9. UNIVERSAL MACHINE

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The universal machine was added to the EMTP by H.K. Lauw and W.S. Meyer [137,140], to be able to study various types of electric machines with the same model. It can be used to represent 12 major types of electric machines:

(1) synchronous machine, three-phase armature;

(2) synchronous machine, two-phase armature;

(3) induction machine, three-phase armature;

(4) induction machine, three-phase armature and three-phase rotor;

(5) induction machine, two-phase armature;

(6) single-phase ac machine (synchronous or induction), one-phase excitation;

(7) same as (6), except two-phase excitation;

(8) dc machine, separately excited;

(9) dc machine, series compound (long shunt) field;

(10) dc machine, series field;

(11) dc machine, parallel compound (short shunt) field;

(12) dc machine, parallel field.

The user can choose between two interfacing methods for the solution of the machine equations with the rest of the network. One is based on compensation, where the rest of the network seen from the machine terminals is represented by a Thevenin equivalent circuit, and the other is a voltage source behind an equivalent impedance representation, similar to that of Section 8.5, which requires prediction of certain variables.

The mechanical part of the universal machine is modelled quite differently from that of the synchronous machine of Section 9. Instead of a built-in model of the mass-shaft system, the user must model the mechanical part as an equivalent electric network with lumped R, L, C, which is then solved as if it were part of the complete electric network. The electromagnetic torque of the universal machine appears as a current source in this equivalent network.

9.1 Basic Equations for Electrical Part

Any electric machine has essentially two types of windings, one being stationary on the stator, the other rotating on the rotor. Which type is stationary and which is rotating is irrelevant in the equations, because it is only the relative motion between the two types which counts. The two types are:

(a) Armature windings (windings on "power side" in BPA Rule Book). In induction and (normally) in synchronous machines, the armature windings are on the stator. In dc machines, they are on the rotor, where the commutator provides the rectification from ac to dc.

(b) Windings on the field structure ("excitation side" in BPA Rule Book). In synchronous machines the field structure windings are normally on the rotor, while in dc machines they are on the rotor, either in the form of a short-circuited squirrel-cage rotor, or in the form of a wound rotor with slip-ring connections to the outside. The proper term is "rotor winding" in this case, and the term "field structure winding" is only used here to keep the notation uniform for all types of machines.

These two types of windings are essentially the same as those of the synchronous machine described in Section 8.1. It is therefore not surprising that the system of equations (8.9) and (8.10) describe the behavior of the universal machine along the direct and quadrature axes as well. The universal machine is allowed to have up to 3 armature windings, which are converted to hypothetical windings d, q, 0a ("a" for armature) in the same way as in Section 8.1. The special case of single-phase windings is discussed in Section 9.3. The field structure is allowed to have any number of windings D1, D2,...Dm on the direct axis, and any number of windings Q1, Q2,...Qn on the quadrature axis, which can be connected to external circuits defined by the user. In contrast to Section 8, the field structure may also have a single zero sequence winding 0f ("f" for field structure), to allow the conversion of three-phase windings on the field structure (as in wound-rotor induction machines) into hypothetical D, Q, 0-windings.

With these minor differences to the synchronous machine of Section 8 in mind, the voltage equations for the armature windings in d, q-quantities become

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = - \begin{bmatrix} R_a & 0 \\ 0 & R_a \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \lambda_d \\ \lambda_q \end{bmatrix} + \begin{bmatrix} -\omega\lambda_q \\ +\omega\lambda_d \end{bmatrix} \tag{9.1a}$$

with ω being the angular speed of the rotor referred to the electrical side, and in zero sequence,

$$v_{0a} = -R_a i_{0a} - d\lambda_{0a}/dt \tag{9.1b}$$

The voltage equations for the field structure windings are

$$\begin{bmatrix} v_{D1} \\ v_{D2} \\ \cdot \\ \cdot \\ \cdot \\ v_{Dm} \end{bmatrix} = - \begin{bmatrix} R_{D1} & & & & & \\ & R_{D2} & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & R_{Dm} \end{bmatrix} \begin{bmatrix} i_{D1} \\ i_{D2} \\ \cdot \\ \cdot \\ \cdot \\ i_{Dm} \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \lambda_{D1} \\ \lambda_{D2} \\ \cdot \\ \cdot \\ \cdot \\ \lambda_{Dm} \end{bmatrix} \tag{9.2a}$$

$$\begin{bmatrix} v_{Q1} \\ v_{Q2} \\ \cdot \\ \cdot \\ \cdot \\ v_{Qm} \end{bmatrix} = - \begin{bmatrix} R_{Q1} & & & & & \\ & R_{Q2} & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & R_{Qm} \end{bmatrix} \begin{bmatrix} i_{Q1} \\ i_{Q2} \\ \cdot \\ \cdot \\ \cdot \\ i_{Qm} \end{bmatrix} - \frac{d}{dt} \begin{bmatrix} \lambda_{Q1} \\ \lambda_{Q2} \\ \cdot \\ \cdot \\ \cdot \\ \lambda_{Qm} \end{bmatrix} \tag{9.2b}$$

and

$$v_{0f} = -R_{0f}\, i_{0f} - \frac{D\,\lambda_{0f}}{dt} \tag{9.2c}$$

The flux-current relationships on the two axes provide the coupling between the armature and field structure sides,

$$\begin{bmatrix} \lambda_d \\ \lambda_{D1} \\ \lambda_{D2} \\ . \\ . \\ . \\ \lambda_{Dm} \end{bmatrix} = \begin{bmatrix} L_d & M_{dD1} & M_{dD2} & \cdots & M_{dDm} \\ M_{dD1} & L_{D1} & M_{D1D2} & \cdots & M_{D1Dm} \\ M_{dD2} & M_{D1D2} & L_{D2} & \cdots & M_{D2Dm} \\ . & . & . & . & . \\ M_{dDm} & M_{D1Dm} & M_{D2Dm} & \cdots & L_{Dm} \end{bmatrix} \begin{bmatrix} i_d \\ i_{D1} \\ i_{D2} \\ . \\ . \\ . \\ i_{Dm} \end{bmatrix} \tag{9.3a}$$

$$\begin{bmatrix} \lambda_q \\ \lambda_{Q1} \\ \lambda_{Q2} \\ . \\ . \\ . \\ \lambda_{Qn} \end{bmatrix} = \begin{bmatrix} L_q & M_{qQ1} & M_{qQ2} & \cdots & M_{qQn} \\ M_{qQ1} & L_{Q1} & M_{Q1Q2} & \cdots & M_{Q1Qn} \\ M_{qQ2} & M_{Q1Q2} & L_{Q2} & \cdots & M_{Q2Qn} \\ . & . & . & . & . \\ M_{qQn} & M_{Q1Qn} & M_{Q2Qn} & \cdots & L_{Qn} \end{bmatrix} \begin{bmatrix} i_q \\ i_{Q1} \\ i_{Q2} \\ . \\ . \\ . \\ i_{Qn} \end{bmatrix} \tag{9.3b}$$

with both inductance matrices being symmetric. The zero sequence fluxes on the armature and field structure side are not coupled at all,

$$\lambda_{0a} = L_{0a}\, i_{0a} \tag{9.3c}$$
$$\lambda_{0f} = L_{0f}\, i_{0f} \tag{9.3d}$$

The universal machine has been implemented under the assumption that the self and mutual inductances in Eq. (9.3a) and (9.3b) can be represented by a star circuit if the field structure quantities are referred to the armature side, as shown in Fig. 9.1. This assumption

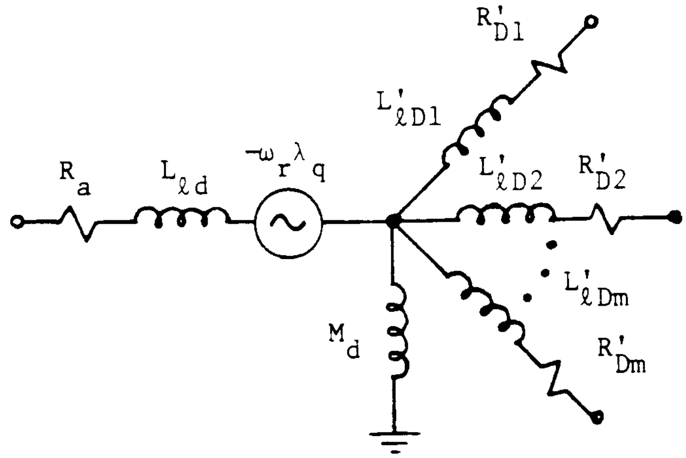


Fig. 9.1 - Star circuit representation of coupled windings in direct axis (analogous in quadrature axis)

implies that there is only one mutual (or main) flux which links all windings on one axis (λ_m in Fig. 8.18), and that the leakage flux of any one winding is only linked with that winding itself. Strictly speaking, this is not always true. For example, part of the leakage flux of the field winding (λ_{lf} in Fig. 8.18) could go through the damper winding as well, but not through the armature winding, which leads to the modified star circuit of Fig. 8.23 (synchronous machines) or Fig. 9.2 (induction machines). The data for such models with unequal mutual inductances is seldom available, however (e.g., Fig. 8.23 requires Canay's characteristic reactance, which is not available from standard test data). The star circuit is therefore a reasonable assumption in practice. At any rate, the code could easily be changed to work with the self and mutual inductances of Eq. (9.3) instead of the star circuit of Fig. 9.1.

