1. INTRODUCTION TO THE SOLUTION METHOD USED IN THE EMTP

This manual discusses by and large only those solution methods which are used in the EMTP. It is therefore not a book on the complete theory of solution methods for the digital simulation of electromagnetic transient phenomena. The developers of the EMTP chose methods which they felt are best suited for a general-purpose program, such as the EMTP, and it is these methods which are discussed here. For analyzing specific problems, other methods may well be competitive, or even better. For example, Fourier transformation methods may be preferable for studying wave distortion and attenuation along a line in cases where the time span of the study is so short that reflected waves have not yet come back from the remote end.

The EMTP has been specifically developed for power system problems, but some of the methods have applications in electronic circuit analysis as well. While the developers of the EMTP have to some extent been aware of the methods used in electronic circuit analysis programs, such as TRAC or ECAP, the reverse may not be true. A survey of electronic analysis programs published as recently as 1976 [22] does not mention the EMTP even once.

Computer technology is changing very fast, and new advances may well make this manual obsolete by the time it is finished. Also, better numerical solution methods may appear as well, and replace those presently used in the EMTP. Both prospects have been discouraging for the writer of this manual; what has kept him going is the hope that those who will be developing better programs and who will use improved computer hardware will find some useful information in the description of what exists today.

Digital computers cannot simulate transient phenomena continuously, but only at discrete intervals of time (step size Δt). This leads to truncation errors which may accumulate from step to step and cause divergence from the true solution. Most methods used in the EMTP are numerically stable and avoid this type of error build-up.

The EMTP can solve any network which consists of interconnections of resistances, inductances, capacitances, single and multiphase π-circuits, distributed-parameter lines, and certain other elements. To keep the explanations in this introduction sample, only single-phase network elements will be considered and the more complex multiphase network elements as well as other complications will be discussed later. Fig. 1.1 shows the details of a larger network just for the region around node 1. Suppose that voltages and currents have already been computed at time instants 0, Δt, $2\Delta t$, etc., up to $t-\Delta t$, and that the solution must now be found at instant t. At any instant of time, the sum of the currents flowing away from node 1 through the branches must be equal to the injected current i_1:

$$i_{12}(t) + i_{13}(t) + i_{14}(t) + i_{15}(t) = i_1(t) \tag{1.1}$$

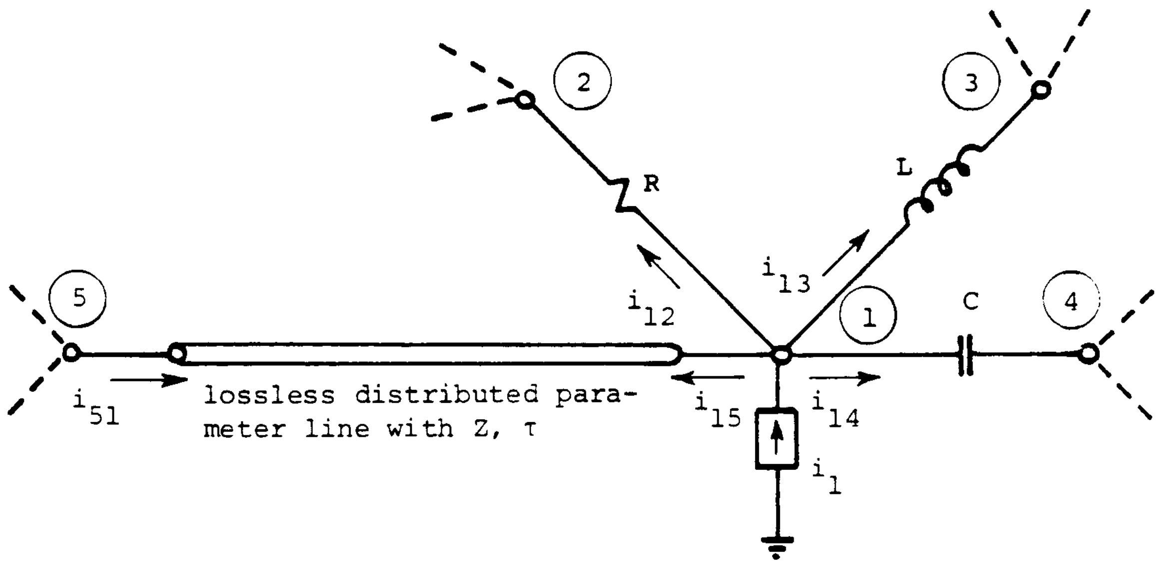

Fig. 1.1 - Details of a larger network around node no. 1

Node voltages are used as state variables in the EMTP. It is therefore necessary to express the branch currents, i_{12}, etc., as functions of the node voltages. For the resistance,

