left[\begin{array}{ccc|ccc}
L_{d*} & 0 & 0 & L_{df*} & L_{dD*} & 0 \\
0 & L_{q*} & 0 & 0 & 0 & L_{qQ*} \\
0 & 0 & L_{0*} & 0 & 0 & 0 \\
\hline
L_{fd*} & 0 & 0 & L_{f*} & L_{fD*} & 0 \\
L_{Dd*} & 0 & 0 & L_{Df*} & L_{D*} & 0 \\
0 & L_{Qq*} & 0 & 0 & 0 & L_{Q*}
\end{array}\right]
\begin{bmatrix} -i_{d*} \\ -i_{q*} \\ -i_{0*} \\ \hline i_{f*} \\ i_{D*} \\ i_{Q*} \end{bmatrix}
$$

so we have:

$$
\begin{cases}
L_{df*} = L_{fd*} = X_{ad*} \\
L_{dD*} = L_{Dd*} = X_{ad*} \\
L_{qQ*} = L_{Qq*} = X_{aq*} \\
L_{fD*} = L_{Df*} = M_{R*}
\end{cases}
$$

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۲.۵ معادلات شار دور (ادامه)

Finally we have flux linkage eqn. in p. u.

$$\begin{bmatrix}\boldsymbol{\Psi}_{dq0}\\ \boldsymbol{\Psi}_{fDQ}\end{bmatrix}=\begin{bmatrix}\Psi_d\\ \Psi_q\\ \Psi_0\\ \Psi_f\\ \Psi_D\\ \Psi_Q\end{bmatrix}=\begin{bmatrix}X_d & 0 & 0 & X_{ad} & X_{ad} & 0\\ 0 & X_q & 0 & 0 & 0 & X_{aq}\\ 0 & 0 & X_0 & 0 & 0 & 0\\ X_{ad} & 0 & 0 & X_f & M_R & 0\\ X_{ad} & 0 & 0 & M_R & X_D & 0\\ 0 & X_{aq} & 0 & 0 & 0 & X_Q\end{bmatrix}\begin{bmatrix}-i_d\\ -i_q\\ -i_0\\ i_f\\ i_D\\ i_Q\end{bmatrix}$$

$$=\begin{bmatrix}\boldsymbol{X}_{SS} & \boldsymbol{X}_{SR}\\ \boldsymbol{X}_{RS} & \boldsymbol{X}_{RR}\end{bmatrix}\begin{bmatrix}-\mathbf{i}_{dq0}\\ \mathbf{i}_{fDQ}\end{bmatrix}$$

- X-matrix is symmetric, constant and sparse. The syn. mach. char. parameters (p. u. values) are existed explicitly.

- Under common flux assumption (subscript * is neglected), we can see:

$$M_R = X_{ad}$$

$$\begin{cases}X_d = X_1 + X_{ad}\\ X_f = X_{f1} + X_{ad}\\ X_D = X_{D1} + X_{ad}\end{cases}$$

$$\begin{cases}X_q = X_1 + X_{aq}\\ X_Q = X_{Q1} + X_{aq}\end{cases}$$

- <u>**Clear physical meanings;**</u>
- <u>**Useful for deriving equivalent circuit.**</u>

۲.۵ -۱ – کمیت های مبنا در مدار آرمیچر(استاتور) و مدارهای رتور

- **Base values for stator circuit**

S_{aB} = 3phase nominal apparent power of machine (VA)

u_{aB} = Peak value of armature nominal phase voltage (V)

i_{aB} = Peak value of armature nominal line current (A)

$$S_{aB} = \left(3/2\right) u_{aB} . i_{aB}$$

$$u_{aB} = \omega_B \psi_{aB} = R_{aB} i_{aB}$$

f_B = Nominal frequency of machine (Hz)

$\omega_B = 2\pi f_B$ (elec. rad/sec) , $\omega_{mB} = \left(1/P_p\right)\omega_B$ (mech. rad/sec) , P_p = Pole pairs

$R_{aB} = u_{aB}/i_{aB}$ (ohm) , $L_{aB} = R_{aB}/\omega_B$ (H) , $\psi_{aB} = L_{aB} . i_{aB} = \dfrac{u_{aB}}{\omega_B}$ (wb-turn)

$$T_B = \frac{S_{aB}}{\omega_{mB}} = \left(3/2\right)\left(P_p\right)\psi_{aB} . i_{aB}$$

۵. معادلات ماشین سنکرون در چارچوب dq0 و سیستم پریونیت

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۲.۵ -۱ – کمیت های مبنا در مدار آرمیچر(استاتور) و مدارهای رتور (ادامه)

♦ **Base values for rotor circuits according to common flux assumption based on L_{ad} and mutual inductances equalization in p. u.**

$$\begin{cases} L_{df}\cdot i_{fB}=L_{ad}\cdot i_{aB} \\ L_{dD}\cdot i_{DB}=L_{ad}\cdot i_{aB} \\ L_{qQ}\cdot i_{QB}=L_{aq}\cdot i_{aB} \end{cases} \Rightarrow \begin{cases} i_{fB}=\left(L_{ad}/L_{df}\right)\cdot i_{aB} \\ i_{DB}=\left(L_{ad}/L_{dD}\right)\cdot i_{aB} \\ i_{QB}=\left(L_{aq}/L_{qQ}\right)\cdot i_{aB} \end{cases}$$

$$\begin{cases} u_{fB}\cdot i_{fB}=S_{aB}\Rightarrow u_{fB}=\dfrac{S_{aB}}{i_{fB}}\ ,\ \psi_{fB}=\dfrac{u_{fB}}{\omega_B} \\ R_{fB}=\dfrac{u_{fB}}{i_{fB}}=\dfrac{S_{aB}}{i^2_{fB}}\ ,\ L_{fB}=\dfrac{\psi_{fB}}{i_{fB}}=\dfrac{u_{fB}}{\omega_B\cdot i_{fB}}=\dfrac{R_{fB}}{\omega_B} \end{cases}$$

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۲.۵ -۱ – کمیت های مبنا در مدار آرمیچر(استاتور) و مدارهای رتور (ادامه)

- **Base values for rotor circuits according to common flux assumption based on L_{ad} and mutual inductances equalization in p. u.**

$$\begin{cases} u_{DB} \cdot i_{DB} = S_{aB} \Rightarrow u_{DB} = \dfrac{S_{aB}}{i_{DB}} \,,\ \psi_{DB} = \dfrac{u_{DB}}{\omega_B} \\ R_{DB} = \dfrac{u_{DB}}{i_{DB}} = \dfrac{S_{aB}}{i^2{}_{DB}} \ ,\ L_{DB} = \dfrac{\psi_{DB}}{i_{DB}} = \dfrac{u_{DB}}{\omega_B \cdot i_{DB}} = \dfrac{R_{DB}}{\omega_B} \end{cases}$$

$$\begin{cases} u_{QB} \cdot i_{QB} = S_{aB} \Rightarrow u_{QB} = \dfrac{S_{aB}}{i_{QB}} \,,\ \psi_{QB} = \dfrac{u_{QB}}{\omega_B} \\ R_{QB} = \dfrac{u_{QB}}{i_{QB}} = \dfrac{S_{aB}}{i^2{}_{QB}} \ ,\ L_{QB} = \dfrac{\psi_{QB}}{i_{QB}} = \dfrac{u_{QB}}{\omega_B \cdot i_{QB}} = \dfrac{R_{QB}}{\omega_B} \end{cases}$$