With the star circuit representation of Fig. 9.1, the flux-current equations (9.3a) can be simplified to

$$\lambda_d = L_{\ell d}\, i_d + \lambda_{md}$$

$$\lambda'_{D1} = L'_{\ell D1}\, i'_{D1} + \lambda_{md} \tag{9.4a}$$

$$\vdots$$

$$\lambda'_{Dm} = L'_{\ell Dm}\, i'_{Dm} + \lambda_{md}$$

with

$$\lambda_{md} = M_d\, (i_d + i'_{D1} \ldots + i'_{Dm}) \tag{9.4b}$$

where the prime indicates that field structure quantities have been referred to the armature side with the proper turns ratios between d and D1, d and D2,...d and D_m. All referred mutual inductances are equal to M_d in this representation, and the referred self inductances of Eq. (9.3a) are related to the leakage inductances of the star branches by

$$
\begin{aligned}
L_d &= L_{\ell d} + M_d \\
L'_{D1} &= L'_{\ell D1} + M_d \qquad (9.5)\\
&\cdot \\
&\cdot \\
L'_{Dm} &= L'_{\ell Dm} + M_d
\end{aligned}
$$

The voltage equations (9.1) and (9.2) are valid for referred quantities as well, if R_{D2}, i_{D2},... are replaced by R_{D2}', i_{D2}',... The quadrature axis equations are obtained by replacing subscripts d, D in the direct axis equations with q, Q.

In the BPA EMTP Rule Book, the turns ratios are called "reduction factors," and the process of referring quantities to the armature side is called "reduction" (referring quantities from one side to another is discussed in Appendix IV.3).

9.2 Determination of Electrical Parameters

By limiting the universal machine representation to the star circuit of Fig. 9.1, the input parameters are simply the resistances and leakage inductances of the star branches and the mutual inductance, e.g., for the direct axis,

$$
\begin{aligned}
&R_a\,, \quad L_{\ell d} \\
&R'_{D1}\,, \quad L'_{\ell D1} \\
&\cdot \\
&\cdot \\
&\cdot \\
&R'_{Dm}\,, \quad L'_{\ell Dm} \\
&M_d\,,
\end{aligned}
$$

(analogous for the quadrature axis), and for the zero sequence on the armature and field structure side,

$$
\begin{aligned}
&L_{0a}\,, \\
&R'_{0f}\,, \quad L'_{0f}
\end{aligned}
$$

If the armature leakage inductance L_ℓ is known instead of the mutual inductance, then find M from Eq. (9.5),

$$
M_d = L_d - L_{\ell d}\,, \qquad M_q = L_q - L_{\ell q}
$$

If neither L_ℓ nor M is known, then use a reasonable estimate. The BPA EMTP Rule Book recommends

$$L_{\ell d} = 0.1\,L_d\;, \qquad L_{\ell q} = 0.1\,L_q \tag{9.6}$$

which seems to be reasonable for round rotor synchronous machines, while for salient pole machines the factor is closer to 0.2 than to 0.1. Compared to the large value of the magnetizing inductance of transformers, the value of the mutual (or magnetizing) inductance M_d, M_q from Eq. (9.6) (90% of self inductance) is relatively low because of the air gap in the flux path.

Compared to the (m + 1) (m + 2) / 2 inductance values in Eq. (9.3a), the star circuit has only m + 2 inductance values. For the most common machine representation with 2 field structure windings, Eq. (9.3a) requires 6 values, compared to 4 values for the star circuit. This means that the star circuit is not as general as Eq. (9.3a), but this is often a blessing in disguise because available test or design data is usually not sufficient anyhow to determine all self and mutual inductances (see requirement of obtaining an extra inductance value X_c in Section 8.2).

As already discussed for the synchronous machine in Section 8.2, the resistances and self and mutual inductances (or the star branch inductances here) are usually not available from calculations or measurements. If the universal machine is used to model a synchronous machine, then the data conversion discussed in Section 8.2 can be used (input identical to synchronous machine model in version M32 and later).

For three-phase induction machines, the data may be given in phase quantities. If so, Eq. (8.11) must be used to convert them to d, q, 0-quantities,

$$\begin{aligned} L_d = L_q &= L_s - M_s \\ L_o &= L_s + 2M_s \end{aligned}$$

with L_s = self inductance of one armature winding,

M_s = mutual inductance between two armature windings (BPA Rule Book uses opposite sign for M_s).

L_m in Eq. (8.11) is zero for an induction machine, where the saliency term defined in Eq. (8.5) does not exist. The same conversion is used if the rotor windings are three-phase. The mutual inductance between stator and rotor follows from Eq. (8.10).

$$M_{d-Di} = \frac{\sqrt{3}}{\sqrt{2}}\,M_{a-Di}$$

(same for q-axis), with $M_{a\text{-}Di}\cos\beta$ being the mutual inductance between armature winding 1 and rotor winding Di (i = 1,...m), as defined in Eq. (8.5). Note that the factor √3/√2 changes the turns ratio; if the turns ratio between phase 1 and the rotor winding is 1:1, it changes to √3:√2 in d, q, 0-quantities (see also Section 8.2). This extra factor must be taken into account when rotor quantities are referred to the stator side.

For modelling three-phase induction machines, a modified universal machine with its own data conversion routine has recently been developed by Ontario Hydro [138]. It uses the standard NEMA specification data to find the resistances and self and mutual inductances of the equivalent circuit. It is beyond the scope of this treatise to describe the conversion routine in detail. Essentially, the field structure (which is the rotor in the induction machine) has two windings to represent the rotor bars as well as the eddy currents in the deep rotor bars of large machines, or the double-squirrel cage rotor in smaller machines. Since there is no saliency, d- and q-axis parameters are identical. The assumption of equal mutual inductances (or the star circuit) is dropped, and the equivalent circuit of Fig. 9.2 is used instead. Not surprisingly, this equivalent circuit is identical with that of the synchronous machine in Fig. 8.23, because a synchronous

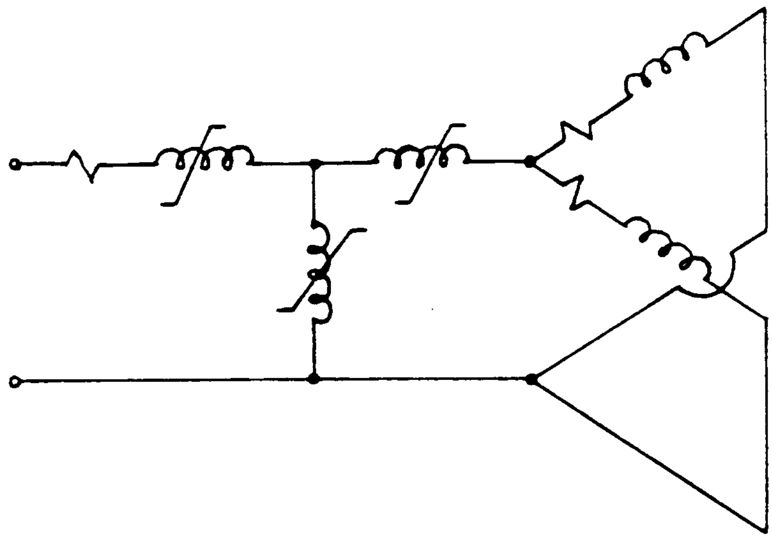
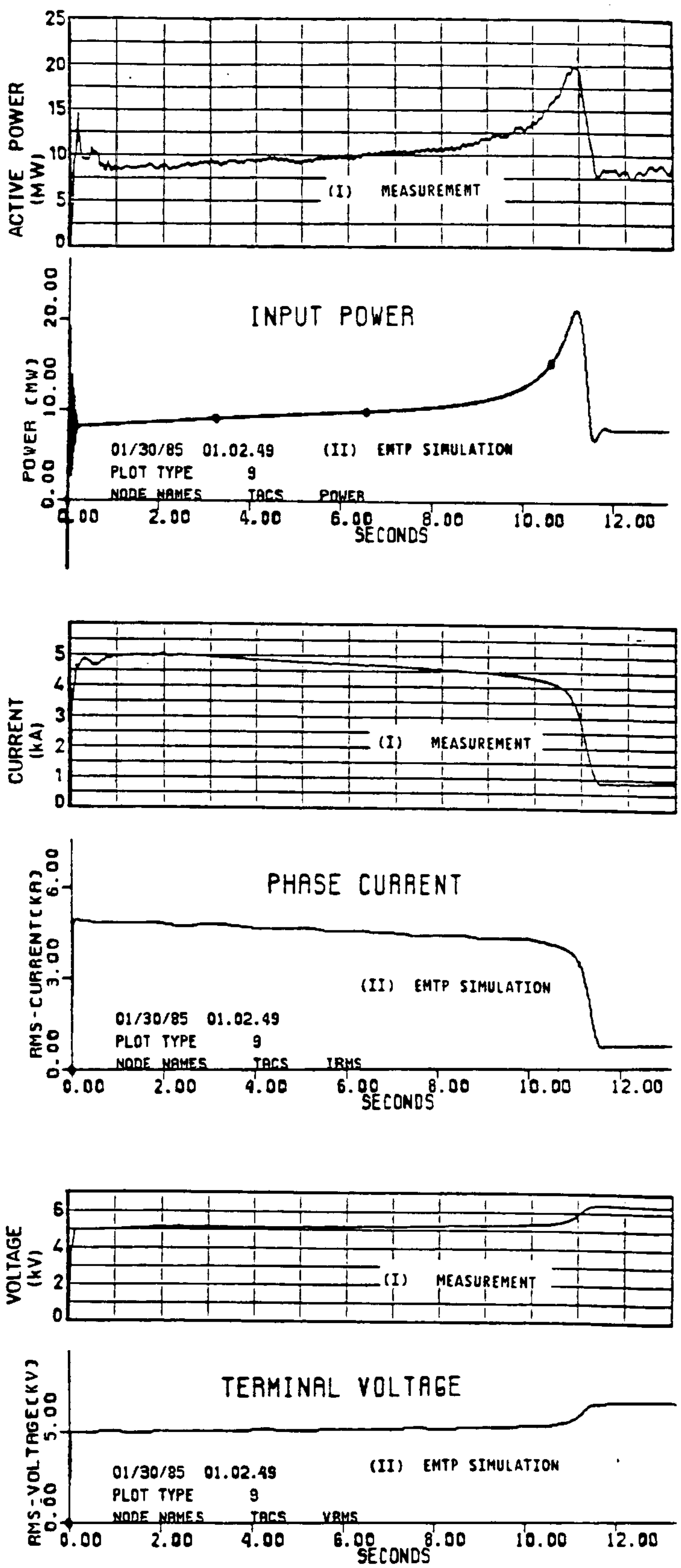


