APPENDIX VI - TRANSIENT AND SUBTRANSIENT PARAMETERS OF SYNCHRONOUS MACHINES

The derivations are the same for the direct and quadrature axis. They will therefore only be explained for the direct axis. Furthermore, it is assumed that field structure quantities have been rescaled (in physical or p.u. quantities) in such a way that the mutual inductances among the three windings d, f and D are all equal, as explained in Section 8.2, except that the subscript "m" (fore modified) is dropped from Eq. (8.15a), to simplify the notation. The equations with this simpler notation are then

$$\begin{bmatrix} \lambda_d \\ \lambda_f \\ \lambda_D \end{bmatrix} = \begin{bmatrix} L_d & M & M \\ M & L_{ff} & M \\ M & M & L_{DD} \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_D \end{bmatrix} \qquad \text{(VI.1)}$$

and

$$-\begin{bmatrix} d\lambda_f /dt \\ d\lambda_D /dt \end{bmatrix} = \begin{bmatrix} R_f & 0 \\ 0 & R_D \end{bmatrix} \begin{bmatrix} i_f \\ i_D \end{bmatrix} + \begin{bmatrix} v_f \\ 0 \end{bmatrix} \qquad \text{(VI.2)}$$

In the past, it has often been assumed that the damper windings can be ignored for the transient effects, which are associated with the open-circuit or short-circuit time constants T_{do}' or T_d'. In earlier EMTP versions, this assumption was made for the definition of the transient reactance X'_d with Eq. (VI.4), while for the definition of the time constants the damper winding effects were always included. In later EMTP versions, the definition of the time constants as well as of the transient reactance takes damper winding effects into account.

VI.1 Transient Parameters with Only One Winding on the Field Structure

If there is no damper winding, or if the damper winding were to be ignored, then there is only the field winding f on the field structure[1]). The field current i_f can then be eliminated from the second row of Eq. (VI.1)

$$i_f = \frac{\lambda_f}{L_{ff}} - \frac{M}{L_{ff}} i_d$$

which, when inserted into the first row, produces

[1])This is true for the direct axis. In the quadrature axis, the analogous assumption is that either the g- or the Q- winding is missing.

$$\lambda_d = (L_d - \frac{M^2}{L_{ff}})\, i_d + \frac{M}{L_{ff}} \lambda_f \qquad \text{(VI.3)}$$

The flux λ_f cannot change instantaneously after disturbance, and can therefore be regarded as constant at first. The transient inductance which describes the flux/current relationship in the armature immediately after the disturbance is therefore

$$\lambda_d' = L_d - \frac{M^2}{L_{ff}} \qquad \text{(VI.4)}$$

The open-circuit time constant T_{do}', which describes the rate of change of flux λ_f for open-circuit conditions ($i_d = 0$) is obtained from Eq. (VI.2) as

$$T_{do}' = L_{ff} / R_f \qquad \text{(VI.5)}$$

As shown in the next section, the definitions of both L_d' and T_{do}' change in the presence of a damper winding.

VI.2 Subtransient and Transient Time Constants with Two Windings on the Field Structure

The open-circuit time constants are found by solving the equations for the currents i_f, i_D. By substituting the last two rows of Eq. (VI.1),

$$\begin{bmatrix} \lambda_f \\ \lambda_D \end{bmatrix} = \begin{bmatrix} M \\ M \end{bmatrix} i_d + \begin{bmatrix} L_{ff} & M \\ M & L_{DD} \end{bmatrix} \begin{bmatrix} i_f \\ i_D \end{bmatrix} \qquad \text{(VI.6)}$$

into Eq. (VI.2), and by setting $i_d = 0$ for the open-circuit condition, we get

$$\begin{bmatrix} \dfrac{di_f}{dt} \\ \dfrac{di_D}{dt} \end{bmatrix} = \frac{i}{L_{ff} L_{DD} - M^2} \begin{bmatrix} -L_{DD} & M \\ M & -L_{ff} \end{bmatrix} \left[\begin{bmatrix} R_f & 0 \\ 0 & R_D \end{bmatrix} \begin{bmatrix} i_f \\ i_D \end{bmatrix} + \begin{bmatrix} v_f \\ 0 \end{bmatrix} \right] \qquad \text{(VI.7)}$$

