APPENDIX IV - ACTUAL VALUES VERSUS PER-UNIT QUANTITIES

The use of per-unit quantities has been customary for so many years in the electric power industry, that it is not always recognized that actual values can be used just as easily, and that the per-unit system may have outlived its usefulness. This writer sees no advantages in working with per-unit quantities, and feels much more comfortable with actual values.

The widespread use f per-unit quantities probably started with network analyzers in the 1930's. For power flow and short-circuit studies on network analyzers, per-unit quantities offered two advantages, namely scaling of impedances to values available on the analyzer, and the possibility of representing transformers as simple series impedances as long as their turns ratio was identical to the ratio of the base voltages on the two sides. Somewhat similar arguments for per-unit quantities could be made in the early days of digital computers with fixed-point arithmetic, where the order of magnitude of intermediate and final results had to be about the same. On modern computers with floating-point arithmetic, there is no reason, however, why one shouldn't work directly with actual values.

IV.1 Per-Unit Quantities

A per-unit quantity is the ratio of the actual value of a quantity to the base value of the same quantity [76, p. 482]. It has been customary to use one common base power S_{base} (apparent power) for the entire system (typically 100 MVA), and a different base voltage for each voltage level (e.g., V_{base} = 115 kV and V_{base} = 230 kV in a 115/230 kV system) as the base values. Then the per-unit quantities in a single-phase network are

$$
\begin{aligned}
I_{p.u.} &= I_{actual} \cdot \frac{V_{base}}{S_{base}} \\
V_{p.u.} &= \frac{V_{base}}{V_{base}} \\
Y_{p.u.} &= Y_{actual} \cdot \frac{V_{base}^2}{S_{base}} \\
Z_{p.u.} &= Z_{actual} \cdot \frac{S_{base}}{V_{base}^2}
\end{aligned}
\tag{IV.1}
$$

It may be safest to use these single-phase equations for three-phase networks as well. In wy-connected equipment, S_{base} would be the single-phase base power of one winding (e.g., 100/3 MVA) and Vase would be the base voltage across each winding, namely the phase-to-ground base voltage (e.g., 113/$\sqrt{3}$ kV and 230/$\sqrt{3}$ kV). In delta-connected equipment, S_{base} would again be the single-phase base power of each winding (e.g., 100/3 MVA), whereas the base voltage V_{base} across each winding would now be the phase-to-phase base voltage (e.g., 115 kV and 23 kV).

The following, well-known formulas with three-phase base values were developed for positive sequence power flow studies, where the distinction between wye- and delta-connections gets lost in the conversion from three-phase representations to equivalent single-phase representations for balanced operation:

$$I_{p.u.} = I_{actual} \cdot \sqrt{3}\ \frac{V_{base-phase-to-phase}}{S_{base-three-phase}}$$

$$V_{p.u.} = \frac{V_{actual}}{V_{base}}\ (numerator and\ denominator either\ both\ phase-to-phase or\ both phase-to-ground)$$

$$Y_{p.u.} = Y_{actual} \cdot \frac{(V_{base-phase-to-phase})^2}{S_{base-three-phase}} \qquad \text{(IV.2)}$$

$$Z_{p.u.} = \frac{1}{Y_{p.u.}}$$

Eq. (IV.2) cannot only be used for the conversion of positive sequence parameters, but for negative and zero sequence parameters as well, as shown in the example of Section IV.3.

Per-unit quantities, as ratios of actual to base values, are meaningless if the base values are not listed as part of the data as well. For example, the positive sequence series impedance of an overhead line is fully described by three actual values,

$$R'_{pos} + j\omega L'_{pos} = 0.05 + j0.40\ \Omega/km,\quad f = 60\ Hz$$

or if R'_{pos} and L'_{pos} are independent of frequency, by two values,

$$R'_{pos} = 0.05\ \Omega/km,\quad L'_{pos} = 1.061\ mH/km$$

On the other hand, the record for per-unit quantities consists of 5 values,

$R'_{pos} + j\omega L'_{pos} = 9.45 \bullet 10^{-5} + j75.61 \bullet 10^{-5}$ p.u., f = 60 Hz, S_{base} = 100 MVA (three phase), V_{base} = 230 kV (phase-to-phase).

With R'_{pos} and L'_{pos}, the frequency could be dropped from the record, but the time base should then be added,

$R'_{pos} = 9.45 \bullet 10^{-5}$ p.u., $L'_{pos} = 20.06 \bullet 10^{-7}$ p.u., S_{base} = 100 MVA (three-phase), V_{base} = 230 kV (phase-to-phase), t_{base} = 1s.