Fig. 9.2 - Equivalent circuit of induction machine with deep rotor bars

machine becomes an induction machine if the field winding is shorted. In contrast to the standard universal machine, saturation is included in the leakage inductance branch of the armature as well, and another nonlinear inductance is added between the star point and the star branches of the field structure windings. Fig. 9.3 shows comparisons between measurements and simulation results with this modified universal machine model [138, 139], for a case of a cold start-up of an induction-motor-driven heat transfer pump (1100 hp, 6600 V). Excellent agreement with the field test results is evident for the whole start-up period, which proves the validity of the modified universal machine model over the whole range of operation.



(a) active power input

(b) phase current
(RMS values)

(c) terminal voltage
(RMS values)

Fig. 9.3 - Comparison between field test and simulation results for starting up an induction motor with a heat transfer pump [138, 139]. Reprinted by permission of G.J. Rogers and D. Shirmohammadi

9.3 Transformation to Phase Quantities

Eq. (9.1) to (9.3) completely describe the universal machine in d, q, 0-quantities, irrespective of which type of machine it is. To solve these machine equations together with the rest of the network, they must be transformed to phase quantities. It is in this transformation where the various types of machines must be treated differently. Fortunately it is possible to work with the same transformation matrix for all types, by simply using different matrix coefficients.

For the case of a three-phase synchronous machine, the transformation has already been shown in Eq. (8.7). If this equation is rewritten for the armature quantities only, then[1]

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_{0a} \end{bmatrix} = [T]^{-1} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \quad \textit{identical for } [v],\ [i] \tag{9.7a}$$

with

$$[T]^{-1} = \frac{\sqrt{2}}{\sqrt{3}} \begin{bmatrix} \cos\beta & \cos(\beta-120°) & \cos(\beta+120°) \\ \sin\beta & \sin(\beta-120°) & \sin(\beta+120°) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{9.7b}$$

being an orthogonal matrix, which means that

$$[T] = [T]^{-1}_{transposed} \tag{9.7c}$$

The rotor position β is related to the angular speed ω of the rotor by

$$\omega = d\beta / dt \tag{9.7d}$$

The transformation matrix $[T]^{-1}$ can be rewritten as a product of two matrices [137],

$$[T]^{-1} = [P(\beta)]^{-1}\ [S]^{-1} \tag{9.8a}$$

with

---

[1]In [137] and [139], $[T]^{-1}$ is called [T]; similarly, [P] and [S] are called $[P]^{-1}$ and $[S]^{-1}$ there.

$$[P(\beta)]^{-1} = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9.8b}$$

$$[S]^{-1} = \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix} \tag{9.8c}$$

each being orthogonal again,

$$[P] = [P]^{-1}_{transposed} \, , \qquad [S] = [S]^{-1}_{transposed} \tag{9.8d}$$

The first transformation with the $[S]^{-1}$-matrix replaces the three-phase coils (displaced by 120° in space) by the three equivalent coils d and q (perpendicular to each other) and 0 (independent by itself). This is the same transformation matrix used for α, β, 0-components in Eq. (4.48), except for a sign reversal of the β-quantities. The second transformation with $[P]^{-1}$ makes the d, q-axes rotate with the same speed as the field poles, so that they become stationary when seen from the field structure. The field structure quantities are not transformed at all.

This approach with two transformations can be applied to any type of machine. For a three-phase induction machine with a three-phase wound rotor, both the armature and field structure quantities are transformed with $[S]^{-1}$ to get equivalent windings on the d- and q-axes, while the transformation with $[P]^{-1}$ is only applied to the armature side. For direct current machines, there is not transformation at all for both the armature and field structure side.

For two-phase armature windings displaced by 90° in space, the windings are already on the d, q-axes. Therefore

$$[S_2]^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{9.9a}$$

and

$$[P_2]^{-1} = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \tag{9.9b}$$

with the zero sequence winding missing.

For single-phase armature windings, there is only flux along one axis, or

$$[S_1]^{-1} = 1 \qquad (9.10a)$$

and

$$[P_1]^{-1} = \cos\beta \qquad (9.10b)$$

with both the quadrature axis and zero sequence winding missing.

The EMTP uses only one transformation matrix $[S]^{-1}$ and $[P]^{-1}$ for all cases, and makes the distinction by resetting the coefficients c_1, c_2, c_3 in these matrices,

$$[S]^{-1} = \begin{bmatrix} c_1 + c_2 + c_3\sqrt{2/3} & -c_3/\sqrt{6} & -c_3/\sqrt{6} \\ 0 & -c_2 - c_3/\sqrt{2} & c_3/\sqrt{2} \\ c_3/\sqrt{3} & c_3/\sqrt{3} & c_3/\sqrt{3} \end{bmatrix} \qquad (9.11)$$

with

$c_3 =$ 1 for three-phase ac windings, and $c_3 = 0$ otherwise,

$c_2 =$ 1 for two-phase ac windings, and $c_2 = 0$ otherwise,

$c_1 =$ 1 for single-phase ac windings and dc machines, and $c_1 = 0$ otherwise.

Since $[S]^{-1}$ in Eq. (9.11) degenerates into 2 x 2 and 1 x 1 matrices for two-phase and single-phase windings, its inverse cannot be found by inversion. Using Eq. (9.8d) instead of inversion works in all cases, however. The matrix in Eq. (9.11) is slightly different from that in [137], because it is assumed here that only phases 1, 2 exist for two-phase machines, and only phase 1 exists for single-phase machines. In [137], phase 1 is dropped for two-phase machines, and phases 1 and 2 are dropped for single-phase machines.

For ac armature windings, $[P]^{-1}$ of Eq. (9.8b) is used, realizing that the zero sequence does not exist in the two-phase case, and that the zero sequence as well as the q-winding do not exist in the single-phase case. For dc armature windings, there is not second transformation with $[P]^{-1}$.

9.4 Mechanical Part

In contrast to the synchronous machine model, the universal machine does not have a built-in model for the mechanical part. Instead, the user must convert the mechanical part into an equivalent electric network with lumped R, L, C, which is then treated by the EMTP as if it were part of the overall electric network. The electromagnetic torque of the universal machine appears as a current source injection into the equivalent electric network.