The field winding voltage v_f is the forcing function in this equation, while the open-circuit time constants must be the negative reciprocals of the eigenvalues of the matrix relating the current derivatives to the currents in Eq. (VI.7)[2]. They are therefore found by solving

$$\left(-R_f \frac{L_{DD}}{L_{ff} L_{DD} - M^2} + \frac{1}{T} \right) \left(\frac{-R_D L_{ff}}{L_{ff} L_{DD} - M^2} + \frac{1}{T} \right) - \frac{R_f R_D M^2}{(L_{ff} L_{DD} - M^2)^2} = 0$$

---

2) The theory is explained in Appendix I.1, where it is shown that there will be two modes of the oscillations defined by terms multiplied with $e^{\lambda 1 t}$ and $e^{\lambda 2 t}$ (λ = eigenvalues). Since the eigenvalues are real and negative here, their negative reciprocals define the two time constants.

for T. The results are

$$\begin{matrix} T_{do}^{\prime} \\ T_{do}^{\prime\prime} \end{matrix} = \frac{1}{2}\left(\frac{L_{ff}}{R_f} + \frac{L_{DD}}{R_D}\right) \pm \frac{1}{2}\sqrt{\left(\frac{L_{ff}}{R_f} - \frac{L_{DD}}{R_D}\right)^2 + 4\frac{M^2}{R_f\,R_D}} \qquad \text{(VI.8)}$$

with the positive sign of the root for T_{do}', and negative sign for T_{do}''. For some derivations, the sums and differences of these two time constants are more useful because of their simpler form,

$$T_{do}^{\prime} + T_{do}^{\prime\prime} = \frac{L_{ff}}{R_f} + \frac{L_{DD}}{R_D} \qquad \text{(VI.9a)}$$

$$T_{do}^{\prime}\,T_{do}^{\prime\prime} = \frac{L_{ff}\,L_{DD} - M^2}{R_f\,R_D} \qquad \text{(VI.9b)}$$

For the short-circuit time constants, i_d in Eq. (VI.6) is no longer zero. Instead, we express it as a function of i_f, i_D and $\underline{\lambda}_d$ with the first row of Eq. (VI.1),

$$M\,i_d = \frac{M}{L_d}\lambda_d - \frac{M^2}{L_d}i_f - \frac{M^2}{L_d}d_D \qquad \text{(VI.10)}$$

which, when inserted into (VI.6) and (VI.2), produces

$$\begin{bmatrix} \dfrac{di_f}{dt} \\ \dfrac{di_D}{dt} \end{bmatrix} = \frac{i}{L_{ffs}\,L_{DDs} - M_s^2}\begin{bmatrix} -L_{DDs} & M_s \\ M_s & -L_{ffs} \end{bmatrix}\left[\begin{bmatrix} R_f & 0 \\ 0 & R_D \end{bmatrix}\begin{bmatrix} i_f \\ i_D \end{bmatrix} + \begin{bmatrix} v_f + \dfrac{M}{L_d}\cdot\dfrac{d\lambda_d}{dt} \\ \dfrac{M}{L_d}\cdot\dfrac{d\lambda_d}{dt} \end{bmatrix}\right] \qquad \text{(VI.11)}$$

with subscript "s" added to define the inductances modified for short-circuit conditions,

$$L_{ffs} = L_{ff} - M^2/L_d, \quad L_{DDs} = L_{DD} - M^2/L_d, \quad M_s = M - M^2/L_d \qquad \text{(VI.12)}$$

