APPENDIX II - RE-INITIALIZATION AT INSTANTS OF DISCONTINUITY

The numerical oscillations which occur in the voltages across inductances at points of discontinuities in di/dt, or in currents through capacitances at points of discontinuities in dv/dt, oscillate around the correct answer. These numerical oscillations can therefore be eliminated from the output if the output is smoothed, e.g., for the voltage across an inductance,

$$v_L(t)_{smoothed} = \frac{1}{2}\,[v_L(t) + v_L(t-\Delta t)] \tag{II.1}$$

If this smoothing algorithm is not just applied to the output, but added directly into the trapezoidal rule solution method, then we obtain

$$v_L(t)_{smoothed} = L\,\frac{i(t) - i(t-\Delta t)}{\Delta t} \tag{II.2}$$

which is simply the backward Euler method (Appendix I.9). B. Kulicke [15] recognized that such a backward difference quotient can be used to restart the solution process smoothly after a discontinuity, with the correct jumps in v_L across L, or in i_C through C. The backward Euler method does have absolute numerical stability, but it is not as accurate as the trapezoidal rule. It is therefore only used to restart the solution with new initial conditions. B. Kulicke also recognized that it is best to use half the step size with the backward Euler method to make the matrix [G] needed for that backward difference solution identical with the matrix [G] of Eq. (I.8), which is needed for the trapezoidal rule solution after the discontinuity anyhow. In what follows, Kulicke's method of re-initialization is explained in detail for the inductance; the derivations for the capacitance equations are analogous. There are three steps in Kulicke's method, namely

(a) interpolation to obtain variables at the point of discontinuity,

(b) network solution at $\Delta t/2$ after the discontinuity for the sole purpose of re-initialization,

(c) re-initialization of history terms at the point of discontinuity.

These three steps are then followed by the normal trapezoidal rule solution method.

(a) Interpolation

Assume that current is to be interrupted at current zero in a circuit breaker. The EMTP solution will give us answers at points 1, 2, 3 (Fig. II.1), with current zero crossing being discovered at point 3. Kulicke then uses linear interpolation to locate the zero crossing at point 0, and then calculates the values of all variables and history terms at that point 0, again with linear interpolation. The solution is then restarted at point 0, with the same Δt as before, but the uniform spacing along the time axis will be disturbed at that point, which would have to be recognized in the output. For Kulicke's method to work, e.g., by re-solving the network in point 3 with the switch open, is unclear at this time, and may require more work than linear interpolation. Interpolation would also help to eliminate overshooting of knee-points in piecewise linear elements.

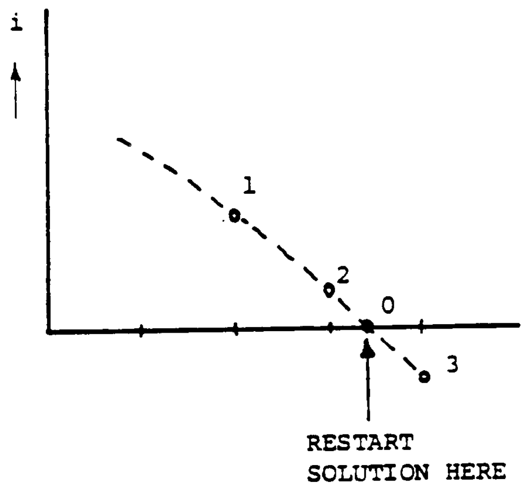

Fig. II.1 - Linear interpolation to locate point of discontinuity

(b) Network solution at $t_o+\Delta t/2$

Let us call the instant of discontinuity t_o, with the argument t_{o-} used for quantities before the jump, and t_{o+} for quantities after the jump (Fig. II.2). Let us also look at the jump in di/dt across an inductance, which is caused by the switch opening. Since no jump can occur in this

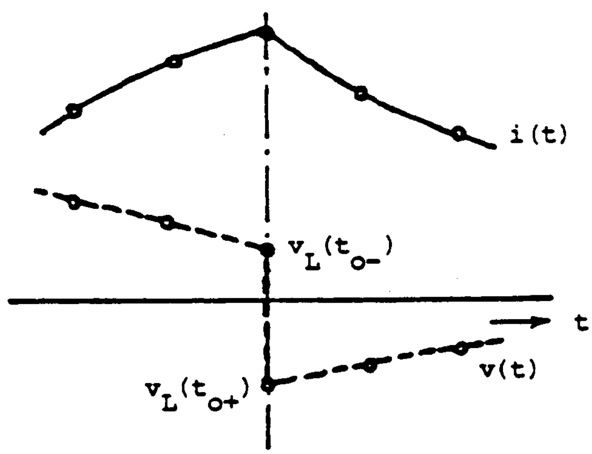

Fig. II.2 - Voltage and current at point of discontinuity

current, we know that

$$i(t_{o+}) = i(t_{o-}) \qquad \text{(II.3)}$$

If we now use the backward difference quotient of Eq. (II.2) to solve the network at $t_o+\Delta t/2$, then we obtain

$$i(t_o + \frac{\Delta t}{2}) = \frac{\Delta t}{2L} v_L(t_o + \frac{\Delta t}{2}) + i(t_{o-}) \qquad \text{(II.4)}$$

which is the same as Eq. (1.3a), except that the history term is now simply $i(t_{o-})$. For capacitance,the analogous equation would be

$$i_c(t_o + \frac{\Delta t}{2}) = \frac{2C}{\Delta t} v(t_o + \frac{\Delta t}{2}) - \frac{2C}{\Delta t} v(t_{o-}) \qquad \text{(II.5)}$$

again with a modified history term of $-2C*v(t_{o-})/\Delta t$ in this case. The solution at $t_o+\Delta t/2$ is therefore found in the usual way with Eq. (1.8b), after [G] has been modified to account for switch opening or for whatever caused the discontinuity, and after it has been re-triangularized. Notice that this matrix change and re-triangularization process is required anyhow, even if Kulicke's re-initialization method is not used. The only difference for this extra solution is in the right-hand side, since the history term is now $i(t_{o-})$ instead of hist from Eq. (1.3b), with an analogous modification of the capacitance history term.

(c) Re-initialization of history terms at t_{o+}

The extra network solution at $t_o+\Delta t/2$ is made for the sole purpose of re-initializing variables at t_{o+}. For the inductance, assuming a linear change in current between t_{o+} and $t_o+\Delta t/2$, the voltage at t_{o+} simply becomes

$$v_L(t_{o+}) = v_L(t_o + \Delta t/2) \qquad \text{(II.6)}$$

which would then be used in Eq. (1.3b) to calculate the history term required for the next, normal solution at $t_o+\Delta t$, for which the triangularized matrix has already been obtained in step (b).

Similarly, assuming a linear change of voltage across capacitances, the current at t_{o+} simply become

$$i_C(t_{o+}) = i_C(t_o + \Delta t/2) \qquad \text{(II.7)}$$