Table 9.1 describes the equivalence between mechanical and electrical quantities. For each mass on the shaft system, a node is created in the equivalent electric network, with a

Table 9.1 - Equivalence between mechanical and electrical quantities

Mechanical			Electrical		
T	(torque acting on mass)	[Nm]	i	(current into node)	[A]
ω	(angular speed)	[rad/s]	v	(node voltage)	[V]
θ	(angular position of mass)	[rad]	q	(capacitor charge)	[C]
J	(moment of inertia)	[kgm^2]	C	(capacitance to ground)	[F]
K	(stiffness coefficient or spring constant)	[Nm/rad]	1/L	(reciprocal or inductance)	[1/H]
D	(damping coefficient)	[Nms/rad]	1/R	(conductance)	[S]

(1 Nm = 0.73756 lb-ft; 1 kgm^2 = 23.73 lb-ft^2)

capacitor to ground with value J for the moment of inertia. If there is damping proportional to speed on this mass, a resistor with conductance D is put in parallel with the capacitor (D_i in Eq. (8.31)). If there is a mechanical torque acting on that mass (turbine torque on generators, mechanical load on motors), a current source is connected to that node (positive for turbine torque, negative for load torque). If there are two or more masses, inductors are used to connect adjacent shunt capacitors, with their inductance values being equal to 1/K (reciprocal of stiffness coefficient of the shaft coupling between two masses). If there is damping associated with this shaft coupling, then the inductor is paralleled with a resistor whose conductance value is D ($D_{i,k}$ in Eq. (8.31)). The electromagnetic torque is automatically added to the proper node as a current source by the EMTP.

Fig. 9.4 summarizes the equivalence between the mechanical and electric components. Representing the mechanical system by an equivalent electric network can provide more

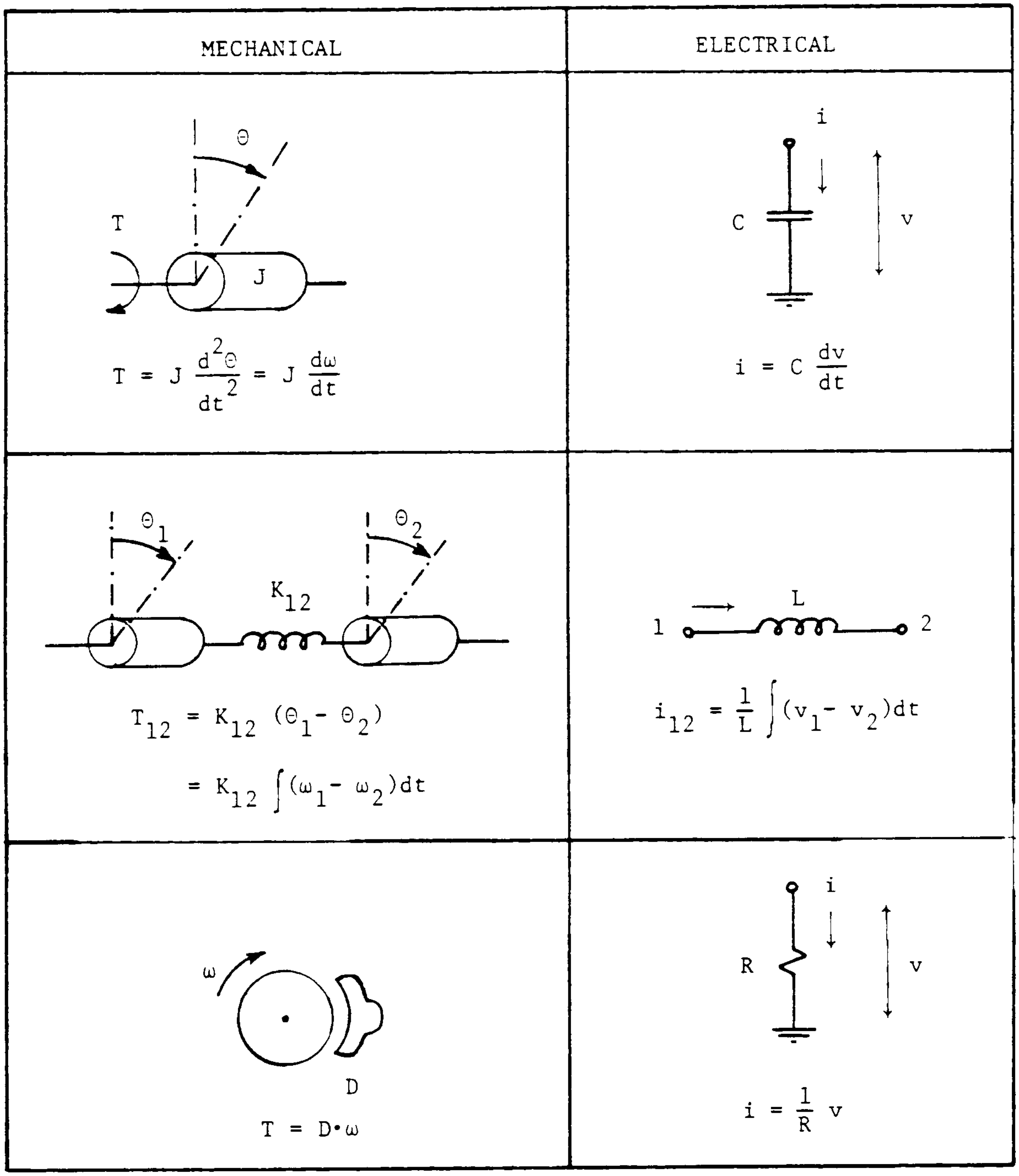


Fig. 9.4 - Equivalence between mechanical and electric components

flexibility than the built-in model of the synchronous machine of Section 8. With this approach it should be easy to incorporate gear boxes, distributed-parameter modelling of rotors, etc. The EMTP further provides for up to three universal machines sharing the same mechanical system.

9.5 Steady-State Representation and Initial Conditions

The steady-state representation of the ac-type universal machine is based on the assumption that the network to which it is connected is balanced and linear. Only positive sequence quantities are used in the initialization, and negative and zero sequence quantities are ignored if there are unbalances. The initialization procedure could obviously be extended to handle unbalanced conditions as well, along the lines discussed in Section 8, but this extension has been given low priority so far.

9.5.1 Three-Phase Synchronous Machine

For three-phase synchronous machine representations, any positive sequence voltage source behind any positive sequence impedance can be used, as long as it produces the desired terminal voltages and currents when solved with the rest of the network. For simplicity, a three-phase symmetrical voltage source directly at the terminals is used for the steady-state solution. If the current (or active and reactive power output) from that solution is not what the user wants, then the power flow iteration option of the EMTP can be used, which will iteratively adjust the magnitude and angle of the three-phase voltage source until the desired active and reactive power output (or some other prescribed criteria) have been achieved. Once the terminal voltages and currents are known, the rest of the electrical machine variables are initialized in the same way as described in Section 8.4.1.

If the excitation system is represented by an electric network (rather than constant v_f), then the EMTP performs a second ac steady-state solution for the excitation systems of all universal machines, with the field currents i_f being treated as current sources $I_f \cos(\omega_f t)$, with ω_f being an angular frequency which is so low that i_f is dc for practical purposes. This trick is used because the EMTP cannot find an exact dc steady-state solution at this time (the network topology for dc solutions is different from that of ac steady-state solution; inductances would have to be treated as closed switches, capacitances as open switches, etc.).

From the initialization of the electrical variables, the electromagnetic torque $T_{mech\text{-}gen}$ on the mechanical side is known from Eq. (8.32b) as well. These torques are used as current sources $i(t) = T_{mech\text{-}gen} \cos(\omega_{mech} t)$ in the equivalent networks which represent the mechanical systems of all universal machines, with ω_{mech} again being an angular frequency so low that i(t) is practically dc. The EMTP then performs a third ac steady-state solution for the initialization of the mechanical system quantities. Note that this three-step initialization procedure is direct, and does not require either predictions or iterations.

9.5.2 Two-Phase Synchronous Machine

Armature currents in two-phase machines with equal amplitudes and displacements of 90° produce a rotating magnetic field in the same way as symmetrical three-phase armature currents displaced by 120°. As long as this condition is met (which is the balanced or positive sequence condition for two-phase machines), the initialization is identical with the three-phase case after proper conversion to d, q, 0-quantities. If the phase quantities are

$$i_1(t) = |I| \cos(\omega_s t + \alpha)$$
$$i_2(t) = |I| \cos(\omega_s t + \alpha - 90°) \tag{9.12}$$

with ω_s being the (synchronous) frequency of the supply network, then the d, q, 0-quantities are obtained with $[S_2]^{-1}$ and $[P_2]^{-1}$ from Eq. (9.9) with $\omega = \omega_s$ as

$$i_d = |I| \sin(\alpha - \delta)$$
$$i_q = |I| \cos(\alpha - \delta) \tag{9.13}$$

where δ is the angle between the position of the quadrature axis and the real axis of the ac phasor representation. Eq. (9.13) is indeed identical with Eq. (8.41) for the balanced three-phase machine, except for a factor of $\sqrt{3}/\sqrt{2}$ there.