Adding the time base may seem superfluous, but there are stability programs which use cycles (of 60 Hz) as a time base, in which case $L'_{pos} = 12.03 \bullet 10^{-5}$ p.u., t_{base} = 1/60 s.

IV.2 Conversion from One Base to Another

If per-unit data is to be exchanged among utilities and manufacturers, then it is important to include the base values, especially if one party customarily uses base values which are different from those used by the other party. For example, a transformer manufacturer lists the short-circuit input impedance in per-unit based on the voltage and power nameplate ratings of the transformers,

$$Z_{p.u.}^{manufacturer} = Z_{actual} \cdot \frac{S_{nameplate}}{(V_{nameplate})^2} \tag{IV.3}$$

In a particular case, these base values might be 660 MVA (three-phase) and 241.5 kV (phase-to-phase). Utility companies generally use different base values (e.g., S_{base} = 100 MVA, V_{base} = 230 kV). By solving Eq. (IV.3) for Z_{actual} and using Eq. (IV.2) to get back to per-unit quantities, one obtains

$$Z_{p.u.}^{utility} = Z_{p.u.}^{manufacturer} \cdot \left(\frac{V_{nameplate}}{V_{base}}\right)^2 \cdot \frac{S_{base}}{S_{nameplate}} \tag{IV.4}$$

Obviously, the per-unit quantity of the manufacturer will be quite different from the one used by the utility company.

In general, the formulas for conversion from base "1" to base "2" are

$$\begin{aligned} I_{p.u.}^{base\ 2} &= I_{p.u.}^{base\ 1} \cdot \frac{S_{base\ 1}}{S_{base\ 2}} \cdot \frac{V_{base\ 2}}{V_{base\ 1}} \\ V_{p.u.}^{base\ 2} &= V_{p.u.}^{base\ 1} \cdot \frac{V_{base\ 1}}{V_{base\ 2}} \\ Y_{p.u.}^{base\ 2} &= Y_{p.u.}^{base\ 1} \cdot \frac{S_{base\ 1}}{S_{base\ 2}} \cdot \left(\frac{V_{base\ 2}}{V_{base\ 1}}\right)^2 \end{aligned} \tag{IV.5}$$

IV.3 Actual Values Referred to One Side Transformer

The advantage of representing transformers as simple series impedances with per-unit quantities, as long as their turns ratio is identical to the ratio of the base voltages, exists with actual values as well, if the quantities on one side are referred to the other side. In the example of Fig. IV.1, quantities on the low voltage side are referred to the high voltage side with

$$\begin{aligned} I_{high} &= I_{low} \cdot \frac{15}{241.5} \\ V_{high} &= V_{low} \cdot \frac{241.5}{15} \end{aligned} \tag{IV.6}$$

$$\begin{aligned} Y_{high} &= Y_{low} \cdot \left(\frac{15}{241.5}\right)^2 \\ Z_{high} &= Z_{low} \cdot \left(\frac{241.5}{15}\right)^2 \end{aligned}$$

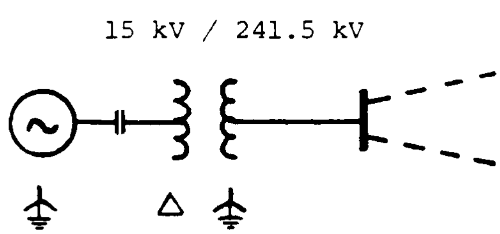
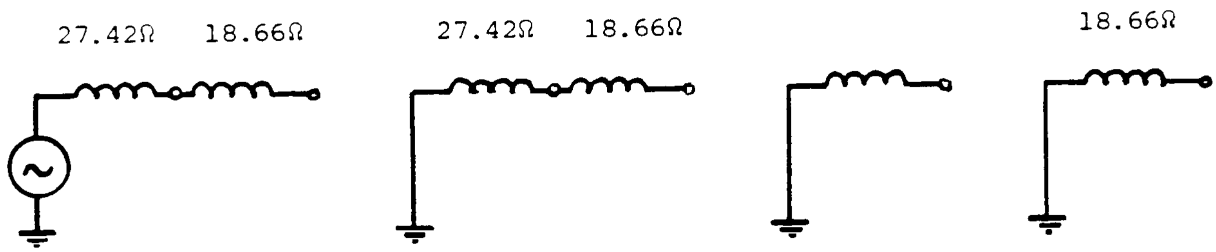


Fig. IV. 1 - Generator with step-up transformer. Generator data: $X_d'' = X_q'' = 10\%$ based on rating of 13.8 kV and 180 MVA. Transformer data: $X_{pos} = X_{zero} = 8\%$ based on rating of 15/241.5 kV and 250 MVA