۳.۵ خلاصه مدل ماشین سنکرون در چارچوب dq0 و سیستم پریونیت

- **The p. u. equations for syn. mac. (volt., flux linkage, and motion eqn's) have the same format as the MKS unit equations:**

$$\begin{bmatrix} \boldsymbol{u}_{dq0} \\ \boldsymbol{u}_{fDQ} \end{bmatrix} = p\begin{bmatrix} \boldsymbol{\Psi}_{dq0} \\ \boldsymbol{\Psi}_{fDQ} \end{bmatrix} + \begin{bmatrix} \boldsymbol{S}_{dq0} \\ \boldsymbol{0} \end{bmatrix} + \begin{bmatrix} \boldsymbol{r}_{dq0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{r}_{fDQ} \end{bmatrix}\begin{bmatrix} -\boldsymbol{i}_{dq0} \\ \boldsymbol{i}_{fDQ} \end{bmatrix}$$

where $\boldsymbol{S}_{dq0} = \left(-\omega\psi_q \quad \omega\psi_d \quad 0\right)^{\mathrm{T}} \quad p = d/dt_*$

$$\begin{bmatrix} \boldsymbol{\Psi}_{dq0} \\ \boldsymbol{\Psi}_{fDQ} \end{bmatrix} = \begin{bmatrix} \Psi_d \\ \Psi_q \\ \Psi_0 \\ \Psi_f \\ \Psi_D \\ \Psi_Q \end{bmatrix} = \begin{bmatrix} X_d & 0 & 0 & X_{ad} & X_{ad} & 0 \\ 0 & X_q & 0 & 0 & 0 & X_{aq} \\ 0 & 0 & X_0 & 0 & 0 & 0 \\ X_{ad} & 0 & 0 & X_f & M_R & 0 \\ X_{ad} & 0 & 0 & M_R & X_D & 0 \\ 0 & X_{aq} & 0 & 0 & 0 & X_Q \end{bmatrix}\begin{bmatrix} -i_d \\ -i_q \\ -i_0 \\ i_f \\ i_D \\ i_Q \end{bmatrix}$$

$$= \begin{bmatrix} \boldsymbol{X}_{SS} & \boldsymbol{X}_{SR} \\ \boldsymbol{X}_{RS} & \boldsymbol{X}_{RR} \end{bmatrix}\begin{bmatrix} -\mathbf{i}_{dq0} \\ \mathbf{i}_{fDQ} \end{bmatrix}, \qquad M_R = X_{ad}$$

$$\begin{cases} 2H_*\dfrac{d\omega_*}{dt_*} = T_{m*} - T_{e*} \\ \dfrac{d\delta}{dt_*} = \omega_* - 1 \end{cases}$$

where $T_{e*} = \Psi_{d*}i_{q*} - \Psi_{q*}i_{d*}$

- **In stability analysis, time is preferred in seconds. The corresponding rotor motion equation is:**

$$\begin{cases} 2H(\mathrm{sec.})\dfrac{d\omega_*}{dt(\mathrm{sec.})} = T_{m*} - T_{e*} \\ \dfrac{d\delta}{dt(\mathrm{sec.})} = (\omega_* - 1)\cdot\omega_B \end{cases}$$

۳.۵ خلاصه مدل ماشین سنکرون در چارچوب dq0 و سیستم پریونیت (ادامه)

- **If we prefer time in sec. and speed in rad./sec, then rotor motion eqn. will be:**

  ($M \stackrel{\Delta}{=} \dfrac{2H(\text{sec})}{\omega_B(\text{rad/sec})} = \dfrac{H}{\pi f_S}(\text{sec}^2)$ **: mac. intertia.)**

$$\begin{cases} M\dfrac{d\omega(\text{rad/sec.})}{dt(\text{sec.})} = T_{m*} - T_{e*} \\ \dfrac{d\delta}{dt(\text{sec.})} = \omega - \omega_S(\text{rad/sec.}) \end{cases}$$

- **When** $\omega \approx 1\,\text{p.u.}$, $r_a \approx 0$ **, and** $p\Psi_{dq0} \approx 0$ **,**

  we have $T_{m*} - T_{e*} \approx P_{m*} - P_{e*}$

  (widely used in transient stab. analysis)

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۶. مدار معادل محورهای d و q و پارامترهای عملیاتی

- **To make physical concept clear;**
- **To set up the relation between Park’s equation parameters and practical (manufacturer provided) parameters**

۱.۶ مدار معادل محور q و پارامترها

q-axis equivalent circuit and parameters

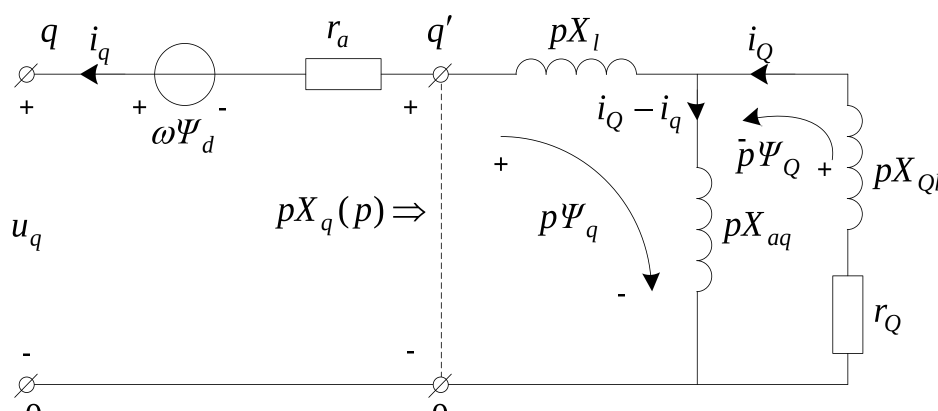
On q-axis, we have:

$$\begin{cases} \psi_q = -X_q i_q + X_{aq} i_Q = -X_l i_q + X_{aq}(-i_q + i_Q) \\ \psi_Q = -X_{aq} i_q + X_Q i_Q = X_{aq}(-i_q + i_Q) + X_{Ql} i_Q \end{cases}$$

and

$$\begin{cases} u_q = p\psi_q + \omega\psi_d - r_a i_q \\ u_Q = p\psi_Q + r_Q i_Q = 0 \end{cases}$$

the corresponding q-axis equivalent circuit is shown in the fig.



۶. مدار معادل محورهای d و q و پارامترهای عملیاتی

۱.۶ مدار معادل محور q و پارامترها (ادامه)

Practical parameters of q-axis used in the simplified syn. mac. model for stability analysis:

- **q-axis syn. reactance**

$$X_q = X_l + X_{aq}$$

where X_l: q-axis stator leakage, X_{aq} : q-axis stator armature reaction reactance(or mutual reactance)

- **q-axis sub-transient reactance X''_q:**

$$X''_q \overset{\Delta}{=} X_l + X_{aq} // X_{Ql} = X_q - \frac{X_{aq}^2}{X_Q}$$

X''_q plays an important role in the system sub-transient process.