9.5.3 Single-Phase Synchronous Machine

Converting a single-phase armature current

$$i_1(t) = |I| \cos(\omega_s t + \alpha) \tag{9.14}$$

into d, q, 0-quantities results in

$$i_d(t) = \frac{1}{2} |I| \sin(\alpha - \delta) - \frac{1}{2} |I| \sin(2\omega_s t + \alpha + \delta)$$
$$i_q = 0 \qquad i_0 = 0 \tag{9.15}$$

with the first term being the dc quantity analogous to the positive sequence effect in three-phase machines, and the second double-frequency term analogous to the negative sequence effect in Eq. (8.53) in three-phase machines. This is a mathematical expression of the fact that an oscillating magnetic field in a single-phase armature winding can be represented as the sum of a constant magnetic field rotating forward at synchronous speed (angular speed = 0 relative to field winding) and a constant magnetic field rotating backwards at synchronous speed (angular speed = 2ω relative to field winding).

Since only the first term in Eq. (9.15) is used in the initialization now, the initial conditions are not totally correct, and it may take many time steps before steady state is reached. The steady-state torque includes a pulsating term very similar to Fig. 8.9 for the case of an unbalanced three-phase synchronous machine. As an alternative to universal machine modelling, the three-phase synchronous machine model of Section 8 could be used for single-phase machines, by keeping two armature windings open-circuited. Unfortunately, the initialization with negative sequence quantities described in Section 8.4.2 is not yet fully correct in the BPA EMTP either, as explained in the beginning of Section 8.4, though it has been implemented in an unreleased version of the UBC EMTP.

9.5.4 DC Machines

The initialization of dc machine quantities is straightforward, and follows the same procedure outlined in

Section 9.5.1. In d, q, 0-quantities, balanced three-phase ac quantities appear as dc quantities. Therefore, there is essentially no difference between the equations of a balanced three-phase synchronous generator and a dc machine.

9.5.5 Three-Phase Induction Machine

In balanced steady-state operation, the angular speed ω of the rotor (referred to the electrical side with Eq. (8.25)) differs from the angular frequency ω_s of the supply network by the p.u. slip s,

$$s = \frac{\omega_s - \omega}{\omega_s} \tag{9.16}$$

The network sees the induction machine as a positive sequence impedance whose value depends on this slip s. The negative and zero sequence impedances are of no interest if the initialization is limited to balanced cases.

Fig. 9.5 shows the well-known equivalent circuit for the balanced steady-state behavior of a three-phase induction machine, which can be found in many textbooks. Its impedance can

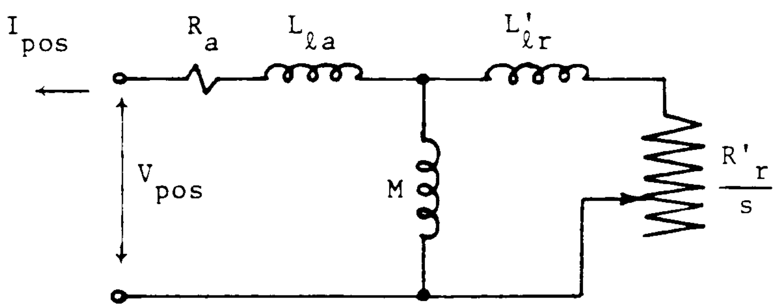


Fig. 9.5 - Conventional equivalent circuit for steady-state behavior of induction machines (subscript a for armature side, subscript r for rotor side)

easily be calculated, and with the relationship between leakage, self and mutual inductances

$$L_{aa} = L_{\ell a} + M$$

$$L'_{rr} = L'_{\ell r} + M \tag{9.17a}$$

becomes

$$Z_{pos} = R_a + j\omega_s L_{aa} - \frac{(j\omega_s M)^2}{\frac{R'_r}{s} + j\omega_s L'_{rr}} \tag{9.17b}$$

This single-phase impedance is used in phases 1, 2, 3 for the steady-state solution, provided there is only one winding on the field structure (rotor).

For the general case of m windings on the field structure, the calculation is slightly more complicated. First, let us assume that the armature currents are

$$i_1(t) = |I| \cos(\omega_s t + \alpha)$$
$$i_2(t) = |I| \cos(\omega_s t + \alpha - 120°)$$
$$i_3(t) = |I| \cos(\omega_s t + \alpha - 240°)$$

in balanced operation. Transformed to d, q, 0-quantities, the currents become

$$\begin{aligned} i_d(t) &= \frac{\sqrt{3}}{\sqrt{2}} |I| \sin(s\omega_s t + \alpha - \delta) \\ i_q(t) &= \frac{\sqrt{3}}{\sqrt{2}} |I| \cos(s\omega_s t + \alpha - \delta) \\ i_o(t) &= 0 \end{aligned} \tag{9.18a}$$

which can be represented as a phasor of slip frequency $s\omega_s$, projected onto the q, d-axes,

$$I_{qd} = \frac{\sqrt{3}}{\sqrt{2}} I_{phase}\, e^{-j\delta} \tag{9.18b}$$

with $I_{phase} = |I|e^{j\alpha}$ being the (peak) phasor current in the ac network solution reference frame, and with the understanding that

$$\begin{aligned} i_q(t) &= R_e\{I_{qd}\, e^{js\omega_s t}\} \\ i_d(t) &= Im\{I_{qd}\, e^{js\omega_s t}\} \end{aligned} \tag{9.18c}$$

All d, q-quantities vary with the slip frequency $s\omega_s$, and can therefore be represented as phasors in the same way as the armature currents.

To obtain the impedance, the rotor currents must first be expressed as a function of armature currents. Since all rotor voltages are zero, Eq. (9.2a) can be rewritten as

$$0 = -[R_r][i_r] - \frac{d}{dt}[\lambda_r] \tag{9.19}$$

with

$$[\lambda_r] = [L_{ra}]\, i_a + [L_{rr}][i_r] \tag{9.20}$$

from Eq. (9.3a) (subscript "r" for rotor or field structure quantities, and "a" for armature quantities). Since there is no saliency in three-phase induction machines, Eq. (9.19) and (9.20) are identical for the d- and q-axes, except that i_a is i_d in one case, and i_q in the other case. The submatrices $[L_{ra}]$ and $[L_{rr}]$ are obtained from the matrix of Eq. (9.3a) by deleting the first row; $[L_{ra}]$ is the first column and $[L_{rr}]$ the m x m-matrix of what is left. If the rotor windings are not shorted, but connected to an R-L network, then $[R_r]$ and $[L_{rr}]$ must be modified to include the resistances and inductances of this connected network (for connected networks with voltage or current sources see Section 9.5.8). Since $[i_r]$ and i_a can both be represented as phasors with Eq. (9.18), the flux in Eq. (9.20) is also a phasor which, after differentiation, becomes

$$\frac{d}{dt}[\Lambda_r] = js\omega_s [L_{ra}] I_{qd} + js\omega_s [L_{rr}][I_r] \tag{9.21}$$

Inserting this into Eq. (9.19) produces the equation which expresses the rotor currents as a function of the armature current phasor,

$$[I_r] = -\{[R_r] + js\omega_s [L_{rr}]\}^{-1} js\omega_s [L_{ra}] I_{qd} \tag{9.22}$$

To obtain the direct axis rotor currents as complex phasor quantities, use $\mathrm{Im}\{I_{q,d}\}$ on the right-hand side of Eq. (9.22), while the use of $\mathrm{Re}\{I_{qd}\}$ will produce the quadrature axis rotor currents.

The next step in the derivation of the impedance is the rewriting of the armature equations (9.1a) in terms of phasor quantities. Since

$$\frac{d}{dt}\Lambda_{qd} = js\omega_s \Lambda_{qd}$$

Eq. (9.1a) becomes

$$V_{qd} = -R_a I_{qd} - js\omega_s \Lambda_{qd} - j\omega\Lambda_{qd}$$

or with $s\omega_s = \omega_s - \omega$ from Eq. (9.16),

$$V_{qd} = -R_a I_{qd} - j\omega_s \Lambda_{qd} \tag{9.23}$$

With the flux from the first row of Eq. (9.3a)

$$\Lambda_{qd} = L_{aa} I_{qd} + [L_{ar}][I_r] \tag{9.24}$$

where $[L_{ar}] = [L_{ra}]^t$, and with Eq. (9.22), Eq. (9.23) finally becomes

$$V_{qd} = -\{(R_a + j\omega_s L_{aa}) - j\omega_s [L_{ar}]\{[R_r] + js\omega_s [L_{rr}]\}^{-1} js\omega_s [L_{ra}]\} I_{qd}$$

Therefore, the positive sequence impedance is

$$Z_{pos} = R_a + j\omega_s L_{aa} - j\omega_s [L_{ar}]\{[R_r] + js\omega_s [L_{rr}]\}^{-1} js\omega_s [L_{ra}] \tag{9.25}$$

If there is only one winding on the rotor, then it can easily be shown that the impedance of Eq. (9.17b) is identical with that of Eq. (9.25), by using the definitions of Eq. (9.17a).