Taking v_f and $d\lambda_d/dt$ as the forcing functions, we obtain the short-circuit time constants as the negative reciprocals of the eigenvalues of the matrix in Eq. (VI.11). Since this equation has the same form as Eq. (VI.7), we can immediately give the answer as

$$\begin{matrix} T_{d}^{\prime} \\ T_{d}^{\prime\prime} \end{matrix} = \frac{1}{2}\left(\frac{L_{ffs}}{R_f} + \frac{L_{DDs}}{R_D}\right) \pm \frac{1}{2}\sqrt{\left(\frac{L_{ffs}}{R_f} - \frac{L_{DDs}}{R_D}\right)^2 + 4\frac{M_s^2}{R_f\,R_D}} \qquad \text{(VI.13)}$$

with the positive sign of the root for T_d', and the negative sign for T_d''. Again, their sums and differences are easier to work with,

$$T_d' + T_d'' = \frac{L_{ffs}}{R_f} + \frac{L_{DDs}}{R_D} \qquad \text{(VI.14a)}$$

$$T_d' \, T_d'' = \frac{L_{ffs} \, L_{DDs} - M_s^2}{R_f \, R_d} \qquad \text{(VI.14b)}$$

There is also a useful relationship between the open- and short-circuit time constants,

$$T_d' \, T_d'' = T_{do}' \, T_{do}'' \frac{L_d''}{L_d} \qquad \text{(VI.14c)}$$

which can easily be derived from Eq. (VI.9b) and (VI.14b) by using the definition for L_d'' given later in Eq. (VI.16).

It is not quite correct to treat $d\lambda_d/dt$ in Eq. (VI.11) as a forcing function, unless R_a is ignored. Only for $R_a = 0$ are the fluxes known from the first two rows of Eq. (8.9) as

$$\lambda_d = \lambda_d(0)\cos\omega t, \quad \lambda_q = \lambda_q(0)\sin\omega t$$

with $v_d=0$, $v_q=0$ because of the short circuit. In practice, R_a is not zero, but very small. Then the fluxes are still known with fairly high accuracy if $\lambda_d(0)$ is replaced by $\lambda_d(0)e^{-\alpha t}$, where

$$\alpha = \frac{1}{T_a} = \frac{\omega \, R_a}{2}\left(\frac{1}{X_d''} + \frac{1}{X_q''}\right) \qquad \text{(VI.14d)}$$

is the reciprocal of the time constant for the decaying dc offset in the short-circuit current [105]. If R_a were unrealistically large, then the time constants could no longer be defined independently for each axis, and the data conversion would be much more complicated than the one described in Section VI.4.

VI.3 Subtransient and Transient Reactances with Two Windings on the Field Structure

The subtransient reactance can easily be defined by knowing that the fluxes λ_f, λ_D cannot change immediately after the disturbance. By treating them as constants, we can express i_f, i_D as a function of i_d with Eq. (VI.6), which after insertion into the first row of Eq. (VI.1), produces

$$\lambda_d = \left(L_d - M^2 \frac{L_{ff}+L_{DD}-2M}{L_{ff}\, L_{DD}-M^2} \right) i_d + \frac{M}{L_{ff}L_{DD}-M^2}\left[(L_{DD}-M)\lambda_f + (L_{ff}-M)\lambda_D\right] \qquad \text{(VI.15)}$$

By definition, the term relating λ_d to i_d must be the subtransient inductance,

$$L_d'' = L_d - M^2 \frac{L_{ff} + L_{DD} - 2M}{L_{ff}\, L_{DD} - M^2} \qquad \text{(VI.16)}$$

To obtain the definition of the transient reactance is more complicated. For many years people have simply assumed that the damper winding currents have already died out after the subtransient period is over, and have used

Eq. (VI.4). Canay has recently shown, however, that this assumption can lead to noticeable errors [104], and that the data conversion is just as easy without this simplification. For the data given in the first IEEE benchmark model for subsynchronous resonance [74], 80% of the current associated with the transient time constant T_d' flows in the field winding, and another 20% in the damper winding after a short-circuit (values obtained while verifying the theory for this section). Ignoring the damper winding for the definition of X_d' would therefore produce errors in the field structure as well as in the armature currents.

