APPENDIX V - RECURSIVE CONVOLUTION

Consider the convoluted integral

$$s(t) = \int_T^\infty f(t-u)e^{-p(u-T)}du \tag{V.1}$$

to be found at time t, with s(t-Δt) already known from the preceding time step. This known value can be expressed as

$$s(t-\Delta t) = e^{p\Delta t} \int_{T+\Delta t}^\infty f(t-u)e^{-p(u-T)}\ du \tag{V.2}$$

by simply substituting a new variable $u_{new} = u + \Delta t$ into Eq. (V.1). At the same time, the integration in Eq. (V.1) can be done in two parts,

$$s(t) = \int_T^{T+\Delta t} f(t-u)e^{-p(u-T)}\ du + \int_{T+\Delta t}^\infty f(t-u)e^{-p(u-T)}\ du$$

which becomes

$$s(t) = \int_T^{T+\Delta t} f(t-u)e^{-p(u-T)}\ du + e^{-p\Delta t} \cdot s(t-\Delta t) \tag{V.3}$$

with Eq. (V.2). Therefore, s(t) is found recursively from s(t-Δt) with a simple integration over one single time step Δt. If we assume that f varies linearly between t-T-Δt and t-T, then [94]

$$s(t) = c_1 \bullet s(t-\Delta t) + c_2 \bullet f(t-T) + c_3 \bullet f(t-T-\Delta t) \tag{V.4}$$

with the three constants

$$c_1 = e^{-p\Delta t}$$

$$c_2 = \frac{1}{p} - \frac{1}{\Delta t\ p^2}(1 - e^{-p\Delta t}) \tag{V.5}$$

$$c_3 = -\frac{1}{p}e^{-p\Delta t} + \frac{1}{\Delta t\ p^2}(1 - e^{-p\Delta t})$$