- **q-axis open-circuit sub-transient time constant T''_{q0}**

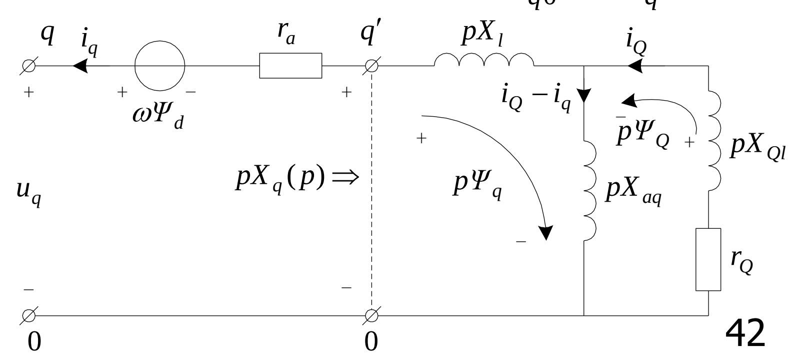
$$T''_{q0} \overset{\Delta}{=} \frac{X_{Ql} + X_{aq}}{r_Q} = \frac{X_Q}{r_Q} \text{ (p.u.)}$$

Note: $T''_{q0}(\text{sec}) = T''_{q0}(\text{p.u.}) \cdot t_B \quad (t_B = \frac{1}{\omega_B})$

- **q-axis short-circuit sub-transient time constant T''_q :**

$$T''_q \overset{\Delta}{=} \frac{X_{Ql} + X_{aq} // X_l}{r_Q} = \frac{X_Q - \frac{X_{aq}^2}{X_q}}{r_Q} \text{ (p.u.)}$$

It can be proved that $\frac{T''_q}{T''_{q0}} = \frac{X''_q}{X_q}$



۲.۶ مدار معادل محور d و پارامترها

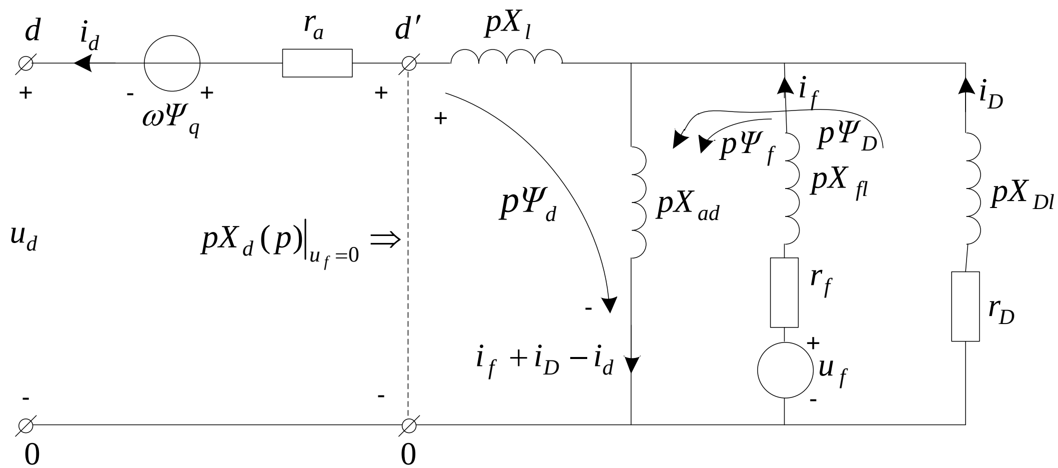
On d-axis, we have:

$$\begin{cases} \psi_d = -X_d i_d + X_{ad} i_f + X_{ad} i_D = -X_l i_d + X_{ad}(-i_d + i_f + i_D) \\ \psi_f = -X_{ad} i_d + X_f i_f + X_{ad} i_D = X_{ad}(-i_d + i_f + i_D) + X_{fl} i_f \\ \psi_D = -X_{ad} i_d + X_{ad} i_f + X_D i_D = X_{ad}(-i_d + i_f + i_D) + X_{Dl} i_D \end{cases}$$

and:

$$\begin{cases} u_d = p\Psi_d - \omega\Psi_q - r_a i_d \\ u_f = p\Psi_f + r_f i_f \\ u_D = p\Psi_D + r_D i_D = 0 \end{cases}$$

the d-axis equivalent circuit can be drawn as shown in the figure



۲.۶ مدار معادل محور d و پارامترها (ادامه)

The practical parameters for d-axis are defined similarly to the q-axis.

- **d-axis syn. reactance:** $X_d = X_l + X_{ad}$

where X_l **:d-axis stator leakage,**

X_{ad} **:d-axis mutual reactance or stator armature reaction reactance.**

- **d-axis transient reactance:**

$$X'_d \overset{\Delta}{=} X_l + X_{ad} // X_{fl} = X_d - \frac{X_{ad}^{\ 2}}{X_f}$$

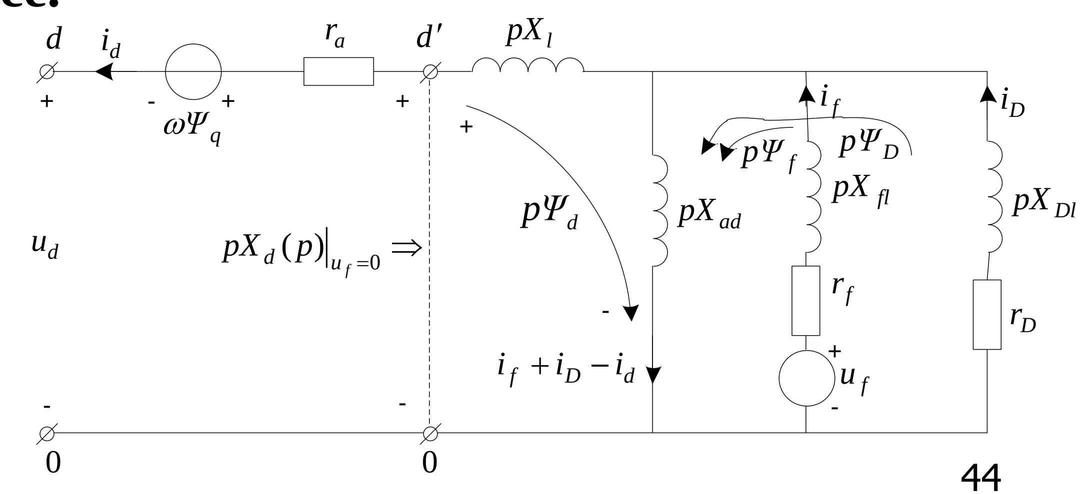
- **d-axis sub-transient reactance:**

$$X''_d = X_l + X_{ad} // X_{fl} // X_{Dl}$$

$$= X_d - \frac{X_{ad}^2 (X_D - 2X_{ad} + X_f)}{X_D X_f - X_{ad}^2}$$

- X'_d **and** X''_d **play an important role in the machine transients and sub-transients.**
- **d-axis open-circuit transient time constant** T'_{d0}**:**

$$T'_{d0} = \frac{X_f}{r_f}$$



۲.۶ مدار معادل محور d و پارامترها (ادامه)

- **d-axis short-circuit transient time constant T_d' :**

$$T_d' = \frac{X_{fl} + X_{ad} // X_l}{r_f} = \frac{X_f - \dfrac{X_{ad}^2}{X_d}}{r_f}$$

It can be proved: $\dfrac{T_d'}{T_{d0}'} = \dfrac{X_d'}{X_d}$

- **d-axis open-circuit sub-transient time constant T_{d0}'':**

$$T_{d0}'' = \frac{X_{Dl} + X_{fl} // X_{ad}}{r_D} = \frac{X_D - \dfrac{X_{ad}^2}{X_f}}{r_D}$$

- **d-axis short-circuit sub-transient time constant T_d'' :**

$$T_d'' = \frac{X_{D1} + X_{f1} // X_{ad} // X_1}{r_D} = \frac{X_{D1} + \left(\dfrac{1}{X_{f1}} + \dfrac{1}{X_{ad}} + \dfrac{1}{X_1}\right)^{-1}}{r_D}$$

It can be proved: $\dfrac{T_d''}{T_{d0}''} = \dfrac{X_d''}{X_d'}$

(T_{d0}', T_d') **and** (T_{d0}'', T_d'') **play an important role in the syn. mac. transients and sub-transients.**

Note: The above time constants are all in p. u. They should multiply $t_B (1/\omega_B)$ to obtain values in seconds.