To summarize: The three-phase induction machine is represented as three single-phase impedances Z_{pos} from Eq. (9.25) in the three phases 1, 2, 3. After the ac network solution of the complete network, the armature currents are initialized with Eq. (9.18b), and the rotor currents with Eq. (9.22). The calculation with Eq. (9.22) is done twice, with the imaginary part of I_{qd} to obtain the direct axis quantities, and with the real part of I_{qd} to obtain the quadrature axis quantities.

As mentioned before, the initialization works only properly for balanced cases at this time. If initialization for unbalanced cases is to be added some day, then the procedures of Section 8.4.2 and 8.4.3 for the synchronous machines should be directly applicable, because negative and zero sequence currents see the field winding as short-circuits. Therefore, there is no difference between synchronous and induction machines in the negative and zero sequence initialization.

9.5.6 Two-Phase Induction Machine

As already discussed in Section 9.5.2 for the two-phase synchronous machine, the equations for balanced operation of a two-phase machine are identical on the d, q-axes with those of the three-phase machine. The only difference is the missing factor $\sqrt{3}/\sqrt{2}$ in the conversion from phase quantities to d, q-quantities.

9.5.7 Single-Phase Induction Machine

The problem is essentially the same as discussed in Section 9.5.3 for the synchronous machine. Only positive sequence values are used now, and the second term in

$$i_d(t) = \frac{1}{2}|I|\sin(s\omega_s t + \alpha - \delta) - \frac{1}{2}|I|\sin((\omega_s + \omega)t + \alpha + \delta) \tag{9.26}$$

is presently ignored.

9.5.8 Doubly-Fed Induction Machine

If the rotor (field structure) windings are connected to an external network with ac voltage and/or current sources, then the EMTP will automatically assume that their frequency is equal to the specified slip frequency $s\omega$, and ignored the frequency values given for these sources.

Feeding the rotor windings from sources requires two modifications to the procedure of Section 9.5.5. In these modifications, it is assumed that the external network is represented by a Thevenin equivalent circuit, with voltage sources $[V_{Thev}]$ behind an impedance matrix $[Z_{Thev}]$ defined at slip frequency.

First, the rotor impedance matrix $[R_r] + js\omega_s[L_{rr}]$ must be modified to include the external impedances,

$$[Z_{rr}^{mod}] = [R_r] + js\omega_s[L_{rr}] + [Z_{Thev}] \tag{9.27}$$

This modification must be done twice, for the direct axis quantities and for the quadrature axis quantities. Since

$[Z_{Thev}]$ is in general different for the two axes, $[Z_{rr}^{mod}]$ is no longer the same on both axes.

Secondly, the left-hand side of Eq. (9.19) is no longer zero, but $[V_{Thev}]$. This will change Eq. (9.22) into

$$[I_r] = -[Z_{rr}^{mod}]^{-1}\{[V_{Thev}] + js\omega_s[L_{ra}]I_{qd}\} \tag{9.28}$$

Again, this calculation must be done twice. For the direct axis, use $Im\{I_{qd}\}$ and the direct axis values $[Z_{rr}^{mod}]$ and $[V_{Thev}]$, for the quadrature axis $Re\{I_{qd}\}$ and quadrature axis values $[Z_{rr}^{mod}]$ and $[V_{Thev}]$.

With these two modifications, the steady-state model of the induction machine is no longer a passive impedance Z_{pos}, but becomes a three-phase voltage source $[E_{source}]$ behind three single-phase impedance branches

$$Z_{pos}^{mod} = R_a + j\omega_s L_{aa} - j\omega_s[L_{ar}][Z_{rr}^{mod}]^{-1} js\omega_s[L_{ra}] \tag{9.29}$$

The voltage source is found by calculating the direct axis contribution,

$$E_d = j\omega_s[L_{ar}][Z_{rr-d}^{mod}]^{-1}[V_{Thev-d}] \tag{9.30a}$$

and the quadrature axis contribution,

$$E_q = j\omega_s[L_{ar}][Z_{rr-q}^{mod}]^{-1}[V_{Thev-q}] \tag{9.30b}$$

and then transforming to phase quantities,

$$\begin{aligned} E_{source-1} &= \frac{\sqrt{2}}{\sqrt{3}}e^{j\delta}(E_q + jE_d) \\ \textit{with} \quad E_{source-2} &= E_{source-1}\cdot e^{-j120°} \\ \textit{and} \quad E_{source-3} &= E_{source-1}\cdot e^{+j120°} \end{aligned} \tag{9.30c}$$

Once the ac steady-state solution of the complete network has been obtained, the d, q, 0-armature currents are initialized with Eq. (9.18b), while the rotor currents are initialized with Eq. (9.28).

9.6 Transient Solution with Compensation Method

For the transient solution with the compensation method, the machine differential equations (9.1) and (9.2) are first converted to difference equations with the trapezoidal rule of integration. Then Eq. (9.1) becomes

$$\begin{bmatrix} v_d(t) \\ v_q(t) \\ v_{0a}(t) \end{bmatrix} = -\begin{bmatrix} R_a & 0 & 0 \\ 0 & R_a & 0 \\ 0 & 0 & R_a \end{bmatrix}\begin{bmatrix} i_d(t) \\ i_q(t) \\ i_{0a}(t) \end{bmatrix} - \frac{2}{\Delta t}\begin{bmatrix} \lambda_d(t) \\ \lambda_q(t) \\ \lambda_{0a}(t) \end{bmatrix} + \begin{bmatrix} -\omega(t)\lambda_q(t) \\ +\omega(t)\lambda_d(t) \\ 0 \end{bmatrix} + \begin{bmatrix} hist_d \\ hist_q \\ hist_{0a} \end{bmatrix} \tag{9.31a}$$

with the history terms known from the preceding time step,

$$\begin{bmatrix} hist_d \\ hist_q \\ hist_{0a} \end{bmatrix} = -\begin{bmatrix} v_d(t-\Delta t) \\ v_q(t-\Delta t) \\ v_{0a}(t-\Delta t) \end{bmatrix} - \begin{bmatrix} R_a & 0 & 0 \\ 0 & R_a & 0 \\ 0 & 0 & R_a \end{bmatrix}\begin{bmatrix} i_d(t-\Delta t) \\ i_q(t-\Delta t) \\ i_{0a}(t-\Delta t) \end{bmatrix} + \frac{2}{\Delta t}\begin{bmatrix} \lambda_d(t-\Delta t) \\ \lambda_q(t-\Delta t) \\ \lambda_{0a}(t-\Delta t) \end{bmatrix} +$$

$$\begin{bmatrix} -\omega(t-\Delta t)\lambda_q(t-\Delta t) \\ +\omega(t-\Delta t)\lambda_d(t-\Delta t) \\ 0 \end{bmatrix} \tag{9.31b}$$

The field structure equations (9.2) on the direct axis become

$$\begin{bmatrix} v_{D1}(t) \\ \cdot \\ \cdot \\ \cdot \\ v_{Dm}(t) \end{bmatrix} = -\begin{bmatrix} R_{D1} & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & R_{Dm} \end{bmatrix}\begin{bmatrix} i_{D1}(t) \\ \cdot \\ \cdot \\ \cdot \\ i_{Dm}(t) \end{bmatrix} - \frac{2}{\Delta t}\begin{bmatrix} \lambda_{D1}(t) \\ \cdot \\ \cdot \\ \cdot \\ \lambda_{Dm}(t) \end{bmatrix} + \begin{bmatrix} hist_{D1} \\ \cdot \\ \cdot \\ \cdot \\ hist_{Dm} \end{bmatrix} \tag{9.32a}$$

with the known history terms

$$\begin{bmatrix} hist_{D1} \\ \cdot \\ \cdot \\ \cdot \\ hist_{Dm} \end{bmatrix} = -\begin{bmatrix} v_{D1}(t-\Delta t) \\ \cdot \\ \cdot \\ \cdot \\ v_{Dm}(t-\Delta t) \end{bmatrix} - \begin{bmatrix} R_{D1} & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & R_{Dm} \end{bmatrix}\begin{bmatrix} i_{D1}(t-\Delta t) \\ \cdot \\ \cdot \\ \cdot \\ i_{Dm}(t-\Delta t) \end{bmatrix} + \frac{2}{\Delta t}\begin{bmatrix} \lambda_{D1}(t-\Delta t) \\ \cdot \\ \cdot \\ \cdot \\ \lambda_{Dm}(t-\Delta t) \end{bmatrix} \tag{9.32b}$$