Adkins [105] and others derive the transient reactance with Laplace transform techniques. First, Eq. (VI.2) is solved for the currents, after replacing the fluxes with Eq. (VI.6), which leads to the s-domain expression for their sum,

$$M\left(I_f(s)+I_D(s)\right) = \frac{-sM^2(R_f+sL_{ff}+R_D+sL_{DD}-2sM)}{(R_f+sL_{ff})\,(R_D+sL_{DD})-s^2M^2}\; I_d(s) \;+ f\left(V_f(s)\right)$$

where $f(V_f)$ is some function of the field voltage which is not of interest here. Inserting this into the first row of Eq. (VI.1) produces

$$\Lambda_d(s) = \left(L_d - \frac{sM^2(R_f+sL_{ff}+R_D+sL_{DD}-2sM)}{(R_f+sL_{ff})\,(R_D+sL_{DD})-s^2M^2}\right) I_d(s) \;+ f\left(V_f(s)\right)$$

with the expression in parentheses being the operational inductance $L_d(s)$,

$$\Lambda_d(s) = L_d(s)\, I_d(s) + f(V_f(s)) \tag{VI.17}$$

Through some lengthy manipulations it can be shown that it has the simple form

$$L_d(s) = L_d\,\frac{(1+sT_d')\,(1+sT_d'')}{(1+sT_{do}')\,(1+sT_{do}'')} \tag{VI.18}$$

The basic definition of L_d' and L_d'' in the IEEE and IEC standards is

$$\frac{1}{L_d(s)} = \frac{1}{L_d} + \left(\frac{1}{L_d'}-\frac{1}{L_d}\right)\frac{sT_d'}{1+sT_d'} + \left(\frac{1}{L_d''}-\frac{1}{L_d'}\right)\frac{sT_d''}{1+sT_d''} \tag{VI.19a}$$

in the s-domain, or

$$\frac{1}{L_d(t)} = \frac{1}{L_d} + \left(\frac{1}{L_d'}-\frac{1}{L_d}\right)e^{-t/T_d'} + \left(\frac{1}{L_d''}-\frac{1}{L_d'}\right)e^{-t/T_d''} \tag{VI.19b}$$

in the time domain[3]. The transient reactance can therefore be found by expanding $1/L_d(s)$ from Eq. (VI.18) into partial fractions,

$$\frac{1}{L_d(s)} = \frac{1}{L_d} - \frac{1}{L_d} \cdot \frac{(T_d^{\prime}-T_{do}^{\prime})\,(T_d^{\prime}-T_{do}^{\prime\prime})}{T_d^{\prime}\,(T_d^{\prime}-T_d^{\prime\prime})} \cdot \frac{s\,T_d^{\prime}}{1+s\,T_d^{\prime}} - \frac{1}{L_d} \cdot \frac{(T_d^{\prime\prime}-T_{do}^{\prime})\,(T_d^{\prime\prime}-T_{do}^{\prime\prime})}{T_d^{\prime\prime}\,(T_d^{\prime\prime}-T_d^{\prime})} \cdot \frac{s\,T_d^{\prime\prime}}{1+s\,T_d^{\prime\prime}} \tag{VI.20}$$

and by equating the coefficient of the second term in Eq. (VI.19a), which describes what is read off the oscillogram in the short-circuit test, with the coefficient of the second term in Eq. (VI.20), which describes the mathematical model. Then, with the help of Eq. (VI.14c), we obtain

$$T_d^{\prime}\frac{L_d}{L_d^{\prime}} + T_d^{\prime\prime}\left(1 - \frac{L_d}{L_d^{\prime}} + \frac{L_d}{L_d^{\prime\prime}}\right) = T_{do}^{\prime} + T_{do}^{\prime\prime} \tag{VI.21}$$

for the definition of the transient reactance or inductance.