۶. مدار معادل محورهای d و q و پارامترهای عملیاتی

۳.۶ محاسبه پارامترها در معادلات پارک با استفاده از داده های شرکت سازنده (Manufacturer data)

$$X_l, X_d, X_q, X'_d, X''_d, X''_q, T'_{d0}, T''_{d0}, T''_{q0}$$

The required parameters in Park's equations are (in p.u., $\omega_B = 314.16 \text{ rad/sec}$)

$X_{ad}, X_{aq}, X_f, X_D, X_Q$ and r_f, r_D, r_Q

According to the equivalent circuits of d-q axes, we have:

(1) $X_{ad} = X_d - X_l$

(2) $X_{aq} = X_q - X_l$

(3) Since $X'_d = X_d - \dfrac{X_{ad}^2}{X_f}$

so $X_f = \dfrac{X_{ad}^2}{X_d - X'_d}$

(4) Since

$$X''_d = X_d - \frac{X_{ad}^2 (X_D - 2X_{ad} + X_f)}{X_D X_f - X_{ad}^2}$$

so

$$X_D = \frac{X_{ad}^2 (X_d - X''_d - 2X_{ad} + X_f)}{X_f (X_d - X''_d) - X_{ad}^2}$$

۶. مدار معادل محورهای d و q و پارامترهای عملیاتی

۶.۳ محاسبه پارامترها در معادلات park با استفاده از داده های شرکت سازنده (ادامه)

(5) Since $X''_q = X_q - \dfrac{X_{aq}^2}{X_Q}$

so $X_Q = \dfrac{X_{aq}^2}{X_q - X''_q}$

(6) From $T'_{d0} = \dfrac{X_f}{\omega_B r_f}\,(\text{sec})$

we have $r_f = \dfrac{X_f}{\omega_B \cdot T'_{do}\,(\text{sec})}$

(7) Form $T''_{d0} = \dfrac{X_{Dl} + (X_{fl} // X_{ad})}{\omega_B r_D}(\text{sec})$

we can calculate:

$$r_D = \frac{X_{Dl} + X'_d - X_l}{\omega_B T''_{d0}(\text{sec})}$$

(8) From $T''_{q0} = \dfrac{X_Q}{\omega_B r_Q}(\text{sec})$

we have $r_Q = \dfrac{X_Q}{\omega_B T''_{q0}(\text{sec})}$

(If T'_d, T''_d, T''_q rather than $T'_{d0}, T''_{d0}, T''_{q0}$ are known, we can use relations among them to convert T'_d, T''_d, T''_q into $T'_{d0}, T''_{d0}, T''_{q0}$ first.)

۶. مدار معادل محورهای d و q و پارامترهای عملیاتی

۴.۶ مقادیر نوعی پارامترهای عملیاتی (ارائه شده توسط شرکت‌های سازنده)

Parameter		Hydraulic Units	Thermal Units
Synchronous Reactance	X_d	0.6 - 1.5	1.0 - 2.3
	X_q	0.4 - 1.0	1.0 - 2.3
Transient Reactance	X'_d	0.2 - 0.5	0.15 - 0.4
	X'_q	-	0.3 - 1.0
Subtransient Reactance	X''_d	0.15 - 0.35	0.12 - 0.25
	X''_q	0.2 - 0.45	0.12 - 0.25
Transient OC Time Constant	T'_{d0}	1.5 - 9.0 s	3.0 - 10.0 s
	T'_{q0}	-	0.5 - 2.0 s
Subtransient OC Time Constant	T''_{d0}	0.01 - 0.05 s	0.02 - 0.05 s
	T''_{q0}	0.01 - 0.09 s	0.02 - 0.05 s
Stator Leakage Inductance	X_l	0.1 - 0.2	0.1 - 0.2
Stator Resistance	R_a	0.002 - 0.02	0.0015 - 0.005

۷. مدل های گذرای الکترومکانیکی ماشین سنکرون (مدل عملی/ساده سازی شده)

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۱.۷ مقدمه

- **If the 0-axis component is considered independently, then the d, q, f, D, Q windings and rotor motion equations will constitute a 7th -order syn. mac. model.**
- **The Park's equation considering stator winding transients is called "*Electromagnetic Transient (EMT) model*", with very small time constant.**
- **The network components connecting to the stator windings should also be described by *differential equations (DE)*. The overall system model is extremely complicated and it is not used in power system stability analysis.**
- **In simplification, the most commonly used and important assumption is to assume $p\Psi_d = p\Psi_q = p\Psi_0 = 0$ and set ω=1 p.u. in stator volt. eqn. Thus:**
  The stator volt. eqns. can be Linear AE (algebraic eqns.) and network eqns. can be in AE as well in Y_{bus}-matrix format.

۷. مدل های گذرای الکترومکانیکی ماشین سنکرون (مدل عملی/ساده سازی شده)

۱.۷ مقدمه (ادامه)

- **The system model *<u>with the stator transients neglected</u>* ($p\Psi_d = p\Psi_q = p\Psi_0 = 0$) is called "*<u>electromechanical model</u>*", or *<u>practical</u>* / *<u>simplified model</u>*.**
- **In simplified model, some rotor var.'s in Park's equation (e.g. Ψ_{fDQ}, u_f) are referred to the stator side (easy to analyze and measure) become practical var.'s:**
  - **stator excitation voltage** $E_f \overset{\Delta}{=} \frac{X_{ad}}{r_f} u_f \propto u_f$
  - **stator q-axis transient volt. (volt. behind X'd)** $E'_q \overset{\Delta}{=} \frac{X_{ad}}{X_f} \Psi_f \propto \Psi_f$
- **Rotor currents in Park's equation (e.g. i_f, i_D, i_Q) are eliminated using their flux eqns.**
- **In the simplified model, the stator winding Ψ_d and Ψ_q will be eliminated using their flux eqns., and u_{dq}, i_{dq} will remain.**
- **In stator volt. eqns.: assume $\omega=1$**

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۱.۷ مقدمه (ادامه)

- **The syn. mac. per unit parameters (e.g.** $X_d, X_q, X'_d, X''_d, X'_q, X''_q, T'_{d0}, T''_{d0}, T'_{q0}, T''_{q0}$**) will be used in the simplified models which is easy for data preparation and analysis.**
- **Further assumptions can be made which result in even lower order syn. mac. models widely used in stability study**
- **We are going to introduce three simplified models:**
  - **Neglect D, Q damping winding, assume** $p\Psi_d = p\Psi_q = 0$ **and** $\omega \approx 1$ **in the stator volt. eqn., and consider field winding transients and rotor dynamics --- *<u>third order model</u>* with** E'_q, ω, δ **as state variables.**
  - **Assume** $p\Psi_d = p\Psi_q = 0$ **and** $\omega \approx 1$ **in the stator volt. eqn., and consider f, D,Q winding transients --- *<u>fifth order model</u>* with** $E'_q, E''_q, E''_d, \omega, \delta$ **as the state variables.**
  - **Classical model with ω, δ as state variables only.**
- ***The simplified syn. mac. models are widely used in p. s. stability and control. We should understand their physical nature according to the derivation. We should keep the assumptions in mind and know the limitation of the models.***

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۲.۷ مدل مرتبه سوم

- **The simplest dyn. model which can *<u>includes the excitation system dynamics</u>*.**
- **The assumptions made:**
  - **Neglect stator transients i.e.** $p\Psi_d = p\Psi_q = 0$
  - **Assume** $\omega \approx 1$ **p.u. in stator volt. eqn.**
  - **Neglect D, Q damping windings. Only f-winding remains on the rotor!**
- **New variable defined:**
  - **Stator excitation voltage** $E_f \overset{\Delta}{=} \dfrac{X_{ad}}{r_f} u_f \propto u_f$
  - **Stator q-axis open-circuit volt. (volt. behind** X_d **): to be eliminated later**

    $$E_q \overset{\Delta}{=} X_{ad} i_f \propto i_f \qquad (u_q = E_q - X_d i_d)$$
  - **stator q-axis transient voltage (also 'voltage behind** X'_d **')**

    $$E'_q \overset{\Delta}{=} \frac{X_{ad}}{X_f} \Psi_f \propto \Psi_f \qquad (u_q = E'_q - X'_d i_d)$$

Later when see E_f, E_q, E'_q ***we should consider they are associated with*** u_f, i_f, Ψ_f

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۲.۷ مدل مرتبه سوم (ادامه)

Third order model (cont.)