$$i_{12}(t) = \frac{1}{R}\{v_1(t) - v_2(t)\} \tag{1.2}$$

For the inductance, a simple relationship is obtained by replacing the differential equation

$$v = L\frac{di}{dt}$$

with the central difference equation

$$\frac{v(t) + v(t-\Delta t)}{2} = L\frac{i(t) - i(t-\Delta t)}{\Delta t}$$

This can be rewritten, for the case of Fig. 1.1, as

$$i_{13}(t) = \frac{\Delta t}{2L}\{v_1(t) - v_3(t)\} + hist_{13}(t-\Delta t) \tag{1.3a}$$

with $hist_{13}$ known from the values of the preceding time step,

$$hist_{13}(t-\Delta t) = i_{13}(t-\Delta t) + \frac{\Delta t}{2L}\{v_1(t-\Delta t) - v_3(t-\Delta t)\} \tag{1.3b}$$

The derivation for the branch equation of the capacitance is analogous, and leads to

$$i_{14}(t) = \frac{2C}{\Delta t}\{v_1(t) - v_4(t)\} + hist_{14}(t-\Delta t) \tag{1.4a}$$

with $hist_{14}$ again known from the values of the preceding time step,

$$hist_{14}(t-\Delta t) = -i_{14}(t-\Delta t) - \frac{2C}{\Delta t}\{v_1(t-\Delta t) - v_4(t-\Delta t)\} \tag{1.4b}$$

Readers fresh out of University, or engineers who have read one or one too many textbooks on electric circuits and networks, may have been misled to believe that Laplace transform techniques are only useful for "hand solutions" or rather small networks, and more or less useless for computer solutions of problems of the size typically analyzed with the EMTP. Since even new textbooks perpetuate the myth of the usefulness of Laplace transforms, Appendix I has been added for the mathematically-minded reader to summarize numerical solution methods for linear, ordinary differential equations.

For the transmission line between nodes 1 and 5, losses shall be ignored in this introduction. Then the wave equations

$$-\frac{\partial v}{\partial x} = L'\frac{\partial i}{\partial t}$$

$$-\frac{\partial i}{\partial x} = C'\frac{\partial v}{\partial t}$$

where

L', C' = inductance and capacitance per unit length[1],

x = distance from sending end,

have the well-known solution due to d'Alembert:

$$i = F(x - ct) - f(x + ct)$$
$$v = ZF(x - ct) + Zf(x + ct) \tag{1.5a}$$

with

F(x - ct)
f(x = ct)] = functions of the composite expressions x - ct and x + ct,

Z = surge impedance $Z = \sqrt{L'/C'}$ (constant),

c = velocity of wave propagation (constant).

If the current in Eq. (1.5a) is multiplied by Z and added to the voltage, then

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[1]The prime is used on L', C' to distinguish these distributed parameters from lumped parameters L, C.

$$v + Zi = 2ZF(x - ct) \tag{1.5b}$$

Note that the composite expression v + Zi does not change if x - ct does not change. Imagine a fictitious observer travelling on the line with wave velocity c. The distance travelled by this observer is $x = x_0 + ct$ (x_0 = location of starting point), or x - ct is constant. If x - ct is indeed constant, then the value of v + Zi seen by the observer must also remain constant. With travel time

$$\tau = \text{line length} / c ,$$

an observer leaving node 5 at time t - τ will see the value of $v_5(t - \tau) + Zi_{51}(t - \tau)$, and upon arrival at node 1 (after the elapse of travel time τ), will see the value of $v_1(t) - Zi_{15}(t)$ (negative sign because i_{15} has opposite direction of i_{51}). But since this value seen by the observer must remain constant, both of these values must be equal, giving, after rewriting,

$$i_{15}(t) = \frac{1}{Z} v_1(t) + hist_{15}(t - \tau) \tag{1.6a}$$

where the term $hist_{15}$ is again known from previously computed values,

$$hist_{15}(t-\tau) = -\frac{1}{Z} v_5(t-\tau) - i_{51}(t-\tau) \tag{1.6b}$$

Example: Let $\Delta t = 100\ \mu s$ and $\tau = 1$ ms. From equations (1.6) it can be seen that the known "history" of the line must be stored over a time span equal to τ, since the values needed in Eq. (1.6b) are those computed 10 time steps earlier. Eq. (1.6) is an exact solution for the lossless line if Δt is an integer multiple of τ; if not, linear interpolation is used and interpolation errors are incurred. Losses can often be represented with sufficient accuracy by inserting lumped resistances in a few places along the line, as described later in Section 4.2.2.5. A more sophisticated treatment of losses, especially with frequency dependent parameters, is discussed in Section 4.2.2.6.