On the quadrature axis, they are identical in form to Eq. (9.32), except that subscripts D1,…Dm must be replaced by Q1,…Qn. Finally for Eq. (9.2c),

$$v_{of}(t) = -R_{of}\, i_{of}(t) - \frac{2}{\Delta t}\lambda_{of} + hist_{of} \tag{9.33a}$$

with

$$hist_{of} = -v_{of}(t-\Delta t) - R_{of}\, i_{of}(t-\Delta t) - R_{of}\, i_o f(t-\Delta t) + \frac{2}{\Delta t}\lambda_{of}(t-\Delta t) \qquad (9.33b)$$

As explained in Section 12.1.2, the network connected to the armature side of the machine can be represented by the instantaneous Thevenin equivalent circuit equation

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} v_{1-0} \\ v_{2-0} \\ v_{3-0} \end{bmatrix} + \begin{bmatrix} & & \\ & R_{equiv} & \\ & & \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} \qquad (9.34)$$

with a sign reversal for the current compared to Section 12.1.2, to change from a load to source convention. Similarly, if external networks are connected to the field structure windings, they will also be represented by Thevenin equivalent circuits with equations of the form

$$\begin{bmatrix} v_{D1}(t) \\ \cdot \\ \cdot \\ \cdot \\ v_{Dm}(t) \end{bmatrix} = \begin{bmatrix} v_{D1-0} \\ \cdot \\ \cdot \\ \cdot \\ v_{Dm-0} \end{bmatrix} + \begin{bmatrix} & & \\ & R_{D-equiv} & \\ & & \end{bmatrix} \begin{bmatrix} i_{D1}(t) \\ \cdot \\ \cdot \\ \cdot \\ i_{Dm}(t) \end{bmatrix} \qquad (9.35a)$$

$$\begin{bmatrix} v_{Q1}(t) \\ \cdot \\ \cdot \\ \cdot \\ v_{Qn}(t) \end{bmatrix} = \begin{bmatrix} v_{Q1-0} \\ \cdot \\ \cdot \\ \cdot \\ v_{Qn-0} \end{bmatrix} + \begin{bmatrix} & & \\ & R_{Q-equiv} & \\ & & \end{bmatrix} \begin{bmatrix} i_{Q1}(t) \\ \cdot \\ \cdot \\ \cdot \\ i_{Qn}(t) \end{bmatrix} \qquad (9.35b)$$

and

$$v_{of}(t) = v_{of-o} + R_{of-equiv}\, i_{of}(t) \qquad (9.35c)$$

The external network connected to the first three field structure windings is represented by a three-phase Thevenin equivalent circuit (Section 12.1.2.3), whereas the external networks connected to the rest of the field structure windings are represented by single-phase Thevenin equivalent circuits (Section 12.1.2.1). This limitation results from the fact that the BPA EMTP could handle M-phase Thevenin equivalent circuits only for M $\leq$ 3 at the time the Universal Machine was first implemented. In practice, this limitation should not cause any problems because the field structure windings are usually connected to separate external networks. An exception is the three-phase wound rotor of induction machines, which is the reason why a three-phase equivalent circuit was chosen for the first three rotor windings.

The solution of the machine equations is then roughly as follows:

(1) Solve the complete network without the universal machines. Extract from this solution the Thevenin equivalent open-circuit voltages of Eq. (9.34) and (9.35), as well as the open-circuit voltages of the network which represents and mechanical system.

(2) Predict the rotor speed ω(t) with linear extrapolation.

(3) Transform Eq. (9.34) from phase to d, q, 0-quantities with Eq. (9.7) if the armature windings are ac windings,

$$\begin{bmatrix} v_d \\ v_q \\ v_{oa} \end{bmatrix} = \begin{bmatrix} v_{d-o} \\ v_{q-o} \\ v_{oa-o} \end{bmatrix} + \begin{bmatrix} & & \\ & R_{phase-equiv} & \\ & & \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_{oa} \end{bmatrix} \tag{9.36a}$$

where

$$\begin{bmatrix} v_{d-o} \\ v_{q-o} \\ v_{oa-o} \end{bmatrix} = [T]^{-1} \begin{bmatrix} v_{1-o} \\ v_{2-o} \\ v_{3-o} \end{bmatrix} \quad and \quad [R_{phase-equiv}] = [T]^{-1} [R_{equiv}] [T] \tag{9.36b}$$

For dc armature windings, the Thevenin equivalent circuit is already in the form of Eq. (9.36a) without transformation.

(4) Substitute Eq. (9.36a) into Eq. (9.31a), and substitute Eq. (9.35) into Eq. (9.32). This eliminates the voltages as variables. Then solve the resulting linear equations for the m + n + 4 currents by Gauss elimination, after the fluxes are first replaced by linear functions of currents with Eq. (9.3). Using the star circuit of Fig. 9.1 instead of the more general inductance matrix of Eq. (9.3) simplifies this solution process somewhat.

(5) Calculate the electromagnetic torque on the electrical side,

$$T_{el}(t) = i_q(t)\,\lambda_d(t) - i_d(t)\,\lambda_q(t) \tag{9.37}$$

and convert it to $T_{mech}(t)$ on the mechanical side with Eq. (8.25) if the mechanical system is not modelled as a one pole-pair machine. Use $T_{mech}(t)$ as a current source in the Thevenin equivalent circuit which represents the mechanical system and solve it to obtain the speed (as an equivalent voltage). Up to 3 universal machines can share the same mechanical system, because the EMTP uses an M-phase compensation method for $M \leq 3$ (see Section 12.1.2.3).

(6) If the speed calculated in (5) differs too much from the predicted speed, then return to step (3). Otherwise:

(7) Update the history terms of Eq. (9.31b), (9.32b) for d- and q-axes, and (9.33b) for the next time step.

(8) Transform the armature currents from d, q, 0-quantities to phase quantities with Eq. (9.7) (only if the windings are ac windings).

(9) Find the final solution of the complete network by super-imposing the effects of the armature currents, of

the field-structure currents (if they have externally connected networks) and of the current representing the electromagnetic torque in the network for the mechanical system, with Eq. (12.8) of Section 12.

(10) Proceed to the next time step.

Since the variables of the mechanical system usually change much slower than the electrical variables, because of the relatively large moment of inertia of practical machines, the prediction of the speed is fairly good. As a consequence, the number of iterations typically lies between 1 and 3.

Interfacing the solution of the machine equations with the solution of the electric network through compensation offers the advantage that the iterations are confined to the machine equations only. Furthermore, if a small tolerance is used for checking the accuracy of the speed, the solution is practically free of any interfacing error.

The only limitation of the compensation method is the fact that the universal machines must be separated from each other, and from other compensation-based nonlinear elements, through distributed-parameter lines with travel time. Stub lines can be used to introduce such separations artificially, but such stub lines create their own problems. Because of this limitation, a second solution option has been developed, as described in the next section.

9.7 Transient Solution with Armature Flux Prediction

In the transient solution of the synchronous machine of Section 8, essentially voltage sources behind resistances R_a and average subtransient inductances $(L_d'' + L_q'')/2$ are used, with the trapezoidal rule applied to the inductance part. The voltage sources contain predicted currents and the predicted speed.

The prediction-based interface option for the universal machine also uses voltage sources with elements of prediction in them, but just behind resistances R_a, with no inductance part (Fig. 9.6). If we think of R_a as belonging to the electric network and not to the machine, then Eq.

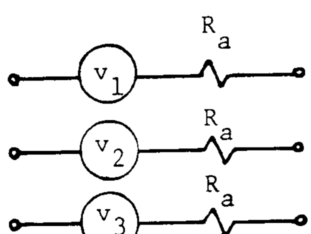


Fig. 9.6 - Thevenin equivalent circuit for universal machine

(9.1) becomes a simple relationship between armature voltages and fluxes,

$$\begin{bmatrix} v_d \\ v_q \\ v_{oa} \end{bmatrix} = -\frac{d}{dt}\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_{oa} \end{bmatrix} + \begin{bmatrix} -\omega\lambda_q \\ +\omega\lambda_d \\ 0 \end{bmatrix} \tag{9.38}$$

The fluxes always change smoothly, in contrast to the voltages which can suddenly jump in case of short-circuits. Therefore, the fluxes are chosen as variables suitable for prediction. Furthermore, the fluxes λ_d, λ_q of induction machines vary sinusoidally with slip frequency during steady-state operation, whereas the fluxes seen from a synchronously rotating reference frame (rotating at the supply frequency ω_s) would remain constant. Because of this, the fluxes seen from a synchronously rotating reference frame are predicted, rather than λ_d, λ_q. This requires a transformation of Eq. (9.38) from the d, q-axes to the synchronously rotating reference frame [140]. Alternatively, one can forget about the original transformation from phase quantities to the d, q-axes altogether, and transform the phase quantities directly to the ds, qs-axes of the synchronously rotating reference frame. That means that $d\beta/dt = \omega$ must be replaced by ω_s, which leads to

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{oa} \end{bmatrix} = -\frac{d}{dt}\begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \\ \lambda_{oa} \end{bmatrix} + \begin{bmatrix} -\omega_s\lambda_{qs} \\ +\omega_s\lambda_{ds} \\ 0 \end{bmatrix} \tag{9.39}$$

The only difference with Eq. (9.38) is the replacement of rotor speed ω by the ac supply frequency ω_s. This simple change works only for the voltage equations; for the flux-current relationships the synchronously rotating reference frame cannot be used because that would make the inductances time-dependent rather than constant.