- **Initial value calc.: (steady-state)**

Since $u_{f0} = i_{f0} \cdot r_f$ **, we have**

$$\boxed{E_{f0} = \frac{X_{ad}}{r_f} u_{f0} = X_{ad} i_{f0} = E_{q0}}$$

From volt. eqn. we also know at the steady state:

$$\boxed{u_{q0} = \Psi_{d0} - r_a i_{q0} = E_{q0} - X_d i_{d0} - r_a i_{q0}}$$

$$(\because\ \Psi_{d0} = -X_d i_{d0} + X_{ad} i_{f0})$$

The above equation is used to calc. initial value of $E_{q0} (= E_{f0})$**.**

- **We know from Park's equation:**

$$\psi_f = -X_{ad} i_d + X_f i_f$$

Multiplying both side by $\frac{X_{ad}}{X_f}$ **, we have:**

$$E'_q \stackrel{\Delta}{=} \frac{X_{ad}}{X_f} \Psi_f = -\frac{X_{ad}^2}{X_f} i_d + X_{ad} i_f$$

Since $X'_d = X_d - \frac{X_{ad}^2}{X_f}$**, so we have:**

$$E'_q = E_q - (X_d - X'_d) \cdot i_d$$

or $\boxed{E'_q - X'_d i_d = E_q - X_d i_d}$

then we see:

$$\boxed{u_{q0} = E'_{q0} - X'_d i_{d0} - r_a i_{q0} = E_{q0} - X_d i_{d0} - r_a i_{q0}}$$

important relation of u_{q0}, E_{q0} **and** E'_{q0} **!**

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۲.۷ مدل مرتبه سوم (ادامه)

Third order model (cont.)

<u>**Deriving third order model**</u>

- **keep** $\boldsymbol{u}_{dq}, \boldsymbol{i}_{dq}$,
- **transform** u_f, i_f, Ψ_f **into** E_f, E_q **and** E_q'
- **use 3 flux linkage equations to eliminate** Ψ_d, Ψ_q **and** $i_f\,(E_q)$.
- **Finally we obtain the model with** $\boldsymbol{u}_{dq}, \boldsymbol{i}_{dq}, E_f, E_q'$ **and** ω, δ, T_M **as 9 variables, and composed of 3 volt. eqn.'s (*d, q, f*) and 2 rotor motion equations.**

We need 4 boundary conditions to solve the problem and they are E_f, T_m ***(input) and 2 network interface equations (d, q windings).***

(1) Stator volt. eqn's:

$$\boxed{\begin{cases} u_d = p\Psi_d - \omega\Psi_q - r_a i_d = X_q i_q - r_a i_d \\ u_q = p\Psi_q + \omega\Psi_d - r_a i_q = E_q' - X_d' i_d - r_a i_q \end{cases}}$$

$$\because\ \Psi_d = -X_d i_d + X_{ad} i_f = -X_d i_d + E_q$$

$$E_q' - X_d' i_d = E_q - X_d i_d$$

$$\therefore\ \Psi_d = -X_d i_d + (E_q' - X_d' i_d + X_d i_d)$$

$$= E_q' - X_d' i_d$$

(2) Field winding volt. eqn.:

From: $u_f = p\Psi_f + r_f i_f$

multiplying $\dfrac{X_{ad}}{r_f}$ **, we have**

$$p\frac{X_{ad}}{r_f}\Psi_f = E_f - E_q$$

۲.۷ مدل مرتبه سوم (ادامه)

Third order model (cont.)

Since $\dfrac{X_{ad}}{r_f}\Psi_f = (\dfrac{X_{ad}}{X_f}\Psi_f)\cdot\dfrac{X_f}{r_f} = E'_q \cdot T'_{do}$

so

$$\boxed{\begin{aligned} T'_{d0}\frac{d}{dt}E'_q &= E_f - E_q \\ &= E_f - (E'_q + (X_d - X'_d)\cdot i_d) \end{aligned}}$$

(3) Rotor motion equations:

$$\boxed{\begin{aligned} T_J\frac{d\omega}{dt} &= T_m - T_e = T_m - (\Psi_d i_q - \Psi_q i_d) \\ &= T_m - [(E'_q - X'_d i_d)\cdot i_q - (-X_q i_q)\cdot i_d] \\ &= T_m - [E'_q i_q - (X'_d - X_q) i_d i_q] \\ \frac{d\delta}{dt} &= \omega - 1 \end{aligned}}$$

Eqn's* (1), (2) *and* (3) *constitute the syn. mac. 3rd order model where time is in p. u.

- **If we want to use 'sec.' for time, the eqn's are (widely used in stab. Study):**

$$\boxed{\begin{cases} u_d = X_q i_q - r_a i_d \\ u_q = E_q - X_d\, i_d - r_a i_q \\ T'_{d0}\dfrac{d}{dt}E'_q = E_f - [E'_q + (X_d - X'_d)\cdot i_d] \\ 2H\dfrac{d\omega}{dt} = T_m - [E'_q\, i_q - (X'_d - X_q)\cdot i_d i_q] \\ \dfrac{d\delta}{dt} = (\omega - 1)\omega_s \end{cases}}$$

where T'_{d0}, t, H **: in sec.,** ω**: in p. u.,**

$\omega_S = 2\pi f_S$ **: rad/sec,** δ**: in rad.**

others: in p. u.

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۲.۷ مدل مرتبه سوم (ادامه)

Third order model

Discussion

- **Initial value calculation:**
  - $\omega_{(0^-)} = \omega_{(0^+)} = 1\,\text{p.u.}$
  - $\dot{E}_{qd} = \dot{V}_G\big|_{0^-} + (r_a + jX_q)\dot{I}_G\big|_{0^-} = E_{qd}\angle\delta_0$ **to determine δ_0 and d, q-axes location.**
  - $u_{dq}\big|_{0-}, i_{dq}\big|_{0-}$ **can be calculated then.**

    Note: $u_{dq}\big|_{0+}, i_{dq}\big|_{0+}$ ***may be different from*** $u_{dq}\big|_{0-}, i_{dq}\big|_{0-}$ ***if there is an operation at*** $t = 0$
  - E'_{q0} **and** $E_{f0} = E_{q0}$ **are calc. according to the initial value relations.**
  - $T_{m0} = T_{e0-} = [E'_q i_q - (X'_d - X_q)\cdot i_d i_q]\big|_{0-}$

- **When we neglect the dynamics of the speed governor and prime mover, T_m = const. = T_{m0}.**
- **In motion eqn., T_m and T_e is often replaced by P_m and P_e --- the output power of the prime mover and gen. in p. u. (assume $\omega \approx 1$ p.u., $r_a = 0$).**
- **We introduce a damping term in rotor motion equation to include the D, Q damping effect.**

$$T_J \frac{d\omega}{dt} = P_m - [E'_q i_q - (X'_d - X_q) i_d i_q] - D(\omega - 1)$$

D is the damping factor, usually in 1~3 p.u.