Laplace transform techniques are downgraded in Appendix I for EMTP implementation, but for the type of analytical work just described they are quite useful. The transient reactance can also be derived using the eigenvalue/eigenvector approach of Eq. (I.5). The starting point for that approach is Eq. (VI.11), which has the general form

$$\left[\frac{dx}{dt}\right] = [A]\ [x]\ +[g(t)]$$

of Eq. (I.1), with the solution

$$[x(t)] = [M]\ [e^{\Lambda t}]\ [M]^{-1}\ [x(0)] + \int_0^t [M]\ [e^{\Lambda(t-u)}]\ [M]^{-1}\ [g(u)]\ du \tag{VI.22}$$

If we treat the variables as deviations from the pre-short-circuit steady-state values, then the initial conditions for these "deviation variables" are zero, and the first term in the above solution with [x(0)] drops out. This is in line with the usual practice of assuming zero initial conditions in Laplace transform techniques. What is of interest then is the expression under the integral. To obtain it, we must first find the eigenvector matrix [M] of

$$[A] = \frac{1}{L_{ffs}L_{DDs} - M_s^2}\begin{bmatrix} -L_{DDs}R_f & M_sR_D \\ M_sR_f & -L_{ffs}R_D \end{bmatrix} \tag{VI.23}$$

which is

[3]These definitions are used to read the inductance and time constant values from the oscillograms of the short-circuit test.

$$[M] = \begin{bmatrix} \dfrac{M_s}{R_f} & \dfrac{L_{ffs}}{R_f} - T_d^{/} \\ \dfrac{L_{DDs}}{R_D} - T_d^{//} & \dfrac{M_s}{R_D} \end{bmatrix} \qquad \text{(VI.24a)}$$

with its inverse

$$[M]^{-1} = \frac{1}{(T_d^{/} - \dfrac{L_{ffs}}{R_f})\ (T_d^{/} - T_d^{//})} \begin{bmatrix} \dfrac{M_s}{R_D} & T_d^{/} - \dfrac{L_{ffs}}{R_f} \\ T_d^{//} - \dfrac{L_{DDs}}{R_D} & \dfrac{M_s}{R_f} \end{bmatrix} \qquad \text{(VI.24b)}$$

That $[M][M]^{-1}$ = unit matrix can easily be verified by knowing that $T_d'' - L_{DDs}/R_D = L_{ffs}/R_f - T_d'$ from Eq. (VI.14a). The forcing function vector [g(t)] is

$$[g(t)] = \frac{M}{L_d T_d^{/} T_d^{//} R_f R_D} \begin{bmatrix} M_s - L_{DDs} \\ M_s - L_{ffs} \end{bmatrix} \frac{d\lambda_d}{dt} \qquad \text{(VI.25)}$$

The matrix with exponentials in Eq. (VI.22) contains the two diagonal elements $e^{-(t-u)/Td'}$ and $e^{-(t-u)/Td''}$. Since we are only interested in the part associated with the transient time constant T_d', we ignore the parts containing T_d'' and obtain

$$[M]\ [e^{\Lambda(t-u)}]\ [M]^{-1} = \frac{1}{T_d^{/} - T_d^{//}} \begin{bmatrix} \dfrac{L_{ffs}}{R_f} - T_d^{//} & \dfrac{M_s}{R_f} \\ \dfrac{M_s}{R_D} & \dfrac{L_{DDs}}{R_D} - T_d^{//} \end{bmatrix} e^{-(t-u)/T_d^{/}} + [a\ 2x2\ matrix] e^{-(t-u)/T_d^{//}} \qquad \text{(VI.26)}$$

Then

$$\begin{bmatrix} i_f - transient \\ i_D - transient \end{bmatrix} = \begin{bmatrix} product\ of\ matrix\ and\ vector \\ from\ (VI.26)\ and\ (VI.25) \end{bmatrix} \cdot \int_0^t e^{-(t-u)/T_d^{/}} \frac{d\lambda_d}{dt} du$$

which produces the 80%/20% split in the two field structure currents for the IEEE benchmark case mentioned at the beginning of this section, when numerical values are inserted. Since

$$i_d = \frac{1}{L_d} \lambda_d - \frac{M}{L_d} (i_f + i_D)$$

the sum of i_f and i_D, after multiplication with $-M/L_D$, will give us the transient part of i_d associated with $T_{d'}$.

$$i_{d\text{-}transient} = -\frac{1}{L_d}\frac{(T_d^{/}-T_{do}^{/})\ (T_d^{/}-T_{do}^{//})}{T_d^{/}\ (T_d^{/}+T_d^{//})}\int_0^t e^{-(t-u)/T_d^{/}}\ \frac{d\lambda_d}{dt}\ du \qquad \text{(VI.27)}$$

By comparing the coefficient in front of the integral with the coefficient of the second term in Eq. (VI.20), we can see that the eigenvalue/eigenvector approach does indeed produce the same definition of the transient inductance as the Laplace transform method.