(a) positive (b) negative (c) zero

Fig. IV.2 - Positive, negative, and zero sequence networks seen from high side

This conversion to the high side is advantageous if the generator and step-up transformer are to be replaced by a Thevenin equivalent circuit seen from the high side. With the data of Fig. IV.1, the positive, negative and zero sequence networks of Fig. IV.2 are obtained as follows: For the transformer,

$$X_{actual} = 0.08\ \frac{241.5^2}{250}\ \Omega = 18.66\ \Omega \quad (\textit{seen high side})$$

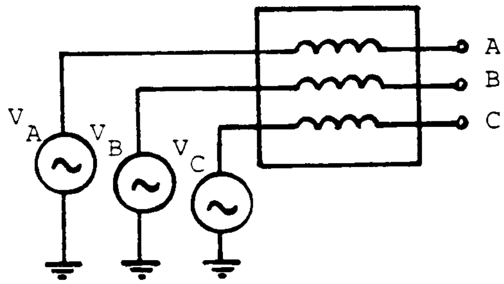
and for the generator,

$$X_{actual} = 0.10\ \frac{13.8^2}{180}\ \Omega = 0.1058\ \Omega \quad (\textit{seen high side})$$

or

$$X_{actual} = 0.1058\ \left(\frac{241.5}{15}\right)^2\ \Omega = 27.42\ \Omega \quad (\textit{seen high side})$$

Note that the delta connection provides a short-circuit for the zero sequence currents (Fig. IV.2(c)). With $X_{pos} = X_{neg} = 46.08\ \Omega$, $X_{zero} = 18.66\ \Omega$, the final three-phase Thevenin equivalent circuit of Fig. IV.3 is obtained by converting the sequence reactances to self and mutual reactances with Eq. (3.4). The amplitude of the Thevenin voltages is set equal to the voltage seen on the high side for the particular operating condition, which may be 230 kV phase-to-phase in a particular case.



3x3 reactance matrix

$$\begin{bmatrix} X_s & X_m & X_m \\ X_m & X_s & X_m \\ X_m & X_m & X_s \end{bmatrix}$$

Fig. IV.3 - Three-phase Thevenin equivalent circuit. Symmetric voltage sources V_A, V_B, V_C with RMS amplitude of 230/√3 kV; x_s = 36.94 Ω, X_m = -9.14 Ω

One could also use per-unit quantities for the Thevenin equivalent circuits of Fig. IV.2, with the transformer ratings as base values. In that case, X = 0.08 p.u. for the transformer, and with Eq. (IV.5),

$$X = 0.10 \bullet (250/180) \bullet (13.8/15)^2 \text{ p.u.} = 0.1176 \text{ p.u.}$$

for the generator. Then, $X_{pos} = X_{neg}$ = 0.19756 p.u., X_{zero} = 0/08 p.u., which leads to $X_{pos} = X_{neg}$ = 46.08 Ω, X_{zero} = 18.66 Ω with S_{base} = 250 MVA (three-phase) and V_{base} = 241.5 kV (phase-to-phase).

IV.4 Advantages of Actual Values

This writer prefers actual values over per-unit quantities for the following reasons:

(1) Confusion may arise with per-unit quantities because the base values are not always clearly stated. This confusion cannot arise with actual values.

(2) The data record is shorter for actual values, as shown in the last paragraph of Section IV.1, even if S_{base} in the per-unit record is left off, with the understanding that it is always 100 MVA.

(3) Actual values are fixed characteristics of a piece of equipment, independent of how this equipment is being used. This is not true for per-unit quantities: If a 500 kV shunt reactor is temporarily used on a 345 kV circuit, its per-unit values based on 500 kV would have to be converted to a base of 345 kV.

(4) Since the ratio of transformer voltage ratings is not always equal to the ratio of base voltages, one has to allow for "off-nominal" turns ratios (unequal 1:1) with per-unit quantities anyhow. If one has to allow for any ratio, then a ratio of 1:1.05 for per-unit quantities is neither easier nor more difficult to handle than a ratio of 15 kV:241.5 kV for actual values. Therefore, one might as well use actual values. Furthermore, the simple series impedance representation of transformers with per-unit ratios of 1:1 (Fig. 3.3(c) with t = 1.0) can seldom be used in EMTP studies. For example, a three-phase bank of single-phase transformers in wye-delta connection would require a 2x2 [Y]-matrix model for each transformer, or alternatively, an equivalent circuit representation with uncouple reactances as shown in Fig. 3.3(b). The case of t = 1.0 offers no advantage whatsoever in that six-branch circuit.

(5) If test data is available in per-unit quantities, e.g., for generators or transformers, then conversions are even necessary for per-unit values, since the base values do in general not agree with the nameplate ratings. Therefore, one might as well convert to actual values. Furthermore, the EMTP does this conversion in most cases anyhow, e.g., in the main program in the case of generators, or in supporting routines in the case of transformers.