۲.۷ مدل مرتبه سوم (ادامه)

Third order model

Third order model is good for *hydro-generator* with *salient* poles.

- **4th order model: for *round* rotor steam-turbine generator, usually a short-circuited *damping winding* (g) is added on the q-axis to consider the the solid rotor effect along the q-axis.**

  (The winding f can take into account the solid rotor effect along the d-axis.)

- **The corresponding 4th order model state variables are $E_d', E_q', \omega, \delta$.**

Here we define $(\because u_g \equiv 0, \therefore E_g = -\frac{X_{aq}}{r_g} u_g \equiv 0)$

$$\boxed{E_d' = -\frac{X_{aq}}{X_g}\Psi_g, \quad E_d = -X_{aq} i_g, \quad T_{q0}' = \frac{X_g}{r_g}}$$

(The negative sign is because q-axis is leading d-axis by 90°, the flux in q-axis will generate emf in the negative direction of d-axis.)

The 4th order model is: (in p.u., except that time is in sec.):

$$\begin{cases} u_d = E_d' + X_q' i_q - r_a i_d \\ u_q = E_q' - X_d' i_d - r_a i_q \\ T_{d0}' p E_q' = E_f - (E_q' + (X_d - X_d') i_d) \\ T_{q0}' p E_d' = -(E_d' - (X_q - X_q') i_q) \\ T_J \dfrac{d\omega}{dt} + D(\omega - 1) = T_m - \\ \qquad [E_q' i_q + E_d' i_d - (X_d' - X_q') i_d i_q] \\ \dfrac{d\delta}{dt} = (\omega - 1)\cdot \omega_B \end{cases}$$

۳.۷ مدل مرتبه پنجم

Fifth order model

- **When the D, Q damping winding fast transients are taken into account, the 3rd model becomes fifth order (f, D, Q winding transients and rotor motion dynamics).**

- **<u>New practical variables:</u>**

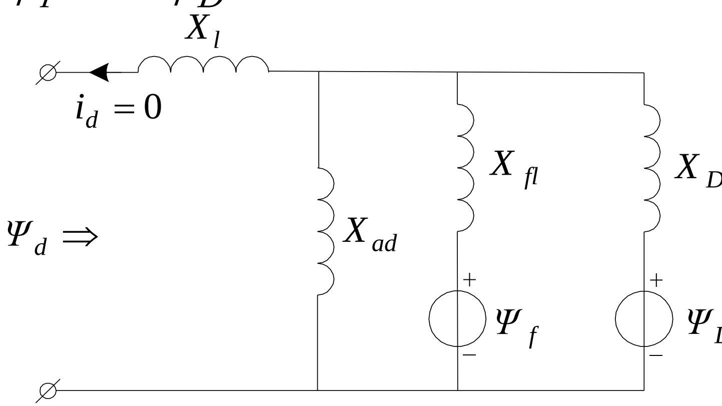
Two new practical variables are introduced in Park's equations. They are:

  - **q-axis open-circuit sub-transient voltage E_q'' (also: voltage behind X''_d):**

From d-axis eq. circuit, we know the ψ_d is: (using superposition)

$$\Psi_d = \frac{\Psi_f}{X_{fl} + X_{ad} // X_{Dl}} \cdot \frac{X_{Dl}}{X_{ad} + X_{Dl}} \cdot X_{ad} + \frac{\Psi_D}{X_{Dl} + X_{ad} // X_{fl}} \cdot \frac{X_{fl}}{X_{ad} + X_{fl}} \cdot X_{ad} = \frac{X_{ad}}{X_f X_D - X_{ad}^2}(X_{Dl}\Psi_f + X_{fl}\Psi_D)$$

which is a linear function of both ψ_f and ψ_D.



۳.۷ مدل مرتبه پنجم (ادامه)

Fifth order model

The corresponding q-axis open-circuit subtransient volt. defined as E''_q should be $(\omega = 1\text{p.u}, r_a i_q = 0, p\Psi_q = 0)$:

$$E''_q = \frac{X_{ad}}{X_f X_D - X_{ad}^2}(X_{Dl}\Psi_f + X_{fl}\Psi_D)$$

It can be proved:

$$u_{q0} = E''_{q0} - X''_d i_{d0} - r_a i_{q0}$$

- **d-axis open-circuit subtransient volt. E''_d (also: "voltage behind X''_q"):**

We define $(\omega = 1\text{p.u}, r_a i_q = 0, p\Psi_q = 0)$:

$$E''_d = -\Psi_q = -\frac{X_{aq}}{X_{aq} + X_{Ql}}\Psi_Q = -\frac{X_{aq}}{X_Q}\Psi_Q$$

It can be proved:

$$u_{d0} = E''_{d0} + X''_q i_{q0} - r_a i_{d0}$$

How to derive the fifth order model

- **assume $p\Psi_q = p\Psi_d = 0$ and $\omega = 1\,\text{p.u.}$ in stator voltage equations.**
- **Ψ_f, Ψ_D, Ψ_Q will be converted into stator-side practical variables of E'_q, E''_q, E''_d , and $u_f \Rightarrow E_f$.**
- **The 5 flux linkage equations are used to eliminate i_f, i_D, i_Q and Ψ_d, Ψ_q.**
- **The var. of u_{dq}, i_{dq} will remain.**

Deriving the fifth order model

- **Derive expressions for eliminating i_f, i_D, i_Q and Ψ_d, Ψ_q from 5 flux eqn's, we finally have:**

$$\Psi_d = E''_q - X''_d \cdot i_d$$

۳.۷ مدل مرتبه پنجم (ادامه)

Fifth order model (cont.)

$$\Psi_q = -X_q \cdot i_q + (-E''_d + \frac{X_{aq}^2}{X_Q} i_q)$$

$$= -E''_d - X''_q \cdot i_q$$

$$E_q = X_{ad} i_f = \frac{X_d - X_l}{X'_d - X_l} E'_q - \frac{X_d - X'_d}{X'_d - X_l} E''_q + \frac{(X_d - X'_d)(X''_d - X_l)}{(X'_d - X_l)} i_d$$

$$i_D = \frac{1}{X_d - X'_d}\left(E'_q - E_q\right) + i_d$$

$$= \frac{1}{X'_d - X_l}\left[E''_q - E'_q + (X'_d - X''_d) i_d\right]$$

$$i_Q = \frac{-1}{X_{aq}}\left[E''_d - (X_q - X''_q) i_q\right]$$

- **Convert** Ψ_f, Ψ_D, Ψ_Q **into** E'_q, E''_q, E''_d **, and** u_f **to** E_f **using:**

$$E'_q = \frac{X_{ad}}{X_f} \Psi_f$$

$$E''_q = \frac{X_{ad}}{X_f X_d - X_{ad}^2} (X_{Dl} \Psi_f + X_{fl} \Psi_D)$$

Therefore

$$\Psi_D = \frac{X'_d - X_l}{X'_d - X''_d} E''_q - \frac{X'' - X_l}{X'_d - X''_d} E'_q$$

$$E''_d = -\frac{X_{aq}}{X_Q} \Psi_Q$$

$$E_f = \frac{X_{ad}}{r_f} u_f$$

۳.۷ مدل مرتبه پنجم (ادامه)

Fifth order model (cont.)