The fluxes λ_{ds}, λ_{qs}, λ_{oa} on the synchronously rotating axes are now predicted linearly,

$$\begin{bmatrix} -\lambda_{ds-pred} \\ \lambda_{qs-pred} \\ \lambda_{oa-pred} \end{bmatrix} = 2\begin{bmatrix} \lambda_{ds}(t-\Delta t) \\ \lambda_{qs}(t-\Delta t) \\ \lambda_{oa}(t-\Delta t) \end{bmatrix} - \begin{bmatrix} \lambda_{ds}(t-2\Delta t) \\ \lambda_{qs}(t-2\Delta t) \\ \lambda_{oa}(t-2\Delta t) \end{bmatrix} \tag{9.40}$$

and the backward Euler method (see Appendix I.9) is then applied to Eq. (9.39),

$$\begin{bmatrix} v_{ds}(t) \\ v_{qs}(t) \\ v_{oa}(t) \end{bmatrix} = -\frac{1}{\Delta t}\begin{bmatrix} \lambda_{ds-pred} - \lambda_{ds}(t-\Delta t) \\ \lambda_{qs-pred} - \lambda_{qs}(t-\Delta t) \\ \lambda_{oa-pred} - \lambda_{oa}(t-\Delta t) \end{bmatrix} + \begin{bmatrix} -\omega_s\lambda_{qs-pred} \\ +\omega_s\lambda_{ds-pred} \\ 0 \end{bmatrix} \tag{9.41}$$

With all quantities on the right-hand side known (either from the preceding time step or from prediction), the terminal voltages are now known, too, and can be transformed back to phase quantities with

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} \cos(\omega_s t) & \sin(\omega_s t) & 0 \\ -\sin(\omega_s t) & \cos(\omega_s t) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} v_{ds}(t) \\ v_{qs}(t) \\ v_{oa}(t) \end{bmatrix} \tag{9.42}$$

The representation of the universal machine as three voltage sources $v_1(t)$, $v_2(t)$, $v_3(t)$ behind resistances R_a

in the complete electric network is only used on the armature side, whereas compensation-based interfaces are still maintained for the field structure windings and for the mechanical system. With this in mind, the solution process works roughly as follows:

(1) With the universal machine represented as voltage sources behind R_a (implemented as current sources in parallel with R_a in the EMTP), solve the complete electric network. Extract from this solution the Thevenin equivalent open-circuit voltages of Eq. (9.35) if there are any external networks connected to the field structure windings (see Section 9.6 for details about three-phase compensation on the first three windings, and single-phase compensation on the rest). Extract as well the open-circuit voltages of the network which represents the mechanical system.

(2) Execute steps (2) to (9) of the compensation-based procedure described in the preceding Section 9.6, except that the armature currents i_1, i_2, i_3 (and i_d, i_q, i_{oa} after transformation with $[T^{-1}]$) are now known from step (1) and used directly in place of the Thevenin equations (9.36) for the armature part. The calculations for the other parts remain unchanged.

(3) Rotate the armature fluxes λ_d, λ_q, λ_{oa} from the d, q-axes to the synchronously rotating ds, qs-axes

$$\begin{bmatrix} \lambda_{ds} \\ \lambda_{qs} \\ \lambda_{oa} \end{bmatrix} = \begin{bmatrix} \cos(\omega_s t - \beta) & -\sin(\omega_s t - \beta) & 0 \\ \sin(\omega_s t - \beta) & \cos(\omega_s t - \beta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_{oa} \end{bmatrix} \tag{9.43}$$

and use them to predict the voltage sources for the next time step with Eq. (9.40) to (9.42). Note that no predictions for the speed and angle are needed here.

(4) Proceed to (1) to find the solution at the next time step.

Experience has shown [140] that this interfacing option is as accurate as the compensation-based interface of Section 9.6. It also requires less computation time. Its numerical stability can be partly attributed to the backward Euler method in Eq. (9.41). As shown in Appendix I.9, the backward Euler method is identical to the trapezoidal rule of integration with critical damping, and is therefore absolutely numerically stable. However, Eq. (9.41) involves predictions as well, and the comparison is therefore not completely correct.

9.8 Saturation

Saturation effects are only represented for the main flux (M_d in Fig. 9.1), except for the special induction machine model of Ontario Hydro, which includes saturation effects in the leakage fluxes as well.

The saturation curve of the universal machine is approximated as two piecewise linear segments for the d-axis, the q-axis, or for both (Fig. 9.7). By using the star circuit of Fig. 9.1,

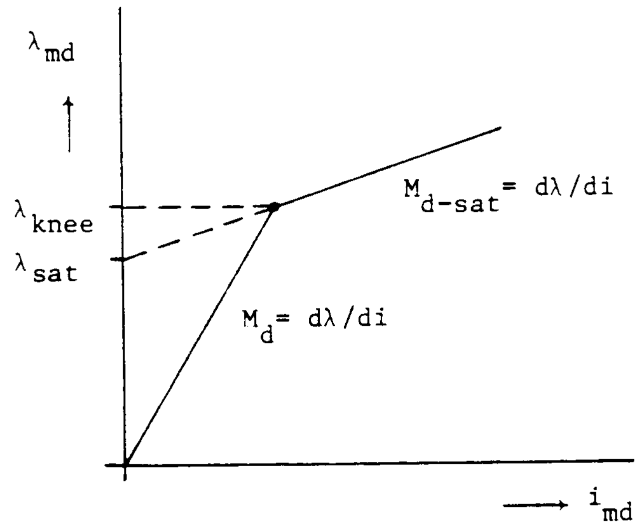


Fig. 9.7 - Piecewise linear inductance

the piecewise linear representation can easily be implemented. Whenever the flux lies above the knee-point value λ_{knee}, the relationship of Eq. (9.4b) in the form of

$$\lambda_{md} = M_d i_{md} \tag{9.44a}$$

is simply replaced by

$$\lambda_{md} = \lambda_{sat} + M_{d-sat} i_{md} \tag{9.44b}$$

on the direct axis, and analogous on the quadrature axis.

Residual flux can be represented as well. In that case, the characteristic of Fig. 9.8 is used. If the absolute value of the flux is less than $\lambda_{residual}$, then the M_d-branch is open-circuited,

$$i_{md} = 0 \qquad if \ \ |\lambda_{md}| < \lambda_{residual} \tag{9.45a}$$

$$\lambda_{md} = \lambda_{residual} + M_d i_{md} \qquad if \ \ \lambda_{residual} \le |\lambda_{md}| \le \lambda_{knee} \tag{9.45b}$$

and

$$\lambda_{md} = \lambda_{sat} + M_{d-sat} i_{md} \qquad if \ \ |\lambda_{md}| > \lambda_{knee} \tag{9.45c}$$

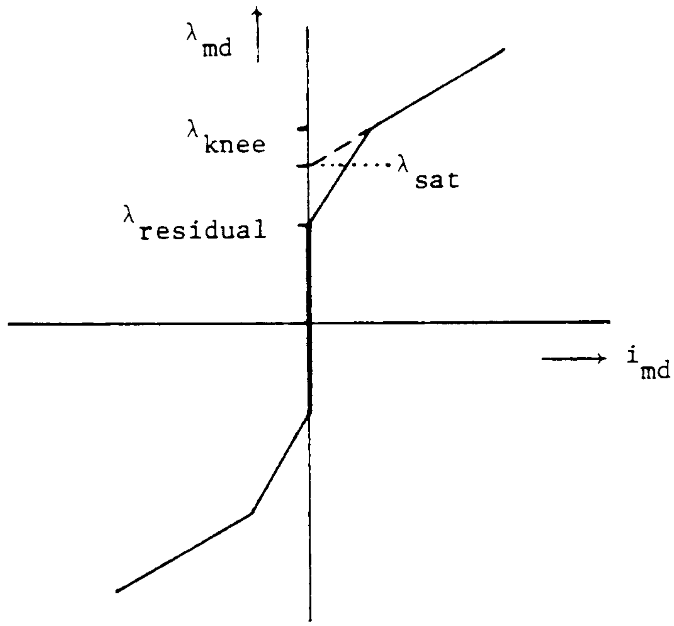


Fig. 9.8 - Residual flux

The decoupled approach of d- and q-axis saturation works reasonably well for salient-pole synchronous machines and for dc machines with a definite field coil in one axis. However, when both the armature and field structures are round with no pronounced saliency, as in most induction machines and in round-rotor synchronous machines, then this decoupled approach leads to unacceptable results. Therefore, a "total saturation" option is available, which uses a solution method very similar to that discussed in Section 8.6.