If Eq. (1.2), (1.3a), (1.4a) and (1.6a) are inserted into Eq. (1.1), then the node equation for node 1 becomes

$$\left(\frac{1}{R} + \frac{\Delta t}{2L} + \frac{2C}{\Delta t} + \frac{1}{Z}\right) v_1(t) - \frac{1}{R} v_2(t) - \frac{\Delta t}{2L} v_3(t) - \frac{2C}{\Delta t} v_4(t) =$$
$$i_1(t) - hist_{13}(t-\Delta t) - hist_{14}(t-\Delta t) - hist_{15}(t-\tau) \tag{1.7}$$

which is simply a linear, algebraic equation in unknown voltages, with the right-hand side known from values of preceding time steps.

For any type of network with n nodes, a system of n such equations can be formed[2],

$$[G]\,[v(t)] = [i(t)] - [hist] \tag{1.8a}$$

[2]Brackets are used to indicate matrix and vector quantities.

with $[G]$ = n x n symmetric nodal conductance matrix,

$[v(t)]$ = vector of n node voltages,

$[i(t)]$ = vector of n current sources, and

[hist] = vector of n known "history" terms.

Normally, some nodes have known voltages either because voltage sources are connected to them, or because the node is grounded. In this case Eq. (1.8a) is partitioned into a set A of nodes with unknown voltages, and a set B of nodes with known voltages. The unknown voltages are then found by solving

$$[G_{AA}][v_A(t)] = [i_A(t)] - [hist_A] - [G_{AB}][v_B(t)] \tag{1.8b}$$

for $[v_A(t)]$.

The actual computation in the EMTP proceeds as follows: Matrices $[G_{AA}]$ and $[G_{AB}]$ are built, and $[G_{AA}]$ is triangularized with ordered elimination and exploitation of sparsity. In each time step, the vector on the right-hand side of Eq. (1.8b) is "assembled" from known history terms, and known current and voltage sources. Then the system of linear equations is solved for $[v_A(t)]$, using the information contained in the triangularized conductance matrix. In this "repeat solution" process, the symmetry of the matrix is exploited in the sense that the same triangularized matrix used for downward operations is also used in the backsubstitution. Before proceeding to the next time step, the history terms hist of Eq. (1.3b), (1.4b) and (1.6b) are then updated for use in future time steps.

Originally, the EMTP was written for cases starting from zero initial conditions. In such cases, the history terms $hist_{13}$, $hist_{14}$ and $hist_{15}$ in Eq. (1.7) are simply preset to zero. But soon cases arose where the transient simulation had to be started from power frequency (50 or 60 Hz) ac steady-state initial conditions. Originally, such ac steady-state initial conditions were read in[3], but this put a heavy burden on the program user, who had to use another steady-state solution program to obtain them. Not only was the data transfer bothersome, but the separate steady-state solution program might also contain network models which could differ more or less from those used in the EMTP. It was therefore decided to incorporate an ac steady-state solution routine directly into the EMTP, which was written by J.W. Walker.

The ac steady-state solution shall again be explained for the case of Fig. 1.1. Using node equations again, Eq. (1.1) now becomes

$$I_{12} + I_{13} + I_{14} + I_{15} = I_1 \tag{1.9}$$

where the currents I are complex phasor quantities $|I| \cdot e^{j\alpha}$ now. For the lumped elements, the branch equations are obvious. For the resistance,

$$I_{12} = \frac{1}{R}(V_1 - V_2) \tag{1.10}$$

[3]This option is still available in the EMTP, but it has become somewhat of a historic relic and has seldom been used after the addition of a steady-state solution routine. For some types of branches, it may not even work ([1], p. 37c).

for the inductance,

$$I_{13} = \frac{1}{j\omega L}(V_1 - V_3) \tag{1.11}$$

and for the capacitance,

$$I_{14} = j\omega C(V_1 - V_4) \tag{1.12}$$

For a line with distributed parameters R', L', G', C', the exact steady-state solution is

$$\begin{bmatrix} I_{15} \\ I_{51} \end{bmatrix} = \begin{bmatrix} Y_{series} + \frac{1}{2} Y_{shunt} & -Y_{series} \\ -Y_{series} & Y_{series} + \frac{1}{2} Y_{shunt} \end{bmatrix} \begin{bmatrix} V_1 \\ V_5 \end{bmatrix} \tag{1.13}$$