<u>Build the fifth order model</u>

(1) Stator volt. eqn’s:

Set $p\Psi_q = p\Psi_d = 0$ **and** $\omega = 1\,\text{p.u.}$

eliminate Ψ_d **and** Ψ_q **we have:**

$$\boxed{\begin{cases} u_d = -\Psi_q - r_a i_d = E_d'' + X_q'' i_q - r_a i_d \\ u_q = \Psi_d - r_a i_q = E_q'' - X_d'' i_d - r_a i_q \end{cases}}$$

(2) Field winding volt. eqn:

From $p\Psi_f = u_f - r_f i_f$**, similarly to the third order model, we get:**

$$\begin{aligned} T_{d0}' pE_q' &= E_f - E_q \\ &= E_f - \frac{X_d - X_l}{X_d' - X_l} E_q' + \frac{X_d - X_d'}{X_d' - X_l} E_q'' \\ &\quad - \frac{(X_d - X_d')(X_d'' - X_l)}{(X_d' - X_l)} i_d \end{aligned}$$

Sometimes we take $i_d \approx \dfrac{E_q' - E_q''}{X_d' - X_d''}$

(Note: this is valid only at i_D**=0), then we have:**

$$\boxed{T_{d0}' pE_q' = E_f - (E_q' - X_{dr} E_q' + X_{dr} E_q'')}$$

where $X_{dr} \approx \dfrac{X_d - X_d'}{X_d'' - X_d'}$

(3) Winding D volt. eqn.:

For $p\Psi_D = -r_D i_D$

since $T_{d0}'' = \dfrac{X_D - \dfrac{X_{ad}^2}{X_f}}{r_D}$ **so** $r_D = \dfrac{(X_d' - X_l)^2}{(X_d' - X_d'')T_{d0}''}$

Eliminating Ψ_D **and** r_D **,we have:**

$$\begin{aligned} T_{d0}'' pE_q'' &= \frac{X_d'' - X_l}{X_d' - X_l} T_{d0}'' pE_q' \\ &\quad - E_q'' + E_q' - (X_d' - X_d'') i_d \end{aligned}$$

۳.۷ مدل مرتبه پنجم (ادامه)

Fifth order model (cont.)

In the system analysis, the effect of pE'_q on E''_q transients is often neglected since $T''_{do} << T'_{do}$. The simplified D-winding eqn. will be

$$\boxed{T''_{d0} pE''_q \approx -E''_q + E'_q - \left(X'_d - X''_d\right)\cdot i_d}$$

(4) Winding Q volt. eqn.: ($u_Q \equiv 0$)

For $p\Psi_Q = -r_Q i_Q$, multiplying both sides by $-\frac{X_{aq}}{r_Q}$, using $T''_{d0} = \frac{X_Q}{r_Q}$ and $E''_d = \frac{X_{aq}}{X_Q}\Psi_Q$,we have:

$$\boxed{\begin{aligned} T''_{q0} pE''_d &= X_{aq} i_Q \\ &= -E''_d + (X_q - X''_q) i_q \end{aligned}}$$

(5) The rotor motion eqn's (p. u.)

$$\boxed{\begin{cases} T_J \dfrac{d\omega}{dt} + D(\omega - 1) = T_m - (\Psi_d i_q - \Psi_q i_d) \\ \qquad = T_m - [E''_q i_q + E''_d i_d - (X''_d - X''_q) i_d i_q)] \\ \dfrac{d\delta}{dt} = \omega - 1 \end{cases}}$$

Discussions:

- **The fifth model has $E'_q, E''_q, E''_d, \omega, \delta$ as state variables. There are 11 variables in the 7 equations: u_{dq}, i_{dq} $E'_q, E''_q, E''_d, E_f, T_m, \omega, \delta$. If E_f, T_m are known from exciter and prime mover models, and two network interface equations are known, the system can be solved in time domain.**

۳.۷ مدل مرتبه پنجم (ادامه)

Fifth order model (cont.)

- **Some relations are used in deriving the d-axis volt. equations. They are:**

$$
\begin{cases}
X'_d - X''_d = \dfrac{X_{ad}^2 X_{f1}^2}{X_f (X_f X_D - X_{ad}^2)} \\
X''_d - X_1 = X_{ad} // X_{f1} // X_{D1} = \dfrac{X_{ad} X_{f1} X_{D1}}{X_f X_D - X_{ad}^2} \\
X'_d - X_1 = X_{ad} // X_{f1} = \dfrac{X_{ad} X_{f1}}{X_f} \\
X_d - X'_d = \dfrac{X_{ad}^2}{X_f} \\
X_d - X''_d = \dfrac{X_{ad}^2 (X_{f1} + X_{D1})}{X_f X_D - X_{ad}^2} \\
X'_d = X_1 + X_{ad} // X_{f1} = X_d - \dfrac{X_{ad}^2}{X_f} \\
X''_d = X_1 + X_{ad} // X_{f1} // X_{D1} = X_1 + \dfrac{X_{ad} X_{f1} X_{D1}}{X_f X_D - X_{ad}^2} \\
\qquad = X_d - \dfrac{X_{ad}^2 (X_{D1} + X_{f1})}{X_f X_D - X_{ad}^2}
\end{cases}
$$

The equations are useful in converting practical parameters to Park's equation parameters or vice versa.

- **When consider two damping windings on the q-axis, i.e. Q and g (they correspond to d-axis D and f windings respectively, except that $u_g \equiv 0, u_f \neq 0$), the fifth model will change into 6th order model. E'_d, E''_d will be defined similar to E'_q and E''_q**

$$
\begin{cases}
E''_d = \dfrac{-X_{aq}}{X_g X_Q - X_{aq}^2} \left(X_{Ql} \Psi_g + X_{gl} \Psi_Q \right) \\
E'_d = -\dfrac{X_{aq}}{X_g} \Psi_g
\end{cases}
$$

۳.۷ مدل مرتبه پنجم (ادامه)

The 6th order model will be (in p. u.):

$$\begin{cases} u_d = E''_d + X''_q i_q - r_a i_d \\ u_q = E''_q - X''_d i_d - r_a i_q \\ T'_{d0} pE'_q = E_f - E_q \approx E_f - \left(E'_q - X_{dr} E'_q + X_{dr} E''_q\right) \\ T'_{q0} pE'_d = -E_d \approx -(E_d - X_{qr} E'_d + X_{qr} E''_d) \\ T''_{d0} pE''_q \approx -E''_q + E'_q - \left(X'_d - X''_d\right) i_d \\ T''_{q0} pE''_d \approx -E''_d + E'_d + \left(X'_q - X''_q\right) i_q \\ T_J \dfrac{d\omega}{dt} = T_m - [E''_q i_q + E''_d i_d - (X''_d - X''_q) i_d i_q] \\ \dfrac{d\delta}{dt} = \omega - 1 \qquad \text{where } X_{dr} = \dfrac{X_d - X'_d}{X''_d - X'_d}; \quad X_{qr} = \dfrac{X_q - X'_q}{X''_q - X'_q} \end{cases}$$

۷. مدل های گذرای الکترومکانیکی ماشین سنکرون (مدل عملی/ساده سازی شده)

۴.۷ مدل کلاسیک

Classical model

- **For the 4th order model , if we set**

$$pE'_d \equiv 0, pE'_q \equiv 0$$

the syn. mac. model will be:

$$\begin{cases} u_d = E'_d + X'_q i_q - r_a i_d \\ u_q = E'_q - X'_d i_d - r_a i_q \\ T_J \dfrac{d\omega}{dt} + D(\omega - 1) = T_m - \\ \qquad [E'_q i_q + E'_d i_d - (X'_d - X'_q) i_d i_q] \\ \dfrac{d\delta}{dt} = \omega - 1 \end{cases}$$

- **Rotor winding transients are neglected.**
- $E'_d = \text{const}$ **and** $E'_q = \text{const}$ **mean** $\Psi_g = \text{const.}, \Psi_f = \text{const.}$ **(no decay).**

- **If we further neglect the *rotor salience* and set** $X'_d = X'_q = X'$ **, the first two equations can be merged as**

$$\hat{u} = u_d + ju_q = (E'_d + jE'_q) - (r_a + jX'_d)(i_d + ji_q)$$
$$= \hat{E}' - (r_a + jX'_d) \cdot \hat{i} \qquad (a)$$

($(\hat{\ })$ means complex number with d-axis as real-axis.)