VI.4 Canay's Data Conversion

Assume that M has been found from either Eq. (8.20a) or (8.20b) (subscript "m" dropped here), and that the four time constants T_{do}', T_{do}", T_d', T_d" are known. If only one pair of time constants as well as X_d', X_d" are known, the other pari can be found from Eq. (8.12). We then obtain the two time constants of the "f-branch" and "D-branch" of Fig. 8.2,

$$T_1 = \frac{L_f}{R_f},\quad T_2 = \frac{L_D}{R_D},\quad with\ L_f = L_{ff} - M,\quad L_D = L_{DD} - M \qquad \text{(VI.28)}$$

by solving the two equations

$$T_1 + T_2 = (T_{do}^{/} + T_{do}^{//})\ \frac{M-L_d}{M} + (T_d^{/}+T_d^{//})\ \frac{L_d}{M} \qquad \text{(VI.29a)}$$

$$T_1T_2 = T_{do}'\ T_{do}''\ (L_{parallel\ MfD}\ /M) \qquad \text{(VI.29b)}$$

with $L_{parallel\ MfD}$ being the inductance of M, L_f, L_D in parallel, which can be shown with Eq. (VI.16) to be

$$L_{parallel\ MfD} = M - L_d + L_d'' \qquad \text{(VI.29c)}$$

Eq. (VI.29a) is obtained by multiplying Eq. (VI.9a) with (1-M/L_d) and then subtracting it from Eq. (VI.14a), while Eq. (VI.29b) is obtained from Eq. (VI.9b) with the definition of L_d" from Eq. (VI.16). Once T_1 and T_2 are known, the inductance of M, L_f in parallel is found,

$$L_{parallel\ Mf} = \frac{M(T_1 - T_2)}{T_{do}^{/} + T_{do}^{//} - (1 + \frac{M}{L_{parallel\ MfD}})T_2} \qquad \text{(VI.30)}$$

This equation is derived from rewriting Eq. (VI.9a) as

$$M\ (\frac{T_1}{L_f} + \frac{T_2}{L_D}) = T_{do}^{/} + T_{do}^{//} - T_1 - T_2$$

and rewriting Eq. (VI.9b) as

$$M\left(\frac{T_2}{L_f} + \frac{T_2}{L_D}\right) = \left(\frac{M}{L_{parallel\ MfD}} - 1\right) T_2$$

which produces M/L_f after subtracting the second from the first equation. After addition of 1 to M/L_f and division by M the reciprocal of $L_{parallel\ Mf}$ follows. Then

$$L_f = (L_{parallel\ Mf} \bullet M) / (M - L_{parallel\ Mf}) \qquad (VI.31a)$$

$$L_D = (L_{parallel\ MfD} \bullet L_{parallel\ Mf}) / (L_{parallel\ Mf} - L_{parallel\ MfD}) \qquad (VI.31b)$$

and

$$R_f = L_f/T_1, \quad R_D = L_D/T_2, \quad L_{ff} = L_f + M, \quad L_{DD} = L_D + M \qquad (VI.32)$$

Table VI.1 compares the results from the approximate data conversion of [74], from the data conversion which ignores the damper winding in the definition of L_d' by using Eq. (VI.4) instead of (VI.21) [106], and from Canay's data conversion. The approximate data conversion produces an incorrect model with $X_d'=0.156$ instead of 0.169 (transient short-circuit currents 8% too large) and with T_{do}' too large while T_{do}'' is too small. The data conversion with the wrong definition of L_d' produces an incorrect model with $X_d'=0.142$ instead of 0.169 (transient short-circuit currents 19% too large), but with correct time constants T_{do}' and T_{do}''. The iterative method mentioned in [74] is correct and produces the same answers as Canay's conversion, except that no procedure is given there on how to perform the iterations.