if the equivalent π-circuit representation of Fig. 1.2 is used, with

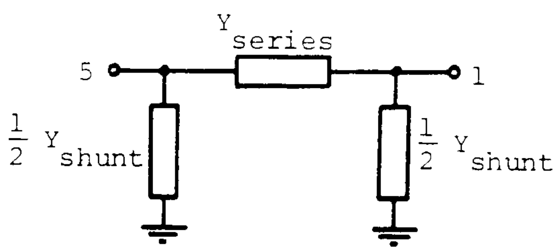

Fig. 1.2 - Equivalent π-circuit for ac steady-state solution of transmission line

$$Y_{series} = \frac{1}{Z_{series}}, \quad with\ Z_{series} = \mathcal{L}(R' + j\omega L')\frac{\sinh(\gamma\mathcal{L})}{\gamma\mathcal{L}}$$

$$\frac{1}{2}Y_{shunt} = \frac{\mathcal{L}}{2}(G' + j\omega C')\frac{\tanh\left(\frac{\gamma\mathcal{L}}{2}\right)}{\frac{\gamma\mathcal{L}}{2}} \tag{1.14}$$

and sometimes equally useful,

$$Y_{series} + \frac{1}{2}Y_{shunt} = \cosh(\gamma\mathcal{L}) \cdot Y_{series}$$

where γ is the propagation constant,

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} \tag{1.15}$$

For the lossless case with R' = 0, and G' = 0, Eq. (1.14) simplifies to

$$Z_{series} = \mathcal{L} \cdot j\omega L' \cdot \frac{\sin(\omega\mathcal{L}\sqrt{L'C'})}{\omega\mathcal{L}\sqrt{L'C'}}$$

$$\frac{1}{2}Y_{shunt} = \frac{\mathcal{L}}{2} \cdot j\omega C' \cdot \frac{\tan\left(\frac{\omega\mathcal{L}}{2}\sqrt{L'C'}\right)}{\frac{\omega\mathcal{L}}{2}\sqrt{L'C'}} \tag{1.16}$$

$$Y_{series} + \frac{1}{2}Y_{shunt} = \cos(\omega\mathcal{L}\sqrt{L'C'}) \cdot Y_{series}$$

If the value of $\omega\mathcal{L}$ is small, typically $\mathcal{L} \leq 100$ km at 60 Hz for overhead lines, then the ratios sinh(x) / x and tanh(x/2) / x/2 in Eq. (1.14), as well as sin(x) / x and tan(x/2) / x/2 in Eq. (1.16) all become 1.0. This simplified π-circuit is usually called the "nominal" π-circuit,

$$Z_{series} = \mathcal{L} \cdot (R' + j\omega L')$$

$$\frac{1}{2}Y_{shunt} = \frac{\mathcal{L}}{2}(G' + j\omega C') \qquad \textit{if } \omega\mathcal{L} \textit{ is small.} \tag{1.17}$$

With the equivalent π-circuit of Fig. 1.2, the branch equation for the lossless line finally becomes

$$I_{15} = (Y_{series} + \frac{1}{2}Y_{shunt})V_1 - Y_{series}V_5 \tag{1.18}$$

Now, we can again write the node equation for node 1, by inserting Eq. (1.10), (1.11), (1.12) and (1.18) into Eq. (1.9),

$$\left(\frac{1}{R} + \frac{1}{j\omega L} + j\omega C + Y_{series} + \frac{1}{2}Y_{shunt}\right)V_1 - \frac{1}{R}V_2 - \frac{1}{j\omega L}V_3 - j\omega C V_4 - Y_{series}V_5 = I_1 \tag{1.19}$$

For any type of network with n nodes, a system of n such equations can be formed,

$$[Y]\,[V] = [I] \tag{1.20}$$

with [Y] = symmetric nodal admittance matrix, with complex elements,

[V] = vector of n node voltages (complex phasor values),

[I] = vector of n current sources (complex phasor values).

Again, Eq. (1.20) is partitioned into a set A of nodes with unknown voltages, and a set B of nodes with known voltages. The unknown voltages are then found by solving the system of linear, algebraic equations

$$[Y_{AA}][V_A] = [I_A] - [Y_{AB}][V_B] \tag{1.21}$$

Bringing the term $[Y_{AB}][V_B]$ from the left-hand side in Eq. (1.20) to the right-hand side in Eq. (1.21) is the generalization of converting Thevenin equivalent circuits (voltage vector $[V_B]$ behind admittance matrix $[Y_{AB}]$) into Norton equivalent circuits (current vector $[Y_{AB}][V_B]$ in parallel with admittance matrix $[Y_{AB}]$).

Norton equivalent circuits (current vector $[Y_{AB}][V_B]$ in parallel with admittance matrix $[Y_{AB}]$).