Since $(f_x + j \cdot f_y) e^{j(\frac{\pi}{2}+\delta)} = f_d + j \cdot f_q$

(f can be u, i or ψ etc.), multiplying the both sides of (a) by $e^{-j(\frac{\pi}{2}+\delta)}$ **, yield:**

$$\boxed{\begin{aligned} \dot{V}_G &= u_x + ju_y = (E'_x + jE'_y) - (r_a + jX'_d)(i_x + ji_y) \\ &= \dot{E}' - (r_a + jX'_d)\dot{I}_G \qquad (*) \end{aligned}}$$

where $\boxed{\dot{E}' = E' \angle \delta', E' = \text{const}}$

۷. مدل های گذرای الکترومکانیکی ماشین سنکرون (مدل عملی/ساده سازی شده)

۴.۷ مدل کلاسیک (ادامه)

Classical model (cont.)

- **The rotor motion equation will be :**

$$\begin{cases} T_J \dfrac{d\omega}{dt} + D(\omega - 1) = P_m - V_G I_G \cos\varphi_G \\ \dfrac{d\delta'}{dt} = \omega - 1 \end{cases} \quad \textbf{(**)}$$

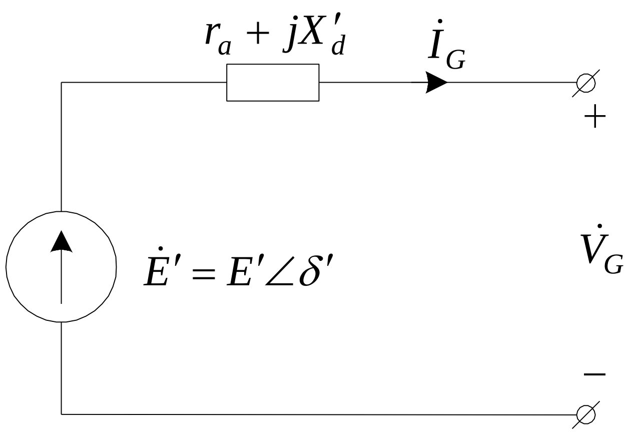
Equations* (*), (**) *constitute the <u>classical model</u>.

- **The significant advantage: with rotor salience neglected, the stator volt. eqn. is now a complex algebraic equation and easy to interface with network Y-matrix eqn.**

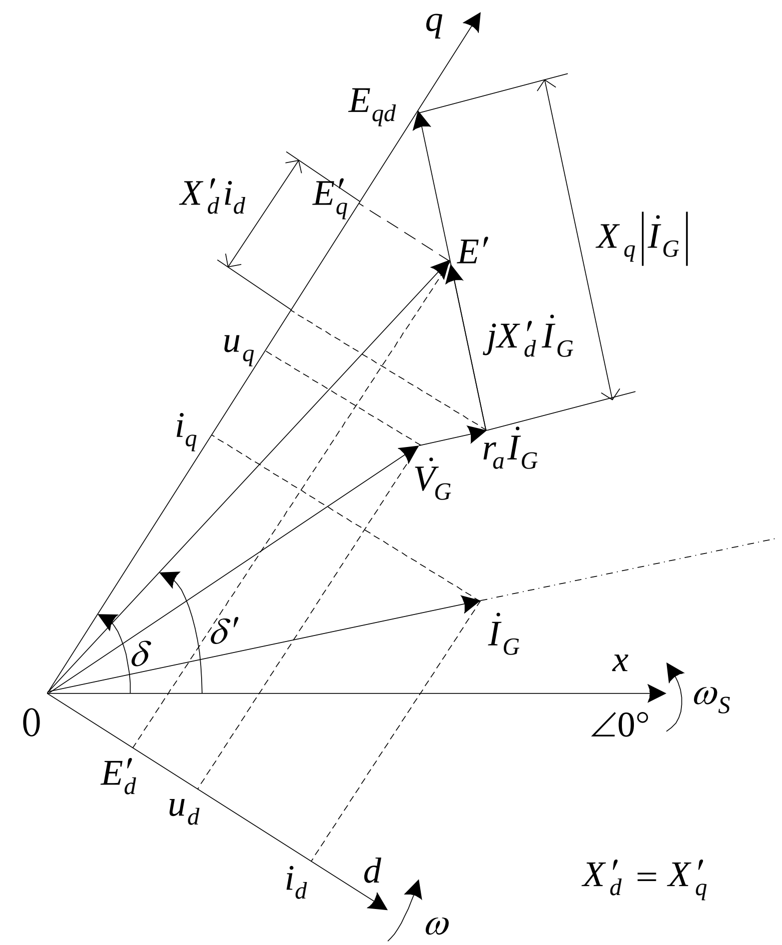
- **In classical model the angle δ' will be used to represent the rotor position and to judge synchronous stability.**
- **The assumption of E'≐const. does not mean the effect of excitation system is neglected. Rather, it means the excitation system is strong enough to keep $E' \approx \text{const.}$**
  - **For the fast-response, high-gain static excitation, the results of using classical model usually are conservative in stab. analysis;**
  - **For the traditional slow-response, low-gain exciter, the results may be optimistic.**

۷. مدل های گذرای الکترومکانیکی ماشین سنکرون (مدل عملی/ساده سازی شده)

۴.۷ مدل کلاسیک (ادامه)



$E' = \text{const.}$



Equivalent circuit and phasor relation for the classical model syn. machine.

۵.۷ خلاصه پارامترهای عملیاتی مدل های ماشین سنکرون

- **Section 1.8 derives several syn. machine practical models. The main assumption is neglecting stator winding transients $p\Psi_d = p\Psi_q = 0$ and set $\omega \approx 1$ in stator voltage equation.**
- **The 3rd and 5th order models are good for hydro-generators, whereas the 4th and 6th order models are good for steam-turbine generators. The classical model is good for analysis with lower accuracy without exciter transients.**
- **We should pay attention to the new variables defined.**
- **The initial values can be calculated by setting $p = d/dt = 0$. This is a pre-disturbance equilibrium point.**
- **The corresponding phasor diagram can be drawn according to the equilibrium point equations.**

۸. خلاصه مدل های ماشین سنکرون

- This chapter started from the "***ideal machine***" assumption, derived the syn. mac. in abc coordinates and in MKS (SI) units first;
- Then the Park's transformation is introduced and the corresponding syn. mac. model in dq0 coordinates and MKS (SI) units is obtained.
- After applying a properly selected 'X_{ad} per unit system', the syn. mac. p. u. model in dq0 coord. are developed. This is indeed an electro-magnetic transients (EMT) model usually called as Park's equations of the syn. mac.
- Stator transients are neglected to obtain simplified/practical syn. mac. 3rd, 4th, 5th, 6th order models and classical model. They are widely used in power system stability analysis.

۸. خلاصه مدل های ماشین سنکرون

- **The practical variables (E'_d, E'_q, E''_d, E''_q etc.) are introduced and practical parameters ($X_d, X_q, X'_d, X'_q, X''_d, X''_q, T'_{d0}, T'_{q0}, T''_{d0}, T''_{q0}$) are used in the simplified models.**
- **Later the interface of the machine with the network will be introduced for overall power system analysis.**
- **The models of excitation systems, speed governors and prime movers will also be introduced and connected with generators (through E_f and T_m).**
- **The syn. mac. is the *heart* of power systems with *extremely complicated* dynamic behavior which influences the *global* system performance and stability.**