(6) All digital computers use floating-point arithmetic nowadays, and therefore accept numbers over a wide range of magnitudes. Therefore, the numbers do not have to be of the same order of magnitude, and a turns ratio of 15 kV:241.5 kV causes no more problems than a turns ratio of 1:1.05.

Sometimes the question is raised whether solutions with per-unit values aren't possibly more accurate than solutions with actual values. Many years ago on computers with fixed-point arithmetic, per-unit values may indeed have produced more accurate than the other. To show this, let us look at the steady-state solution of a single-phase network with nodal equations,

$$[Y_{actual}]\,[V_{actual}] = [I_{actual}] \tag{IV.7}$$

where $[I_{actual}]$ is given, and $[V_{actual}]$ is to be found. In general, the network will have two or more voltage levels, which will be taken into account in $[Y_{actual}]$ with the proper transformer turns ratios. To convert Eq. (IV.7) to per-unit quantities, the base voltages are first defined in the form of a diagonal matrix,

$$[V_{base}] = \begin{bmatrix} V_{base\ 1} & & & & \\ & V_{base\ 2} & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & V_{base\ N} \end{bmatrix} \tag{IV.8}$$

with the possibility of each node having its own base voltage. In reality of course, all nodes within one voltage level would have the same base value. With S_{base} being the same for the entire network, the current and voltage vectors in per-unit and actual values are related by

$$[I_{p.u.}] = (1/S_{base})[V_{base}][I_{base}] \tag{IV.9}$$

$$[V_{actual}] = [V_{base}][V_{p.u.}] \tag{IV.10}$$

Premultiplying Eq. (IV.7) with $[V_{base}]/S_{base}$, and replacing $[V_{actual}]$ with Eq. (IV.10) will produce the per-unit equations

$$[Y_{p.u.}][V_{p.u.}] = [I_{p.u.}] \tag{IV.11}$$

with

$$[Y_{p.u.}] = (1/S_{base})[V_{base}][Y_{actual}][V_{base}] \tag{IV.12}$$

Therefore, the conversion from actual to per-unit values consists of the transformation of the coefficient matrix $[Y_{actual}]$ into $[Y_{p.u.}]$ with Eq. (IV.12). This transformation is very simple since $[V_{base}]$ is a diagonal matrix: Aside from dividing all elements by the constant S_{base}, each row i (i = 1, 2, ... N) is multiplied by $V_{base\text{-}i}$, and each row k (k = 1, 2, ...N) is multiplied by $V_{base\text{-}k}$. This is essentially a scaling operation.

This scaling operation has no influence on the solution process if pivoting[1)] is not used, but it may influence the accumulation of round-off errors. This influence on round-off errors is difficult to assess. For a system of linear equations, the following can however be said [77, p.39]: If scaling is done in such a way that it changes only the exponent of the floating-point number (e.g., by using S_{base} = 128 MVA or 2^7, $V_{base\ 1}$ = 128 kV or 2^7 and $V_{base\ 2}$ = 512 kV or 2^9 on a computer using base 2 for the exponents), and if the order of eliminations is not changed, then the scaled (per-unit) coefficients will have precisely the same mantissas, and all intermediate and final results will have precisely the same number of significant digits. Therefore, it is reasonable to assume that scaling will neither improve nor degrade the accuracy of the solutions. M.D. Crouch of Bonneville Power Administration has shown that this assumption is correct for power flow solutions with 48 bit precision.

IV.5 Per-Unit Voltage with Actual Impedances

Sometimes, overvoltage studies are made with impedances in actual values, but with voltage source amplitudes scaled to 1.0 p.u. or similar values,

$$V_{p.u.} = V_{actual} / V_{base}$$

This produces overvoltages expressed in per-unit, which is often preferred in insulation co-ordination studies. If there are no nonlinear elements in the network, then this approach is quite straightforward. Actual values can be obtained from the per-unit values by multiplying per-unit voltages and currents with V_{base}, and per-unit power with V^2_{base}.

Some care is required, however, if the network contains nonlinearities. For nonlinear resistances or inductances defined point-by-point with pairs of values v, i or ψ, i both values of each pair must be divided by V_{base} in the input data. If the nonlinearities are defined by their piecewise linear slopes R_1, R_2, ... or L_1, L_2, ..., and by the "knee-point" v_1, v_2, ... or ψ_1, ψ_2, ..., only these knee-point values must be divided by V_{vase} in the input data.

[1)]Pivoting is generally not used in the EMTP, except in some subroutines for the inversion of small matrices of couple branches.