To double-check whether Canay's data conversion is indeed correct, a system of seven equations of the form

$$[\, di_{dqo} / dt \,] = [A]\, [i_{dqo}] + [B]\, [v_f]$$

was set up which describes the three-phase short-circuit condition. The values of Table VI.1 were first used to find the matrix [A]. Then the eigenvalues of [A] were determined. The reciprocals of four of the eigenvalues differ from the time constants T_d', T_d'', T_q', T_q'' by no more than 0.05% for realistic values of $R_a=0.004$ p.u., the reciprocal of one eigenvalue agrees with T_a of Eq. (VI.14d) to within 0.1%. Unrealistically large values of R_a would produce errors for reasons explained in Section VI.2; for $R_a=0.04$ p.u., the error would still be only 4% for T_q' and 1% or less for the other time constants.

VI.5 Negative Sequence Impedance

Negative sequence currents in the armature produce a magnetic field which rotates in opposite direction to the field rotation, thereby inducing double frequency currents in the field structure windings. The negative sequence impedance can therefore be obtained by setting $s=j2\omega$ in Eq. (VI.18), and adding the armature resistance R_a to it,

$$Z_{d-neg} = R_a + j\omega L_d \frac{(1 + j2\omega T_d')\,(1 + j2\omega T_d'')}{(1 + j2\omega T_{do}')\,(1 + j2\omega T_{do}'')} \qquad (VI.33)$$

Table VI.1 - Data conversion for direct axis data from [74] (X_d = 1.79 p.u., X_d' = 0.169 p.u., X_d'' = 0.135 p.u., X_ℓ = 0.13 p.u., T_{do}' = 4.3 s, T_{do}'' = 0.032 s, f = 60 Hz).

	Approx.	Wrong L_d' *)	Canay
Conversion results			
X_{ff} (p.u.)	1.6999	1.7036	1.7218
X_{DD} (p.u.)	1.6657	1.6700	1.6655
R_f (p.u.)	0.00105	0.002086	0.001407
R_D (p.u.)	0.00371	0.002045	0.004070
Implied model parameters			
X_d' (p.u.) from (VI.21)	0.1564	0.1416	0.169
T_{do}' (s) from (VI.8)	5.466	4.3	4.3
T_{do}'' (s) from (VI.8)	0.0252	0.032	0.032
T_d' (s) from (VI.13)	0.4744	0.3388	0.4000
T_d'' (s) from (VI.13)	0.0219	0.0306	0.0259
$T_f = L_{ff}/R_f$ (s)	4.300	2.166	3.246
$T_D = L_{DD}/R_D$ (s)	1.192	2.166	1.085

*)For conversion of [106] to work, X_ℓ had to be reduced by 1.4%.

and analogous for the quadrature axis. Then

$$Z_{neg} \approx (Z_{d\text{-}neg} + Z_{q\text{-}neg}) / 2 \qquad \text{(VI.34)}$$

with $R_{neg} = \mathrm{Re}\{Z_{neg}\}$ and $X_{neg} = \mathrm{Im}\{Z_{neg}\}$.

If there is only one winding on the field structure, say only the Q-winding on the q-axis, then

$$Z_{q-neg} = R_a + j\omega L_q \frac{1 + j\ 2\omega T_q^{\prime\prime}}{1 + j\ 2\omega T_{qo}^{\prime\prime}} \qquad \text{(VI.33a)}$$

with

$$T_q'' = (L_q'' / L_q)\, T_{qo}'' \qquad \text{(VI.35b)}$$

Eq. (VI.35a) follows from (VI.33) by setting $T_q' = 0$ and $T_{qo}' = 0$, and Eq. (VI.35b) from $T_q'' = L_{QQs} / R_Q$, with L_{QQs} defined by Eq. (VI.12) and $T_{qo}'' = L_{QQ} / R_Q$.